Obsolescence vs. Deterioration with Embodied Technological Change

Frank C. Wykoff*

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fwykoff@pomona.edu

Department of Economics
Pomona College
425 N. College Avenue
Claremont, CA 91711
(909) 621 – 8303 (office)
(909) 626 – 7604 (home)

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Abstract

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Few issues in economics have been as contentious longer than the role of capital in production and distribution. I apply here the neoclassical model of Solow, Hall, Jorgenson and others in order to address a central aspect of today's debate. When technological change is embodied in new capital, should statistical measures of capital distinguish between obsolescence of old capital and deterioration? This question underlies concerns about how to construct capital aggregates.

I make the case that, from the neoclassical perspective, no distinction should be made between obsolescence and deterioration in constructing capital aggregate measures. However, empirical evidence on this question is unavailable. Core to construction of coherent aggregates is the duality equilibrium condition that the ratios of user-costs equal relative in-use productive efficiencies. I also argue that ordinarily one should not adjust capital-services quantity measures for variations in utilization. I make two arguments. One, a coherent measure of the capital input requires aggregation in comparable units. This requires that adjustments be made both for deterioration and obsolescence. Two, rational decisions by capital users are independent of the obsolescence-deterioration distinction. The first half of the paper deals directly with capital aggregation under embodied technical change. The second deals with behavior, in response to new technological innovations, of a representative, rational user of old capital.

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Introduction

Several stubborn questions continue to plague statistical agencies that produce capital accounts. In this paper I ask how they should measure the capital stock when technological change is embodied in new vintages of capital. Technological change embodied in new capital renders existing capital relatively obsolete. It does not, however, lower the marginal product of old capital. In measuring capital stocks, does one adjust for obsolescence and deterioration in the same way? If so, how does one do so? If not, then is different information needed by statisticians to make the distinction? If one wishes to distinguish the stock of capital from the quantity of capital services used in production in the current period, then what role does utilization play in making these distinctions?

These questions can be addressed from two different but interrelated perspectives. One deals directly with the measurement of capital aggregates. Four economic measures of capital are of interest. (1) The stock of capital K_t is a quantity measure of the productive capacity of physical capital in the relevant economic entity (country, industry, region, firm) in time period-t. (2) The price of capital P_t is the cost of acquiring ownership rights to one unit of the relevant capital stock in time t. This is often referred to as the acquisition price of capital to allow for the possibility that this may be a market price or a shadow price. These first two terms refer to stocks.

The distinctive feature of capital is that it is durable and provides a flow of services over time. Economists are interested in price and quantity flow concepts that have bearing on production decisions and processes. The second two measures refer to flow concepts. (3) The flow of capital services k_t is the quantity of services provided by the stock in producing output in period t. (4) The user-cost of capital c_t is the cost the producer incurs for these services. This user-cost could be a market price, for example the cost of renting the capital good to produce output, or, if the capital is used by its owner then the user-cost is a shadow cost that the owner incurs to derive the services. Clearly, the four concepts, K, P, k, and c, are interrelated. The cost of acquiring capital in period-t is related to the future sequence of user-costs that one expects to incur from the acquired asset, and the capacity-quantity of capital is related to the flows of capital services to be provided in the future. Jorgenson in (1974) presents a model, based on neoclassical capital theory developed by many economists, most notably Solow (1957, 1970), Hall (1968), and Hall and Jorgenson (1963), to specify these interdependencies. Hulten in (1990) integrates the Solow—Jorgenson models raising different implications of various concepts of capital and technological change from other strands of literature and illustrates potential problems encountered when trying to construct these aggregates.

Here I focus on one question: What are the consequences for constructing capital aggregates of technological change embodied in new assets? New, superior vintages render old assets obsolete. At the same time, old assets deteriorate. How do these two forces, obsolescence and deterioration, influence how one aggregates across vintages? This specific question is of immediate policy importance. International statistical agencies, charged with measuring capital, productivity and other aspects of real aggregate economic activity, would like to design a uniform system of capital accounts to facilitate comparisons of performance among countries and regions.

The second perspective on the consequences of embodied technological change deals with how economic agents respond to changes in price resulting from obsolescence vs. changes resulting from deterioration. If a new "better" asset model enters the market, how do producers who are using old models respond? If an asset becomes less productive relative to a new asset, does the producer respond differently than he would in response to the introduction of "better" new models? Answers to the behavioral questions have implications for measurement if one wants measurement of economic aggregates to reflect economic behavior.

Even though the issues raised here are not new and existing literature resolves some of them, disputes about physical capital persist even after years of conflict that at times flared up into contentious debates. Today, interest among economists is relegated mainly to the measurement community margin. Historical disputes about capital have been ideological, theoretical and statistical. To avoid confusion in a field long rife with conflict, I first put this paper in context with respect to relatively recent historical treatment of physical capital. I develop the neoclassical model of capital based primarily on work by Hall, Jorgenson and Hulten building on the Solow analysis of capital in his growth models.

Next, I clarify my use of pertinent terms like depreciation, obsolescence, deterioration, age, and vintage. These terms have been used by different authors to mean different things. Confusion can be avoided if we are clear about what we mean by these terms. I will use these terms as used in the economics literature on depreciation from Hotelling (1925) to Solow (1957, 1971), Hall and Jorgenson (1967), Hall (1968, 1971), Jorgenson (1974), Feldstein and Rothschild (1974), and Hulten (1990) as well as from Wykoff (1970, 2003), Oliner (1993, 1996), Hulten and Wykoff (1975, 1979, 1981, 1996) and Triplett (1996).[1] These definitions are also consistent with definitions by the U.S. Internal Revenue Service for tax purposes.

I then examine embodied technological change in the neoclassical model that I shall refer to as the Jorgenson model of capital (or the J-model).[2] Fourth, I suggest several strategies for measuring the quantity of capital services when technological change is embodied in new capital. I will show the assumptions that underlie different measures as well as conceptual implications of different measures. This paves the way for analysis of producer behavior in response to changes in capital costs resulting from obsolescence compared to

[1] Diewert (2004), Dunn, Domes, Oliner and Sichel (2004) also grapple with the issue of terminology. Diewert reviews some very interesting arcane definitions and builds on some of these terms. He also uses some of these terms differently; and he notes that different uses of these words appear in other literatures.
[2] As noted above, Jorgenson alone did not originate this entire model. Along with Hall and Hulten, he built on the neoclassical model and especially the seminal work of Solow. Many others, including Hicks and Leontief, also contributed to this subject, but I do not intend a formal literature review here. I use the term "Jorgenson" model, because his name is most closely associated with the model I shall be analyzing.

deterioration. This analysis feeds back on how obsolescence and deterioration are treated in the capital accounts. It also leads to some observations about the role of utilization of capital. Finally, I sum up and suggest future lines of research.

Overview of the capital controversy

Few issues in economics have been contentious longer than the role of capital. Karl Marx's (1867) Das Kapital still resonates with those who find capitalism objectionable and aggravates followers of Adam Smith. The issue's most famous flare up among academic economists in the West was the famous Two Cambridge's debate in the 1950s. The principal protagonists, Joan Robinson for Cambridge, England and Robert Solow for Cambridge, Massachusetts, fired salvos across the Atlantic.[3] Robinson argued that marginal analysis of capital is circular, vacuous and misleading. Solow defended marginal analysis of capital as coherent, substantive and useful. The conflict was energized in part by ideological differences among Marxists and capitalists.

A less exotic but important question in the two-Cambridge's debate was: How does one measure capital? Robinson argued that one could compile a list of capital inputs, but aggregation was not sensible. Solow endorsed a neoclassical aggregation procedure based on marginal analysis. Not surprisingly, little was resolved. Eventually, each side tired of the argument, and, as Keynesian economics turned focus away from stocks toward flows, researchers on both sides simply went their own ways.

Disagreement on capital stock measurement issues continued to simmer beneath the surface. It flared up in the 1970s when Dale Jorgenson and Zvi Griliches questioned U.S. Statistical Agencies on their measures of the capital input in the national income and product accounts. The agencies were defended by Edward Denison.[4] As before, the combatants tired with neither side convincing the other. The Jorgenson model now plays a

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[3] Several key articles by Robinson and by Solow appear in the 1971 book of readings on growth edited by Harcourt and Laing.

[4] The debate is in the May 1972 Survey of Current Business, US Commerce Department. See Jorgenson, et al. (1974). Jorgenson and Griliches employed the neoclassical framework to be used in this paper.

major role in design of capital accounts by U.S. statistical agencies, and possibly others, so one would think the issues have been resolved.

Still, a low-grade cold war has been going on within the U.S. statistical agencies and among American academics over closely related issues. At the international level, even less agreement exists. One bone of contention has been whether statistical agencies should measure gross capital stocks or net capital stocks? A related question: Is there one correct capital measure or do different uses of data require different measures? U.S. statistical agencies have settled on producing both gross and net capital stock estimates. Hulten (1990) criticizes this compromise arguing that gross measures employed by the agencies are not sensibly grounded in economic analysis.

Today, even though capital measurement has become rather esoteric, concern with productivity growth makes capital measurement important to public policy. I focus here on a central question in the debate. When technological change in embodied in new capital, should statistical measures of the capital stock distinguish between obsolescence of old capital and deterioration? I suspect that this question underlies current disputes among academic and government economists involving the measurement of the capital input.

Jorgenson on economic depreciation

To model differences in assets' productivity as they age, Jorgenson (1974) developed the efficiency function. At any point in time, the efficiency function, φ_s, is the ratio of the in-use productive efficiency of age-s assets relative to new assets. It satisfies the following conditions:

(1) $\varphi_0 = 1$ and $\varphi_{s+1} - \varphi_s \geq 0 \forall s$

Jorgenson defined the mortality function, m_s, as the decline in efficiency φ with age-s:

(2) $m_s = \varphi_{s+1} - \varphi_s \forall s = 0,1,2,...$

Mortality is the decline in productive efficiency in the asset cohort as it ages, given time.[5] Because efficiency is stationary this is also the rate of decline in productive efficiency as an asset cohort works through its life. If the cohort has a finite maximum life S, then the s-sequence terminates at S. Jorgenson illustrated the efficiency and mortality sequences with several familiar functions: one-hoss-shay, linear, and geometric. Harper later proposed the hyperbolic function in order to allow the efficiency pattern to decline slower than straight line.

In the most tractable case, mortality occurs at a constant geometric rate, δ. In this case,

$$(3) \qquad \varphi_s = (1-\delta)^s \; ; \; m_s = -\delta(1-\delta)^s = -\delta\varphi_s$$

The rate of decline in efficiency of an in-use asset is δ.

Jorgenson applies a "duality condition" to in-use assets by equating the ratio of the user-costs of an age-s asset, c_s, to a new one, to the efficiency function:

$$(4) \quad (c_s / c_0) = \varphi_s$$

Duality results from cost minimization; so, in economic terminology, the efficiency function, φ_s, is the age-sequence of marginal rates of substitution to a new asset or the ratio of marginal products of age-s assets to the marginal product of a new asset:

$$(5) \qquad \varphi_s \equiv MRS_s \equiv MP_s / MP_0 .$$

The duality equilibrium condition is the core of the Jorgenson model, because it connects concepts on the price side of capital, relative user-costs, to the quantity side, relative marginal products.[6] Using duality Jorgenson links physical loss of in-use efficiency φ_s to the decline in the service price, user-cost of the asset as it ages, $(\partial c/\partial s)_{\Delta t=0}$.

The asset price p and the user-cost c are linked through the fundamental price equation of capital theory.

[5] Jorgenson does not refer explicitly to cohorts. I discuss the idea of cohorts instead of individual assets below.

[6] Formally, Hall refers to the price side as the dual to the quantity side of the neoclassical model. The primal defines the quantitative choices made by producers selecting the input mix to produce output and the dual is the equivalent condition on the price side. Duality indicates that the two sides have to be coherent because duality is an equilibrium condition linking relative costs to relative marginal products.

(6) $$p_{0,t} = \sum_{s=0}^{S} \frac{c_{s,t+s}}{(1+r)^s}$$

The acquisition price of a new asset in period-t equals the present discounted value of the future stream of user-costs.[7] If the efficiency function, φ_s, is stationary, so that $\varphi_{s,t} = \varphi_s$ for all t, then we can use duality to replace $c_{s,t+s}$ with $c_{0,t} \times \varphi_s$ in the price equation.

(7) $$p_{0,t} = c_{0,t} \sum_{s=0}^{S} \frac{\varphi_s}{(1+r)^s}$$

Duality has linked quantities to prices because the efficiency function enters both quantity and price measures. This creates the cohesion between prices and quantities imposed by the model.

Economic depreciation is the decline in asset price with age holding time constant.

(8) $$D_{s,t} = p_{s+1,t} - p_{s,t} = \sum_x \frac{c_{s+1+x}}{(1+r)^x} - \sum_x \frac{c_{s+x}}{(1+r)^x}$$

Employing duality and the definition of mortality we can write depreciation as:

(9) $$D_{s,t} = c_{0,t} \sum_x \frac{m_{s+x}}{(1+r)^x}$$

The mortality function while defined on the primal to represent decline in productive efficiency of physical capital also enters the dual via the depreciation equation. Economic depreciation is equal to the present value of the mortality function applied to future flows of user costs. In the geometric case, economic depreciation is:

(10) $$D_{s,t} = -\delta \sum_x \frac{(1-\delta)^{s+x}}{(1+r)^x} = -\delta p_{s,t} ; \frac{D_{s,t}}{p_{s,t}} = -\delta$$

The rate of economic depreciation δ is the same as the rate of decline of in-use productive efficiency. This result is true for the constant rate of depreciation illustration, but not the one hoss shay or the straight line examples.

[7] Like equation (4), this equation expresses an equilibrium condition. If one assumes perfect foresight, then equation (6) amounts to assuming that, in fully operating competitive purchase and rental markets, the purchase price equals the present value of the future sequence of rental prices. Dropping perfect foresight would require development of an expectations generating model for both c and r, and of course both r and expected r could vary by date. We abstract from these issues here.

Several important assumptions are embedded in the Jorgenson model. These assumptions, while strong, make the model tractable, coherent, and econometrically viable. First, the loss of productive efficiency is assumed to depend on age. In reality, physical decay probably reflects intensity of use. Perhaps this is not a major problem for the model. The unit of time need not be age in the sense of calendar time but hours of work completed. The model is the same; just the φ-function depends on a different metric than age. The J-model also assumes perfect foresight. Without perfect foresight, an expectations generating model may alter the interpretation of the equation linking asset prices to user costs, but they will be linked.

A third assumption is that the efficiency function is stationary: φ_s for all-s is independent of date and time. This assumption greatly simplifies the model but is inessential to many of the results. We could replace stationary duality with:

$$(11) \qquad (c_{s,t} / c_{0,t}) = \varphi_{s,t} \forall s .$$

Equation (11) allows $\varphi_{s,t}$, $\varphi_{s,t-1}$, and $\varphi_{s,t+1}$ to differ. Of course,

$$(12) \qquad \varphi_{s,t} \equiv MRS_{s,t} \equiv MP_{s,t} / MP_{0,t}$$

A practical difficulty with using this version of the model to construct capital stocks is that one needs to identify assets by vintage and not simply by age.[8] Statistical agencies would have to compile vintage accounts, a costly endeavor because vintage accounting requires continual updating of the φ-sequences.

A fourth assumption embedded in the efficiency function is that efficiency is built right into the asset. This strong assumption appears to be inconsistent with producers making decisions about frequency of repairs, replacement, scrappage, retrofitting, and so on that producers can surely make.[9] Why does one need this assumption and is it worth the cost in realism? As we noted earlier, the unit of efficiency need not be age but could be hours

[8] Solow (1970) develops a vintage-capital model to allow for this situation. I, like Hall, Jorgenson, Hulten and others, rely heavily on it in what follows.
[9] Feldstein and Rothschild (1974) present many ways in which efficiency can be endogenously influenced while assets remain in use. Of course, a simple model is a good model if it holds up empirically, so reality is not the last word in model assessment.

or some gauge of intensity of use, although such a re-interpretation creates conceptual issues of its own. It also requires significantly different and harder to get data, but in principle would allow the producer to choose intensity of asset use. The Jorgenson model is an econometric model devised to use available data.

Another implication of built-in efficiencies, however, is unavoidable if we wish to construct a capital aggregate. The marginal rates of substitution between age-s and new assets are independent of the marginal products of other inputs. This is the same assumption required to construct a capital aggregate for use in a production function. When can the production function, Q = F(K1, K2, L) be written as Q = F [K(K1, K2), L]; i.e., when is the capital aggregator function, K(K1, K2), independent of the labor input? The answer is that the marginal rate of substitution that determines the weights for K1 and K2 must be independent of the marginal product of the labor input. This assumption is also satisfied by the non-stationary efficiency function.

According to the Jorgenson model, at equilibrium the ratio of the user-costs between age-s and new assets equals the marginal rate of technical substitution between age-s and new assets. Assuming φ is inherent in the asset is equivalent to the assumption needed to obtain a capital aggregate. Of course, this assumption may be wrong, but then a coherent measure of capital aggregation independent of other aspects of the production process does not exist.[10] In this case, measurement is more complex, and appropriate uses of measures in productivity research or policy analysis can be more difficult, because some important summary statistics are no longer reliable.

Economic depreciation with embodied technological change

I define "economic depreciation" here to deal with cohorts of assts rather than individual units. Let **economic depreciation** be the decline in price of one (average) unit of an original cohort of assts with age given date. The rate of economic depreciation is the rate

[10] An important practical application of the model where this assumption may be problematic is the construction of computer stocks over time when technological change is occurring in both hardware and software. See Diewert (2004) and Wykoff (2004).

at which this price declines.[11] The term “depreciation” is often used in other ways; for example, it may refer to the amount of money set aside to prepare for replacement as an asset wears out, amortization. Depreciation of a stock of capital is functionally related to the price concept depreciation and that relationship will be developed below. To accountants depreciation refers to a set of accounting rules for writing down an investment for bookkeeping or tax purposes. But these rules may not accurately reflect economic depreciation. In the J-model depreciation refers only to the decline in price resulting from aging but not the passage of time. The effect on price of the passage of time, holding age constant, is **revaluation**.[12]

Economic depreciation has two components, because as assets age, given time, two distinct types of effects influence price: vintage effects and aging effects. The relationship between age, time and vintage, as pointed out by Hall (1968), imposes an important identification problem on price analysis. Vintage is defined as $v = t - s$ where t is the time period, and s is the cohort’s age. If one takes the partial derivate of price with respect to age holding time constant, then vintage is changing as well: an older asset at a point in time is an earlier vintage. Similarly, the partial derivative of price with respect to time holding age constant advances vintage, because an asset the same age in the next period is a newer vintage.

Since this concept can be elusive, consider the following thought experiment. Suppose we have a room for each year that asset prices are observed and that a label on the door states the year, say 2003. All the prices in room-2003 are prices observed of assets in the year 2003. Within each room we have boxes, one box containing the prices observed for assets of each age. We have a new box, a one-year-old box, and so on. We go to the 2005 room and open the box labeled age-4. Now, suppose someone says lets create a filing system in box-4 for each vintage in order to further distinguish assets by vintage. You start to pull out prices in order to separate them into vintages. Since you are in room-2005

[11] Mathematically economic depreciation at a point in time is the partial derivative of price with respect to age, holding time constant: $\partial p(s,t)/\partial s \,\big|_{\partial t=0}$ where p(s,t) is the price of an asset age-s at time-t.

[12] Revaluation is the partial derivative of price with respect to time, holding age constant: $\partial q(p,t)/\partial t \,\big|_{\partial s=0}$.

and in box age-4, every asset price you pull out will be a vintage 2005-4=2001! You don't need any files. In fact they make no sense because specifying the age box in the date room locks in the vintage. We have only 2 degrees of freedom, not 3. With this identification constraint in mind, we turn to the two components of economic depreciation, deterioration and obsolescence.

Deterioration

Deterioration is a relatively straight forward concept referring to the decline in price of one unit of the cohort of assets as the cohort ages resulting from use, wear and tear, and retirement from service. In principle, deterioration is independent of the specific date. It results from assets getting older and possibly "less efficient". Even if all assets were one-hoss-shays, so that in-use decrepitation is zero, price would fall with age because the cohort is using up some of its capacity both by being consumed and by being retired from service—each one hoss shay is getting closer to that moment when it turns to dust. Triplett, following Griliches, calls this effect **exhaustion**.[13]

Decline in efficiency, while still in-use, is also possible. After all, the one-hoss-shay is imaginary precisely to mock frustrated drivers who complain because parts of their vehicles breakdown. Strictly speaking a decline in efficiency is a decline in the marginal product of the capital and may reflect less accuracy, slower speed, increased operating costs, longer down-times, increased frequency of breakdowns, and so forth. Physicists call this entropy. For all these reasons old assets are usually worse physically than when they were new, and this decline in marginal product is reflected in the price. (We are ignoring here start-up costs and learning curves.) In other words, deterioration is the decline in price that reflects the decline in physical prowess of the original cohort of assets.

[13] Triplett and Griliches are thinking in terms of exhaustion of a specific asset and not a cohort of assets. Age is a proxy for use. Data on intensity of use is rarely available, and the concept of use is, as Hulten (1990) argues, itself ambiguous. The decline in marginal product with age would depend on the intensity of use, the care of the asset, the nature of the work it does, maintenance and repair, the quality of complementary goods, and numerous other factors.

The ratio of the marginal products of a cohort of age-(s+1) to age-s assets is less than one—marginal products decline with age.[14]

Obsolescence

The concept of obsolescence is more subtle than the concept of deterioration. **Obsolescence** is the effect on the price of an average unit of a cohort resulting from the availability of new vintages of the assets with superior characteristics to the older vintage. Nothing physically happens to the used assets *per se*, as it did with deterioration. The in-use marginal physical products are not changed. A newer version is now on the market, so the older ones, while unchanged, are relatively obsolete. Even if the old assets are the same, the environment is different—a better substitute[15] has become available. Obsolescence is a date-specific concept—it reflects unique date-specific characteristics embodied in new assets that are not embodied in older assets.

A 1993-Bordeaux is a unique vintage defined by the date the grapes were grown on the hillsides of Bordeaux.[16] The term **vintage** means the date the grapes were grown and harvested. With wines, of course, new vintages may be better or worse than existing vintages. The same may be true of some types of capital as well. If we run out of quality materials and build new assets that are "not as good," then like wine the older vintages may be superior to newer ones. We've all heard the expression, "they don't make 'em like they used to." Also, classic designs can cause older vintages of certain assets to be extremely valuable—the 1964 Ford Thunderbird for example. Still, as a rule, new assets frequently have features that make them better than pre-existing versions as a result of technological innovations embodied in the new vintages.

For most capital assets the concept of vintage is not as simple as with wine. One cannot, by definition, produce a 1993-vintage wine in 1995; however, with produced capital this is not the case. Just because a new design or new feature is available, one does not have

[14] In continuous time: $\partial[\partial Q/\partial K]/\partial s<0$ where Q is output, K is capital and s is age.
[15] The term "better" here means either higher marginal product or less costly or both.
[16] Ashenfelter, Ashman, and LaLonde (1995) analyze the causes of vintage effects on Bordeaux wines.

to build all new assets with the new best design. Not all automobile models, for instance, produced in 2002 had all the newest characteristics available in 2002. This means that our analogy of a different room for each year and a different box per age implying only one particular vintage, in one sense, does not exactly apply. We could go to room-2004, open box-age-3 and find several different **models** of cars and create a file for each model in the box. Some of the models have new features but some may only have features that were introduced in some earlier vintages. This would appear to violate the Hall Identification Theorem based on $v = t - s$. It also raises the question: If new assets in each year are themselves heterogeneous, how do we define a vintage-2001 car or a vintage-January-2003 Dell Computer?[17]

New features tend to enter markets of complex capital goods gradually over time. Very few new cars have the newest features—usually only the top of the line models do. The same is true of computers and consumer electronic products. A successful feature is gradually built into lower-line models over the years. According to Diewert, this gradual adoption in the market of new goods and of quality improvements is what creates the new-goods and quality-change biases in consumer price indexes. Triplett explains that this tendency of physical-capital markets to produce some models with old features and some with new features, permits researchers to use hedonic methods to isolate the prices of "quality improvements" from inter-temporal changes in the price of a fixed-quality good.

Embodied Technological Change: Two Models

We now allow for embodied technological change in the Jorgenson model by specifying vintage efficiency functions.[18] Consider a simple model in which embodied technological change occurs at a constant rate τ. How do we reflect this rate of technological change in the generic efficiency and mortality functions and in the special geometric case? The ef-

[17] Technological change has been so rapid in computers that vintages vary by month not just year.
[18] See Hulten (1990) for a theoretical presentation of a similar model.

ficiency function is the ratio of the marginal product of age-s assets to new assets, but now, with technological innovation, we must ask to which new assets?

There are now two possible ways to model capital with vintage-specific marginal products. We could re-normalize each year on that year's new assets or we could normalize on new assets in a base year. The first model normalizes the efficiency function in each year on new assets in that year, so that $\varphi_{s,t}$, s = 0, 1, 2, 3, …, the sequence of the ratios of the marginal products of used to new assets in each year, satisfies the property that $\varphi_0 = 1$. It follows that every used asset not embodying the new technology has to be drawn down both for decay and technological obsolescence in order to convert them into units compatible with the new, superior vintage in the next period.

Consider the example with the constant rate of economic depreciation, δ. One can decompose the depreciation rate δ into its two component parts: deterioration γ and obsolescence θ. In the primal, corresponding to deterioration and obsolescence in the dual, are decay and technological obsolescence. Letting $\delta = \gamma + \theta$,[19] so that efficiency is:

$$(13) \qquad \varphi_s = (1-\delta)^s = [1-(\gamma+\theta)]^s$$

The corresponding mortality function is:

$$(14) \qquad m_s = -(\gamma+\theta)[1-(\gamma+\theta)]^s = -(\gamma+\theta)\varphi_s .$$

The mortality of an asset results both from deterioration, decline in the asset's marginal product, and from technological obsolescence. It is necessary to include obsolescence in the mortality function in order to convert old assets into units comparable to technologically-advanced new assets—new (superior vintage) asset equivalents. The improvement in the quality of new assets means a decline in the relative, though not the absolute, marginal product of aged assets.

The second model normalizes on a base vintage. This model explicitly recognizes that new asset efficiency varies by vintage. New asset efficiency is one only in the base pe-

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[19] The depreciation rate δ actually equals $\gamma + \varphi - \gamma\varphi$. It is simpler to ignore the interaction term which is in any case small.

riod. It is less than one before the base and greater than one after the base. To capture the differences in new asset efficiency over time, we normalize on t = 0:[20]

(15) $$\psi_{0,t} = (1-\theta)^{-t}; 0 < \theta < 1; t = \ldots -1, -2, 0, 1, 2, 3$$

If $\theta = .01$, then the sequence of new asset productive efficiencies will be:

				Base t = 0			
t:	-3	-2	-1	0	1	2	3
$\psi_{0,t}$:	…$(1\text{-}.01)^3$,	$(1\text{-}.01)^2$,	$(1\text{-}.01)^1$,	1,	$(1\text{-}.01)^{-1}$,	$(1\text{-}.01)^{-2}$,	$(1\text{-}.01)^{-3}$,…

The normalization function ψ is an efficiency function only in the sense that it equals the marginal rates of substitution between vintage-t new assets and base-vintage new assets.[21] Assuming that decay occurs at rate γ, we write the normalization function of age-s assets in period-t to a new asset in the base period as:

(16) $$\psi_{s,t} = (1-\theta)^{-t}(1-\gamma)^{s}(1-\theta)^{s} = (1-\theta)^{-t}(1-\delta)^{s}; \forall s, t$$

$$\psi_{s,t} = (1-\theta)^{-t}\varphi_{s}$$

The productive efficiency of an age-s asset in year-t relative to a new asset in the base year equals the product of the normalization correction for vintage and the stationary efficiency function φ_s. For example, if the base period is 2002 then normalization of the 4-year-old cohort of assets in 2005 is $\psi_{4,3} = (1\text{-}\theta)^{-3}(1\text{-}\delta)^4 = (1-\theta)^{-3}\varphi_4$.

The age-date tableau of relative efficiencies, normalized on a new base 2000 assets is illustrated in Figure 1. This tableau assumes no changes related to passage of time *per se*; inter-temporal changes in supply and demand may be dealt with separately. Inflation is not relevant because we are talking about relative asset efficiencies not relative costs. We are using a tableau of efficiencies, rather than prices or user costs, in order to analyze the implications for measuring the flow of capital services in the primal that result from obsolescence vs. deterioration changes in the dual.

[20] We defined the rate of technological change to be τ. We could express the normalization in terms of τ. Instead of $(1\text{-}\theta)^{-t}$ we would have $(1+\tau)^t$. It is more convenient to express the normalization in terms of the rate of obsolescence θ. $\tau = \theta/(1\text{-}\theta)$.

[21] Producers making decisions in period-t deal with contemporary marginal rates of substitution between assets of different ages and not between inter-temporal MRS between new assets in different periods.

Figure 1 Tableau asset efficiencies normalized on new vintage 2000 assets*

$$\psi_{s,t}$$

t = date - 2000

Age ↓	Date →				
	2000	2001	2002	2003	2004
0	1	$(1\text{-}\theta)^{-1}$	$(1\text{-}\theta)^{-2}$	$(1\text{-}\theta)^{-3}$	$(1\text{-}\theta)^{-4}$
1	$(1\text{-}\gamma)\ (1\text{-}\theta)$	$1\text{-}\gamma$	$(1\text{-}\gamma)\ (1\text{-}\theta)^{-1}$	$(1\text{-}\gamma)\ (1\text{-}\theta)^{-2}$	$(1\text{-}\gamma)\ (1\text{-}\theta)^{-3}$
2	$(1\text{-}\gamma)^2(1\text{-}\theta)^2$	$(1\text{-}\gamma)^2\ (1\text{-}\theta)$	$(1\text{-}\gamma)^2$	$(1\text{-}\gamma)^2(1\text{-}\theta)^{-1}$	$(1\text{-}\gamma)^2(1\text{-}\theta)^{-2}$
3	$(1\text{-}\gamma)^3(1\text{-}\theta)^3$	$(1\text{-}\gamma)^3(1\text{-}\theta)^2$	$(1\text{-}\gamma)^3\ (1\text{-}\theta)$	$(1\text{-}\gamma)^3$	$(1\text{-}\gamma)^3(1\text{-}\theta)^{-1}$
4	$(1\text{-}\gamma)^4(1\text{-}\theta)^4$	$(1\text{-}\gamma)^4(1\text{-}\theta)^3$	$(1\text{-}\gamma)^4(1\text{-}\theta)^2$	$(1\text{-}\gamma)^4\ (1\text{-}\theta)$	$(1\text{-}\gamma)^4$
5	$(1\text{-}\gamma)^5(1\text{-}\theta)^5$	$(1\text{-}\gamma)^5(1\text{-}\theta)^4$	$(1\text{-}\gamma)^5(1\text{-}\theta)^3$	$(1\text{-}\gamma)^5(1\text{-}\theta)^2$	$(1\text{-}\gamma)^5(1\text{-}\theta)$

* Where s indexes age and t indexes time relative to base 2000. This tableau assumes constant rates of decay γ and of obsolescence θ; no shifts in supply and demand. We normalize on vintage 2000 new assets, $\psi_{0,2000} = 1$.

The history of any cohort of assets runs down a diagonal starting at age-0 in year-t from left to right. For instance, the new period-2001 cohort of assets has the following efficiency sequence through its history:

(s, t+s): (0, 2001) (1, 2002) (2, 2003) (3, 2004)

$\psi_{s,t+s=,v=2001}$: $(1-\theta)^{-1}$, $[(1-\theta)^{-1}(1-\gamma)]$, $[(1-\theta)^{-1}(1-\gamma)^2]$, $[(1-\theta)^{-1}(1-\gamma)^3]$, …

Each cohort decays over its life and obsolescence has no effect directly on the cohort's efficiency relative to the base. Decay depends on age and is independent of vintage.[22] This reflects the assumption that the technology embodied in the 2001 assets remains constant as that asset cohort evolves over its history and that γ and θ do not interact.

At any point in time, say year 2004, assets of different ages have different levels of technology embodied in them reflecting a constant rate of technological change per period. This can be seen by tracing the sequence down a column. For example the 2004 efficiency sequence is:

s: 0 1 2 3 4

$\psi_{s,t=2004}$: $(1-\theta)^{-4}$, $[(1-\gamma)(1-\theta)^{-3}]$, $[(1-\gamma)^2(1-\theta)^{-2}]$, $[(1-\gamma)^3(1-\theta)^{-1}]$, $(1-\gamma)^4$, …

The ψ-function is not an economic efficiency function. The efficiency function is the ratio of in-use marginal products between age-s and new assets in one period. The ratio of marginal products in year-t equals the ratio of two normalization functions, $\psi_{s,t}/\psi_{0,t}$.

In this second model, the in-use productive efficiency function is:[23]

$$(17) \quad \left(\psi_{s,t} \middle/ \psi_{0,t} \right) = (1-\delta)^s = \varphi_s ; \forall s,t$$

Comparing equation (13) to (17), we see that the marginal rates of substitution between age-s and new assets, in each year, are the same in each model. However, the models are different. The second model has two mortality functions not one. If by mortality we mean

[22] Decay rates would be vintage specific if technological change altered decay rates. This elaboration of the model would require that each cohort have its own decay rate, say γ_v. Since we are making a theoretical point here, we abstract from this elaboration.

[23] This efficiency function is the same as in the J-model, because we removed the ψ-normalization by calculating the ψ-ratio.

the difference between in-use productive efficiencies between age-s and age-(s+1) assets given date, then the period-t mortality function is:[24]

$$(18) \qquad m_{s,t} = \varphi_{s+1} - \varphi_s = -\delta(1-\delta)^s = -\delta\varphi_s .$$

If by mortality we mean the loss of in-use productive efficiency over the cohort's history, then the mortality function is:

$$(19) \qquad \mu_{s,t} = \left[\psi_{s+1,t+1} - \psi_{s,t}\right] = -\gamma(1-\theta)^{-t}(1-\delta)^s$$

$$\mu_{s,t} = -\gamma\psi_{s,t} = -\gamma(1-\theta)^{-t}\varphi_s$$

It is now evident that we have two different models on which to base capital aggregation from investment flow data. The efficiency and mortality functions, equations (13)—(14) are based directly on the Jorgenson model. The efficiency function is virtually the same, because $\delta = \gamma + \theta$. The mortality function is the same whether tracing an assets history between periods or comparing assets of different ages at a point in time.

The second model, equations (16)—(19), is different. Asset cohort relative efficiencies are normalized on a base year. This normalization places the in-use efficiency of all assets in every year in the same units—new, base-vintage equivalents. Asset efficiency at a point in time, in-use productive marginal rate of substitution between used and new assets, is the ratio of these normalization functions These ratios generate the same stationary efficiency function as in the first model. There are two possible mortality functions in the second model. Differences in in-use productive efficiency at a point in time are the same as in the J-model; but changes in efficiency over an asset-cohort's history are different. Mortality in the latter sense results only from decay, not from obsolescence. This μ-mortality function only reduces the productive efficiency of used assets that reflect a decline in the absolute value of the marginal product, not the marginal product relative to next period's superior vintage assets. For reasons that will become obvious shortly I call this model the "G-model."

[24] This mortality function is the same as the J-model.

Before we evaluate the two econometric models, we lay out the empirical problem they are intended to solve.

The quantity of aggregate capital services

To estimate or measure capital aggregates, statistical agencies begin with data on current and past investment expenditure flows that have been collected for construction of national income and product accounts. Direct observation of quantities of capital in each year would require annual capital censuses. To my knowledge no capital census has been undertaken.[25] Four capital aggregate measures are useful: the quantity of capital services and their user-costs, and the stock of capital, and the price of acquiring capital.[26]

Let I_t be nominal gross aggregate investment expenditures in period-t. These total expenditures are the sum of the products of the prices and quantities of individual investment goods, j=1, … , n, that were produced, marketed, and sold (or acquired) in period-t:

$$(20) \qquad I_t = \sum_{j=1}^{n} p_{t,j}\iota_{t,j} \equiv p_t \bullet \iota_t$$

The identity indicates that I_t equals the dot product of two vectors, p_t and ι_t. Assume that each vector can be represented by an index. Let price index $p_{0,t}$ be the period-t market price of one new, period-t investment good in period-t dollars. Let $\iota_{0,t}$ be the period-t quantity of new period-t investment goods. I assume the price index is not corrected for inflation and that neither index is corrected for quality change. Heuristically, these terms p_t and ι_t represent the current market price and quantity of the generic new period-t investment good.

The problem now is to derive two quantity measures from the past sequence of prices p and quantities ι of new investment goods. One is the current flow of capital services, k.

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[25] Accountants for various purposes construct book values of capital stocks, but these reflect accounting conventions and not economic processes.

[26] My main focus here reflects my interest in determining the quantitative implications of obsolescence vs. deterioration on service flows, so I do not want to deflect attention by raising complexities related to capacity-stock (or wealth stock) measures or on purchase prices and shadow prices of assets.

The other is the "capacity "stock of capital goods, K.[27] We derive two sets of quantities from the p and ι:

(21) $k_t = f(p, \iota)$ $K_t = F(p, \iota)$

$p \equiv (p_t, p_{t-1}, ..., p_{t-S})$; $\iota \equiv (\iota_t, \iota_{t-1}, ..., \iota_{t-S})$

We will also determine the relationship between asset prices and user-costs, since these will play a role via duality in measuring the quantity aggregates. For convenience we assume that the flow of the quantity of period-t services k_t and the stock of capital K_t are each linear functions of past real investment flows, $\iota_{0,t-s}$.[28]

(22) $k_t = \sum_{s=0}^{S} \omega_{s,t-s} \iota_{t-s}$ $K_t = \sum_{s=0}^{S} w_{s,t-s} \iota_{t-s}$

The weights for both quantity aggregates are vintage specific: they depend both on age-s and on vintage, v = t – s. Assume momentarily that these weights are stationary. In this case we normalize on one new period-t investment good by letting ω_0 and w_0 be one in each period. The first term in the sum, ι_t, the quantity of new investment goods acquired in period t, is, by definition, the quantity of new period-t capital goods, $K_{0,t}$. The proposed normalization on the service flow from one unit of newly acquired capital sets ω_0 to one. This assumes that the quantity of services, k_t, that flow of services equals the quantity of new capital, K_t. This assumption seems quite strong but I think it only acknowledges that we, as economists, ordinarily cannot observe the number of capital services flowing from a unit of capital.

One possible solution to obtaining flow measures distinct from stocks involves modifying the stock for capital utilization. As Hulten (1990) points out, however, it is not clear that a capital good only provides service when it is actually "in use." Even if I don't sit in all the chairs in my office all the time, I have the option value of sitting in each one whenever I want to, so chairs provide services even when not active. Producers know product demand will fluctuate but they require the option-value of producing goods when called for. I do not think we know enough to be able to measure a unit of services distinct from

[27] This aggregate is sometimes called the wealth stock.
[28] Diewert (2004) argues for an index number formula rather than a linear model. He assumes the new investment flows have already been corrected for quality change.

a unit of capital. One could define the quantity of output produced by capital and set to one the quantity produced by new capital, but this just begs the question of what is the output, i.e., service flow. I shall argue below that, once one takes into account expectations, an adjustment for utilization is rarely appropriate.

Another problem remains to be resolved. Recall that ι_v where v = t - s is a quantity measure derived from the flow of new investments in period-v. If we interpret $\varphi_{s,t}$ to be the marginal rate of substitution in between a new and a vintage-v asset that has survived until period-t, then we have to deal with the fact that some of the assets acquired new in year-v have expired or been retired before year-t. There are two ways of dealing with this problem, and each has advantages and disadvantages.

Hulten (1990) proposes that a correction be made to the quantity of assets acquired in year v = t - s. so that the quantity of services in equation (22) is based on the quantity of survivors from the original cohort. He suggests that we replace ι_v with κ_v defined as:

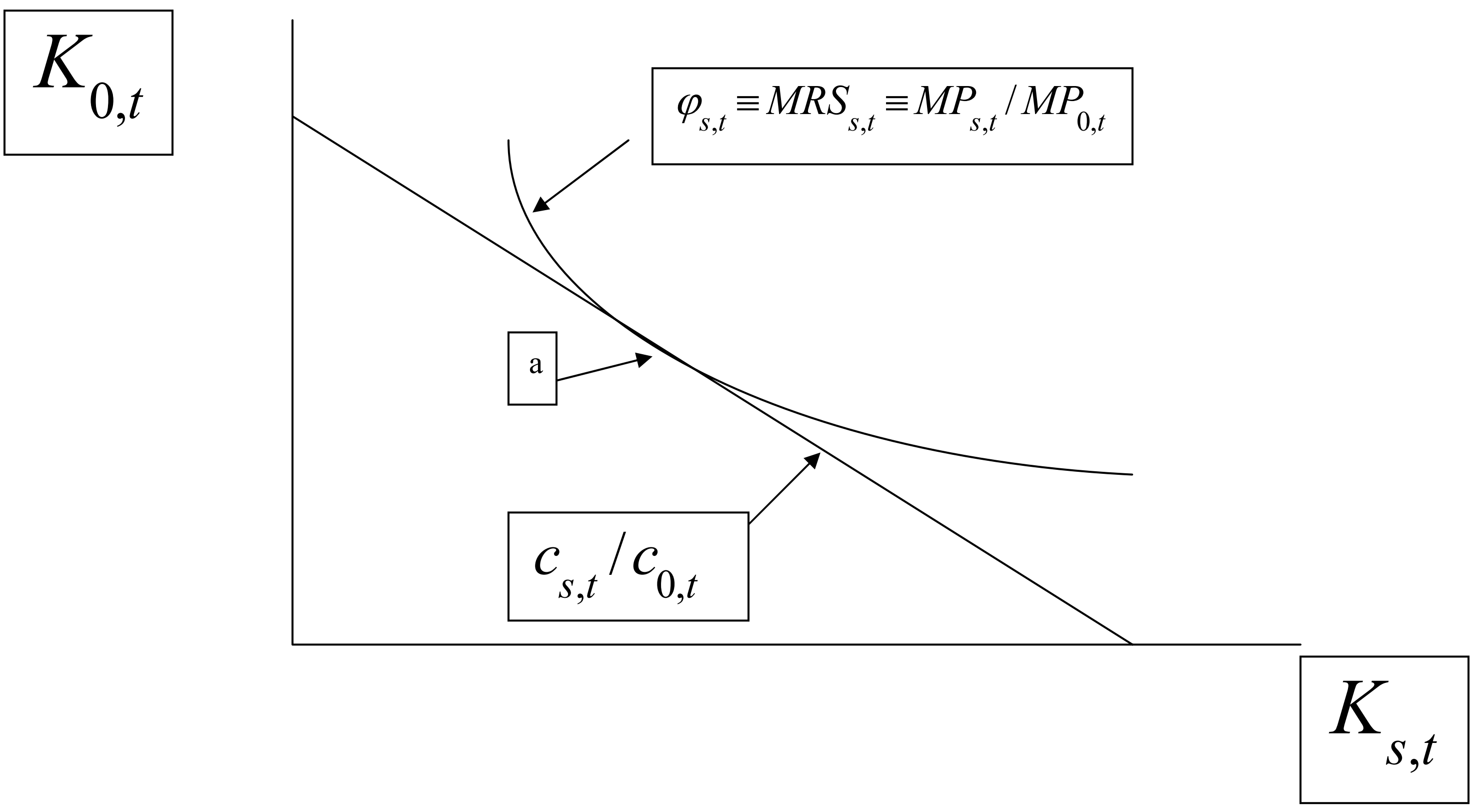
(23) $\kappa_t = \xi_{s,t}\iota_{t-s}$

The ξ-function is a correction for retirements. κ_v now replaces ι_v. The advantage of this approach is that retirement is seen to be a different process than in-use decay and may result from different factors, but this is also its disadvantage. Deterioration, the decline in price (the dual) that reflects physical decay (the primal), combines exhaustion (retirement) and in-use decay. This means that the mortality function combines the two effects so that the duality condition already allows in theory for retirements.

This brings us to the second way of dealing with the retirement problem, the approach used here. Define the φ-function to be the in-use value in period-t of one unit of the original cohort of vintage-v = t-s assets to one unit of the new cohort of assets in period-t. The correction for retirement is included in the φ-function itself so that $\iota_{s,t}$ is not changed. We give this interpretation to the efficiency functions and also to the mortality functions.

We are now prepared to ask what weights, ω and w, we should use? The J-model answers this question with the efficiency function. Figure 2 illustrates that the efficiency function is the sequence, expressed as a function, of marginal rates of technical substitution between age-s and new capital goods at time t = v + s. The duality condition is consistent with the producer selecting point “a” on the isoquant as the necessary condition for cost minimization. Since $\varphi_{s,t}$ is the marginal rate of technical substitution between an age-s and a new asset, in current production, it is the quantity of age-s assets one needs to provide the equivalent output of one new period-t asset—the slope of the isoquant. We do not in this analysis need to know what that output is; we only need to know the slope of the isoquant. Duality equates the slope of the isocost curve to the slope of the isoquant.

Figure 2 Duality is the Cost Minimizing Condition



Two Models for Capital Aggregation

A natural weight for the flow of services from assets of different ages is the marginal rate of substitution between the age-s asset and the new asset in each period. In each model, the J-model and the G-model, the marginal rates of substitution are the same, φ_s, so perhaps in each case $\omega_s = \varphi_s$. However, one purpose of the G-model would be that instead of resetting the efficiency of a new asset to one each period, we let each vintage have the efficiency that reflects its embodied technology relative to technology in the base. In the G-model, then, the ω's are vintage specific ψ-values: $\omega_{s,t} = \psi_{s,t}$ $s = 0, \ldots, S$.

For every s, $\psi_{s,t}$ is different in each period t. In the J-model the φ—weights are the same in each period t. This means that the aggregate quantity of capital services per year will be different in the two models, even though the ratio of the marginal products in each year will be the same in the two models: $MP_s/MP_0 = \psi_{s,t}/\psi_{0,t} = \varphi_s$. Given these distinctions between the two models, we now derive the capital aggregates for both models. We will then begin to assess their relative merits.

The Jorgenson model weights the quantity of in-use services by the efficiency function φ.

$$k_t = \sum_{s=0}^{S} \varphi_s \iota_{t-s} = \sum_{s=0}^{S} (1-\delta)^s \iota_{t-s} \tag{24}$$

The duality condition serves two purposes in this analysis, one theoretical and one practical. The theoretical purpose is to justify the weights of in-use capital, because in theory we are using the decisions of rational cost-minimizing economic agents to convert assets that are heterogeneous by age into new asset equivalents. The practical contribution of the duality condition solves the problem that as economist, though possibly not engineers, we do not directly observe the relative efficiencies, slopes of the isoquants. The dual, equating relative user costs to relative efficiencies, allows us to infer the φ-sequence from the ratios of user-costs—the slope of the isocosts.

The capacity capital stock weights in the J-model can be derived from a parallel structure to the service flows. Corresponding to the in-use efficiency function φ_s is a capacity efficiency function Φ_s. An asset exchange, or capacity, duality condition corresponds to in-use service flow duality condition.

$$\left(\frac{p_{s,t}}{p_{0,t}}\right) = \Phi_{s,t} \tag{25}$$

The relative cost of acquiring an age-s asset compared to the cost of acquiring a new asset in period-t equals the relative capacity of future in-use services in age-s assets to the capacity in new assets, $\Phi_{s,t}$.

A capacity mortality function M_s indicates the decline in capacity between age-s and new assets:

$$M_s = \Phi_{s+1} - \Phi_s = \frac{p_{s+1,t} - p_{s,t}}{p_{0,t}} = \frac{D_{s,t}}{p_{0,t}} \tag{26}$$

The second equality results from duality and the last equality reflects the definition of economic depreciation. In the unique case in which the depreciation rate is constant δ:

$$M_s = -\delta\left(\frac{p_{s,t}}{p_{0,t}}\right) = -\delta\Phi_{s,t}\,. \tag{27}$$

When the economic depreciation rate is constant, δ, the rate of mortality in capacity and the rate of mortality in service flow both equal δ. Φ is a natural weight w in the capacity stock aggregator.

$$K_t = \sum_{s=0}^{S} \Phi_{s,t} \iota_{t-s}\,. \tag{28}$$

In the G-model the service flow weights, the ω's, are the vintage-specific ψ-function values:

$$k_t = \sum_{s=0}^{S} \psi_{s,t} \iota_{t-s} = \sum_{s=0}^{S} (1-\theta)^{-t}(1-\delta)^s \iota_{t-s} \tag{29}$$

Duality in the G-model is:

$$(c_{s,t}/c_{0,0}) = \psi_{s,t}\,. \tag{30}$$

The ψ-function expresses the service price of an age-s asset in period-t relative to a new asset in the base period t=0. The marginal rate of substitution between age-s and new asset services in period-t is:

$$(c_{s,t}/c_{0,t}) = \left(\psi_{s,t}\big/\psi_{0,t}\right) = \varphi_{s,t} \tag{31}$$

Model-G, like model-J, has a capital stock framework that corresponds to its treatment of services. Corresponding to the in-use normalization function ψ is a capacity normalization function Ψ. Both are vintage specific. Capacity duality in the G-model is:

$$(32) \qquad \left(\frac{p_{s,t}}{p_{0,0}}\right) = \Psi_{s,t}$$

Relative asset acquisition costs equal relative productive capacity. Mortality is:

$$(33) \qquad \mathrm{M}_{s,t} = \Psi_{s+1,t+1} - \Psi_{s,t} = \frac{p_{s+1,t+1} - p_{s,t}}{p_{0,0}} = \frac{\Delta_{s,t}}{p_{0,0}}$$

$$\Delta_{s,t} = c_{0,0} \sum_{x=0}^{S} \frac{\mu_{s+x,t+x}}{(1+r)^x}$$

The (capacity) capital stock equation for the G-model is:

$$(34) \qquad K_t = \sum_{s=0}^{S} \Psi_{s,t} \iota_{t-s} .$$

We have now presented two complete models of capital aggregation. In the Jorgenson model, the quantity of capital services is equation (24) and the capacity capital stock is equation (28). In the G-model, the quantity of capital services is equation (29) and the capital stock is equation (34). The final step in developing the models is to derive the inter-temporal changes in the flow of capital services, $\Delta k_t = k_{t+1} - k_t$, and in capital stocks, $\Delta K_t = K_{t+1} - K_t$.

From equation (24) we derive the change in the flow of capital services from period t to period t+1.

$$(35) \qquad \Delta k_t = \sum_{s=0}^{S} \varphi_s \iota_{t+1-s} - \sum_{s=0}^{S} \varphi_s \iota_{t-s} = \iota_{t+1} + \sum_{s=0}^{S} m_s \iota_{t-s}$$

When depreciation is constant at rate δ:

$$(36) \qquad \Delta k_t = \iota_{t+1} - \delta \sum_{s=0}^{S} (1-\delta)^s \iota_{t-s}$$

$$\Delta k_t = \iota_{t+1} - \delta k_t$$

Similarly the change in the capacity capital stock in this model is:

$$(37) \qquad \Delta K_t = \iota_{t+1} + \sum_{s=0}^{S} M_s \iota_{t-s}$$

$$\Delta K_t = \iota_{t+1} - \delta K_t$$

The last equality results from the definition of M_s as $D_{s,t}/p_{0,t}$.

From equation (29) we derive the change in capital services for the G-model.

$$(38) \qquad \Delta k_t = \sum_{s=0}^{S} \psi_{s,t+1} \iota_{t+1-s} - \sum_{s=0}^{S} \psi_{s,t} \iota_{t-s} \,.$$

$$\Delta k_t = \psi_{0,t+1} \iota_{t+1} + \sum_{s=0}^{S} \mu_{s,t} \iota_{t-s}$$

For the constant rates example, we replace $\psi_{s,t}$ and $\mu_{s,t}$ from equation (19):

$$(39) \qquad \Delta k_t = (1-\theta)^{-(t+1)} \iota_{t+1} - \gamma (1-\theta)^{-t} \sum_{s=0}^{S} (1-\delta)^s \iota_{t-s}$$

$$\Delta k_t = (1-\theta)^{-(t+1)} \iota_{t+1} - \gamma k_t$$

The last equality derives from equation (29). Similar analysis leads to the following capacity capital stocks for the G-model.

$$(40) \qquad \Delta K_t = \Psi_{0,t+1} \iota_{t+1} + \sum_{s=0}^{S} (\Psi_{s+1,t+1} - \Psi_{s,t}) \iota_{t-s}$$

$$\Delta K_t = \Psi_{0,t+1} \iota_{t+1} + \sum_{s=0}^{S} M_{s,t} \iota_{t-s}$$

Using capacity duality equation (32) and the equation (33) derivation of $M_{s,t}$ in terms of $\mu_{s,t}$ yields:

$$(41) \qquad \Delta K_t = \Psi_{0,t+1} \iota_{t+1} - \gamma K_t$$

Comparison of equations (36)—(37) to equations (39)—(41) shows the similarities and differences between the two models. In each model the change in the quantity of capital services from period-t to period-t+1 is the sum of two terms—new acquisitions and replacement of lost capacity in old capital. The same is true for changes in the capacity capital stocks. In both models the relative weights of new and used capital services equal the marginal rates of substitution between age-s and new assets, φ_s. In both models the relative weights of new and used capacity capital between age-s and new assets equal the relative capacities of age-s to new assets, $\Phi_{s,t}$ and $\Psi_{s,t}$ respectively.

In the G-model, though, new acquisitions of capital, ι_{t+1} are up-graded by $(1-\theta)^{-(t+1)}$ to allow for embodied technological advances by converting new acquisitions into base period units. In each period the capital stock is measured in "base-period, new-asset, in-use productive-efficiency" equivalents. No change is made to the units of new acquisitions in the J-model. In the J-model in each period, the stock of capital is measured in "contemporary new-asset, in-use productive-efficiency "equivalents.

Replacement requirements, to compensate for mortality of services, are also different in the two models. In the G-model old assets are drawn down to compensate for decay, γ. In the J-model old assets are drawn down to compensate for the sum of decay and obsolescence, $\gamma + \theta$. This sum equals via duality the economic depreciation rate δ. The J-model correction amounts to converting the units of old capital between each period into the next period's new acquisition units. Essentially the units in the two models are different. In the G-model capital is always measured in base period equivalent units and replacement is only for decay. In the J-model each period's capital stock is measured in units of current period new acquisitions, and replacement is for both decay and obsolescence.

One consequence of the vintage based G-model is that throughout its history each vintage decays but loses no value as a result of obsolescence. This is partly illusion, because the relative weights of assets in each period's capital service flow correct both for decay and obsolescence. Consider from Figure 1 the ratio of the weight of a new 2004 asset compared to the weight of a 3-year-old asset in 2004, $[(1+\theta)^4/ (1-\gamma)^3 (1+\theta)]$. The ratio is: $[(1+\theta)/(1-\gamma)]^3$. Each unit of new 2004 capital is worth $(1+\theta)^4$ units of new-2000 capital where as each unit of 3-year-old capital originally was worth $(1+\theta)$ units of new-2000 capital and has since decayed by $(1-\gamma)^3$.

Which method do we want to use, the common base G-model or the revised base J-model? Does it really matter? After all, both models are based on neoclassical theory and both are coherent. The J-model does have a practical advantage. One only need know δ, because decay and technological obsolescence never appear separately either in the primal or in the dual. In the G-model, however, one must be able to decompose δ into γ and θ, because between periods assets are depleted for γ only.

The G-model normalization, that requires a non-stationary efficiency function and a non-stationary mortality function, amounts to counting the number of crackers in one vintage cracker box. Then recalibrating boxes in every other year to the equivalent number of crackers in the base vintage box. The J-model re-defines a cracker box in each period to reflect the number of crackers in a new box in that period. It seems at first blush more reasonable to draw down old boxes only if they lose crackers. This would be decay. This is done in the G-model. Old capital is not penalized for obsolescence. Suppose, however, obsolescence causes old vintages of boxes to lose crackers. How could this be? Well, suppose people toss out some of the old boxes when bigger boxes, that are less costly to use, become available? If they do, then the J-model that draws down old boxes for both decay and obsolescence may be superior to the G-model.

In any case, we cannot even consider the G-model without information about the effects on the old stock of capital resulting from decay separately from technological obsolescence. To make a scientific, rather than a heuristic, choice between the two models requires analysis of obsolescence vs. deterioration. Little research has been done on this topic. If we had such information then perhaps we could determine to what extent, if at all, we should distinguish between deterioration and obsolescence. In the meantime, perhaps we can glean some insight by considering producer behavior.

Producer behavior: Obsolescence vs. deterioration

A long held view in capital theory has been to think that obsolescence, as opposed to deterioration, should not lead to a lower weight on old assets. Joan Robinson once said, “an overcoat is an overcoat." Robert Solow later remarked, "a spade is a spade is a spade." Evidently both meant that the old capital retains its worth even if new, superior vintages enter the market.[29] In more recent years, Zvi Griliches argued that since older vintages of capital retain their in-use productive efficiency when better vintages of capital enter the market, old assets should not be drawn down as a result of technologically induced obsolescence. Griliches illustrated his point by observing that even if new material is introduced by new vintages of young faculty, an older tenured professor is still just as good as he was (provided new research has not shown that what he teaches is wrong.) Is Griliches right and what does his argument imply for out choice of models?

The Griliches Assertion

The conventional wisdom held by some very distinguished giants of the profession, Robinson, Solow and Griliches, means that unless the old assets wear out (decay), they retain their full value even though they are now obsolete relative to new versions of the assets. This must mean that producers who employ used capital respond differently to obsolescence than they do to deterioration. A decline in price resulting from deterioration lowers an asset's value, but a decline in price resulting from obsolescence does not. The argument is that in the case of deterioration, the old asset has physically decayed and is less productive. However in the case of obsolescence, the old asset is physically unchanged in productivity and thus should not be reduced in value. This means in turn that even when we observe a decline in the market price of used assets as a result of obsolescence, their in-use value or productive efficiency, remains the same.

A related question is: If Griliches is correct that one must distinguish obsolescence from deterioration in valuing capital, then, is the Jorgenson duality condition wrong? Clearly obsolescence reduces the relative price of used assets to new. Does it also lower the in-use efficiencies represented by the Jorgenson efficiency sequence? After all these are two important ideas, sacred cows perhaps, in capital theory. Can they coexist, or do we have to choose one over the other? I will analyze this question first assuming duality; then I will drop the duality assumption to explore the consequences.

[29] Michael Harper made this point an NBER summer workshop in July, 2003.

Decisions by a Jorgenson producer

Consider a rational producer initially at a competitive equilibrium, like point-a in Figure 2. This equilibrium satisfies Jorgenson's duality condition equation (3), in which the ratio of the user costs equals the marginal rate of substitution between used and new assets. In equilibrium this result holds for all assets, and for the labor as well as other inputs.

(42) $MRS_{s,0} = c(s)/c(0) \quad MRS_{L,0} = w/c(0) \quad MRS_{x,0} = p_x/c(0)$

L is labor and x is other inputs. MRS is the marginal rate of (technical) substitution. Using marginal products, MP, this also means that

(43) $MP_0/c(0) = MP_s/c(s) = MP_L/w = MP_s/p_x$.

The marginal product per dollar (the numeraire) spent on each input is equal to any other.

We are going to consider how a rational producer under several different sets of circumstances will react to new (presumably superior) technology embodied in new capital. First, however, we should be more explicit about what we mean by technological change embodied in new capital. Does the technological change lower capital costs? Does technological change increase marginal product? Does technological change lower marginal product and increase capital costs or do capital costs stay the same or even fall when the marginal product of new capital increases? Does the technological change per se alter the marginal products of other inputs or leave them unchanged? Is it possible for technological change to increase the average product of new capital but not the marginal product? In the abstract technological change could involve many of these alternatives and we have to be careful to explicate which alternatives are relevant to our analysis.

Economists have long considered the possibility of skill-biased technological change. Economists have also considered models in which new technology is exogenous to the economy and leaves the production relation intact: New technology increases output for each initial combination of inputs without altering their marginal rates of substitution and it does so without an increase in social costs. Technological change of this type is **Hicks neutral** technological change that is also **manna from heaven**. Of course, since it is Hicks neutral it cannot be embodied in new capital. If a technological change is to be embodied in new capital, then it must have one of two properties: It must either change the physical nature of new capital relative to earlier vintages of the new capital, or it must lower the cost of using new capital relative to the cost of earlier vintages. Either of these properties rule-out Hicks neutral technological change.

The manna from heaven property is a little different. If the marginal physical product of new capital rises but the costs stays the same, then this is compatible with manna from heaven. What if the physical nature of the product is the same but costs have fallen? Could this be manna from heaven? Yes it could be, but this would mean the manna has fallen somewhere else—the cost of some input into the production of this capital has fallen but the capital itself is unchanged. We consider either type of technological change—the new capital is the same but less costly to the user, or at the same cost, the new capital is more productive.

The manna from heaven property is not necessary however. In endogenous growth models, by definition, technological innovations have social costs. To what extent are these costs passed on to the buyer of new capital in which new technology is embodied? As Hulten has pointed out the social welfare implications of socially costly innovations as opposed to manna differ. But, here we are considering how a user of capital, who holds old assets, will react to new technology embodied in new capital but unavailable to his used assets. (He cannot retrofit this new technology onto his old capital.) If the capital-user incurs the full costs of the new technology then I would argue that, from his perspective it is not a technological innovation. If the cracker box has one third more crackers but costs one third more, then from the buyer's point of view the new box is no better than the old.[30] Only if the innovation results in a better product per unit of costs does the producer, who uses the capital, encounter a technological innovation in any meaningful sense.

[30] I believe this is the case that Hulten has in mind when he argues that unless there manna from heaven will be treated differently than new technology with exactly offsetting increases in costs.

Technological change embodied in new capital rules out **Harrod neutral** technological change, because Harrod neutral technological change augments the labor input without altering the capital input. Since both old and new capital are by definition included in the capital input the capital input must change. In fact, it is not adequate to simply multiply the stock of capital by some technological growth rate, because the measure of the capital aggregate must itself change. The K-function in equation (8) must change.

Finally, suppose a new invention increases the average product of new capital. Could this occur without altering the marginal rate of substitution between new and used capital? We are considering here technological change embodied in new capital in the sense that the new capital is superior to its older vintages. We mean superior in the sense that the marginal product per dollar is higher than a new vintage was in the previous period. This means that either the new asset cost less to use or it produces more per unit of cost in comparison to old capital. In comparison to the previous period's capital, the product of the new one asset is higher. If the average product has increased then assuming the same marginal product for the old capital, the marginal rate of substitution between new and used capital must have decreased—the ratio of the marginal product of the new asset per dollar must have risen relative to that of used assets.

Whether this lowers the cost per unit of new capital or raises its marginal product, embodied technological change lowers the cost per unit of marginal product of new capital. The result is that the marginal product per dollar of new capital has increased. Assuming no decay and no deterioration, we will assume the ratios of the other inputs, old capital and labor, all remain unchanged,[31] so that,

(44) $MP_0/c(0) > MP_s/c(s) = MP_L/w = MP_x/p_x$.

How does the producer respond to these new circumstances? In a competitive equilibrium in which all inputs are variable, which is the Jorgenson model, the producer will substi-

[31] If all the equalities in equation (44) hold, then we have ruled out skill biased technological change. Since we are interested here in how the producer acts regarding new versus used capital, the possibility that new capital increases the marginal rate of substitution between two other inputs is not pertinent. Skill biased technological change refers to an innovation that alters the relative value of skilled and unskilled workers. This could take place or not and the marginal rate of substitution between new and used capital would still have increased.

tute away from age-s capital, labor and x-inputs toward new capital. The substitution continues until the equilibrium condition equation (43) is re-established. The result will be an increase in the marginal product per dollar of used assets (and other inputs) **relative** to the marginal product per dollar of new assets. It is at the margin that obsolescence results in a substitution away from used assets and other inputs toward new assets. The marginal product of age-s assets rises or the user-cost of an age-s asset falls or both occur. This means that either the quantity of used assets demanded falls or the asset price of used assets falls.

It would appear that in the Jorgenson model, in which duality holds, the value of the used stock of assets falls as a result of obsolescence. I would conclude that Griliches' assertion does not hold up in the Jorgenson model. Of course one may ask: Is the Jorgenson model reasonable? Perhaps Griliches is right. If so, then should we scrap the Jorgenson model? On the one hand, the Jorgenson model is coherent even if it is not consistent with Griliches' view. On the other hand, many critics dislike the equilibrium duality condition arguing that producers are locked into age-s assets. However, an implicit assumption of the Jorgenson model is that producers can substitute new assets for old assets. This is certainly true in some, perhaps many, cases.

If used-asset markets exist, then producers can sell off old assets in the used-asset market. The used market can be more subtle than often acknowledged by critics. Tax breaks, rebates, pollution credits and other inducements may be available which make "resale" of used assets more likely. In the case of cars for example old cars can be donated to charities for tax deductions. In the case of computers, firms may obtain tax benefits by donating used computers to schools. Many used assets are also exported from the US to other countries for various secondary uses. Furthermore, secondary uses for old capital may exist within large firms themselves. In fact all that is necessary is the presence of a down stream use for the old asset. Such a down stream use implies that the old asset is less productive as a direct result of obsolescence. Since many opportunities are available to unload old assets on used markets, the Jorgenson duality condition, while not universal, may be a reasonable approximation of reality.

Decisions assuming no used-asset markets

What if used markets do not exist and producers cannot receive market value for their old assets? In Griliches' famous example, senior Harvard professors are tenured so that even if superior young professors, new vintages, are available, President Summers of Harvard cannot unload his senior faculty in order to hire the new young Ph.D.s. If a person owns a two year old computer, he may not be able to sell it at a price offsetting its in-use value. This would be true in an Ackerlof lemons model for example.[32] What do producers do when they are stuck with the used asset?

Suppose we drop the assumption that used capital is a variable input and treat the old asset as a sunk cost. If the producer does not use it, he may as well throw it away.[33] Does this mean that the Griliches assertion is correct—the in-use value of old capital is unchanged? How will the producer behave if the used asset is a sunk cost? In this short run, the rational producer will only operate at the margin available to him. While he cannot recapture his sunk costs, he does have a number of margins on which to operate. His variable costs include labor and other variable inputs that are used along with capital in production. These would include fuel, maintenance and repair, electricity, ink, and software to name a few. All of these are possible substitutes for new capital when the marginal product per dollar of new capital has risen.

This explains why producers may reduce their secretarial pool as they bring new computers with embodied technological improvements into use. It explains why less skilled workers are laid off when new capital, with a lower marginal product per dollar becomes available. It also explains why maintenance and repair expenditures are increasingly replaced by total replacement of damaged used machines for new ones.

[32] Ackerlof (1970) argued that asymmetric information in which buyers are ignorant of the quality of a used asset results in a market dominated by lemons. Owners of cherries cannot get value in such a market. Of course, the logical extension of the Akerlof model is a world without used asset markets. As Akerlof himself explained owners of used cherries who wish to sell them have an economic incentive to either inform potential buyers or provide a guarantee on quality. Such behavior can be observed in used-asset markets which are often quite vigorous.

[33] I use throw away at no cost here to avoid complications of scrapping old assets for value. I will consider a model in which costs (or possibly benefits) accrue from scrapping old assets.

This still does not quite explain how the producer decides when to abandon the old machine. He will abandon the old machine when the marginal, variable costs of using the old machine exceed its in-use marginal benefit. The marginal benefit of using the old machine is its marginal product. His decision rule then is to continue to use the old machine as long as the following holds,

$$(45) \qquad VMP_s - MC_z \geq 0 \,.$$

VMP_s, the value of marginal product, is product price times the marginal physical product of age-s capital and MC_Z, the marginal cost of variable inputs used with age-s capital in producing output, is the sum of the variable input prices times quantities used.

Is this decision rule influenced by technological change embodied in new capital? While it may not appear to be so, it is. Variable costs of using the old machine include the opportunity costs—the forgone earnings from using the variable inputs at their next best alternative; namely, in our example, matching up continued use of the old machine to acquiring the new machine. Still, the new machine must be purchased[34] and the residual cost of the old machine cannot be realized. When does this producer decide to retire his old machine and replace it with a new one?

The producer will replace the old machine with the new machine when the value of old capital's marginal product minus the variable costs of inputs used in tandem with the old machine is less than the marginal product of the new machine minus both the costs of the old inputs working with new capital plus the user-cost of the new capital.

$$(46) \qquad VMP_0 - [c(0) + MC_z^0] > VMP_s - MC_z^s \,.$$

If product price, p, is unlikely to change, the:

$$(47) \qquad p(MP_0 - MP_s) - (MC_z^0 - MC_z^s) > c(0)$$

or, if input prices other than new capital remain fixed, then,

$$(48) \qquad p\Delta MP_{0,s} - \sum_z p_z \Delta z > c(0)$$

[34] By assuming that used capital is a sunk cost we are not only assuming no resale markets for used assets but also no used-asset rental markets.

where Δ represents the difference between new and used marginal product in the case of MP and quantity of ancillary inputs in the case of z.[35] Fewer z-inputs may be required for use with the better new machine, because embodied technological change may include lower ancillary costs, such as improved energy efficiency. If, however, the same ancillary inputs are required to use the new capital, then the condition is:

$$(49) \qquad p\Delta MP_{0,s} > c(0)$$

The increase in the value of marginal product from switching to new capital must exceed the user-cost of the new capital. Clearly technological change embodied in new capital will influence the decision to stop using old capital even when old capital costs are sunk costs![36]

Obsolescence vs. deterioration revisited

So far we have analyzed the decision process, in the short run and the long run, of a producer who has to decide how to respond when new capital embodies technological advances over used capital. In each instance, under the Jorgenson model and under irreversibility, we have established the conditions in which a rational competitive producer will substitute new for used capital. We could go on to consider other market models, monopoly and so on, but this would not seem to change the substance of the key argument.

In particular, this analysis of producer behavior has an important implication for the assertion that the quantity of old capital in stock measures remains unchanged when new vintages of capital embody technological change. Evidently the decisions made by the rational producer are always made at the margins—the only difference between the short and long run is which margins are operable. In each case it is comparisons of marginal product to price ratios of different inputs that bear on the decisions. In the short run if

[35] Once this marginal condition is met, it should continue to be met during future periods of use provided efficiency sequences decline with age (or use).

[36] Dividing both sides of equation (49) by c(0) results in p $\Delta MP_{o,s}/c(0) > 1$. The left hand side of this expression is Tobin's q. If the incremental benefit of new capital exceeds the replacement cost then the producer will invest in new capital.

condition in equation (42) holds then the producer switches to new capital. In the long run if the condition in equation (49) holds then the producer switches to new capital. In both cases, it is the ratios of marginal products to user-costs that matter.

In the mental experiments above the relative ratios changed as a result of technological improvements embodied in new capital making old capital obsolete. However, what if the initial change had resulted from decay of used capital rather than from obsolescence? It doesn't matter, because the rational producer based his decisions only on changes in ratios regardless of how they came about. A decrease in the marginal product of used capital as a result of decay, reflected on the price side by deterioration, acts the same as an increase in the marginal product per dollar of new capital as a result of obsolescence. I conclude that the obsolescence-deterioration distinction is a red herring.

Implications for Utilization

The above analysis also has implications for adjusting capital stock measures to obtain capital flow measures by allowing for utilization. Berndt, Mori, Sawa and Wood (1990) develop and apply relative utilization weights of different vintages of capital in order to assess the substitutability consequences of the energy price shocks during the 1970s. They show that the US manufacturing sector lowered utilization rates of energy-intense old capital in response to price increases of gasoline. The question I now address is in measuring capital stocks what allowance should one make to allow for variations in utilization rates of different vintages of capital. To make sense of this question I distinguish between resale, retirement, and scrap of used assets. As above, I also consider the short run vs. the long run in which a rational, competitive producer decides how to deal with his used capital.

Equation (47) shows the criteria on which the producer replaces old with new capital when used-asset markets are unavailable so that used-asset costs are sunk. The second term on the right hand side is the difference between the costs of ancillary inputs that must accompany new capital vs. used capital. If new technology results in less energy-intense inputs and energy costs have risen then, as Berndt, Mori, Sawa, and Wood show, the producer is induced to substitute new energy efficient assets for old.

However, in our model producers replace the old asset in its current use with a new asset. The used asset is either resold or, if possible shifted to a lower-valued use. This suggests that the old assets are still in-use but in a different activity and that this distinction is reflected in its lower user-cost. In this case, the utilization rate of old capital does not fall. Its use shifts and the user cost, and consequently the asset price, will decline to reflect this fact. It would be redundant and misleading to correct in addition for variations in utilization.

Of course, at some point down stream the net benefits of an old asset in any use will be less than scrap value and the asset will be scrapped. At this point the asset is retired from service and will no longer appear in the stock. Its weight will have dropped to zero as has its user-cost. This is the retirement case and of course one must allow for retirements.[37]

None of the models considered so far implies lower utilization of old capital per se. Each producer is generating output with one of two technologies. One uses new capital and the other old capital. Each chooses one or the other.

One could establish circumstances in which a producer will under-utilize old assets. In theory, the circumstances would seem to consist of an unexpected change in asset-use conditions when limited options are available to producers and when producers become less certain about future conditions that influence capital inputs. This type of circumstance is exactly the one envisioned and modeled by Berndt, et al. A shock hits energy-intense capital users and leaves them in a less certain environment. It is hard to imagine that capital users in the US had anticipated the decision by OPEC to instantaneously quadruple energy prices. Furthermore it is hard to imagine that firms were able to predict, with the degree of confidence they had before the shock, the path of energy prices after

[37] See Hulten and Wykoff (1981) for a coherent treatment of retirements albeit one calling for limited information on retirements. The limitation reflects the data available for measurement purposes at the time..

OPEC formed and jacked up price for the first time. A sensible reaction of capital users under these conditions is to wait until the future becomes less obscure. In this case the service flow from old capital will fall relative to that firms had expected to employ. This means that a distinction be made between the available service flows reflected in a priori user-costs will have to be corrected for new utilization rates.

However, these conditions are unlikely to persist. They are, by their very nature, transitory—expectations were wrong and confusion rose. Even if expectations are not formed rationally, eventually producers will adjust their expectations, correct their forecasts (and standard errors about those forecasts), and capital asset prices and user costs will again reflect normal conditions. It seems to me that at this point adjustment for utilization should not be employed. Capital stock measures intended to indicate normal conditions, even normal swings in economic activity, business cycles, should not be adjusted for a distinction between service flow and user-cost. At the margin, these are set equal by rational producers.

If over the course of the business cycle, demand unexpectedly falls short, then capital utilization will decline. Even here surely firms in a modern capitalist economy do not find business cycles *per se* unexpected. Ordinarily then, even over the business cycle, a reasonable model underlying measurement of capital would assume that capital operates pretty much as expected. It is only unpredictable surprises followed by a period of increased uncertainty, that drive capital-use away from expectations and therefore away from costs—Keynesian surprises, followed by increased uncertainty about future events, result in utilization rates below that implied by efficiency functions that equal user cost ratios.

This analysis implies that utilization indicators are useful devices for analyzing business cycles resulting from sudden surprises and increased uncertainty. Furthermore, retirements must be built into depreciation estimates. However, utilization rates should not be used in measuring the capital input in a neoclassical framework and it is this framework that, Joan Robinson and P Sraffa notwithstanding,[38] underlie capital measures in national accounts.

Conclusion

The first half of the paper shows that the measurement of capital over time must correct for both deterioration and obsolescence in order to reflect a coherent aggregate—that is, aggregating in like-units. The last half of the paper establishes conditions in which owners of used assets will switch to new assets. The argument is that the decision making process of a rational producer will depend on the same marginal conditions regardless of whether they are changed by obsolescence or deterioration. If the producer is restricted on some margin in the short run then that restriction will take place whether or not his old asset has decayed or become obsolete! Only under unusual circumstances should the in-use services from capital deviate from the efficiency sequence as a result of variations in utilization. The behavioral model also suggests that a sound assumption under ordinary circumstance is that relative user costs equal in-use productive efficiencies, so that the duality condition is a sound lynchpin of capital measurement.

[38] See Harcourt and Laing (1972) which contains articles both by Robinson and Sraffa.

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