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Efficient Search on the Job and the Business Cycle

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Abstract

We develop a tractable model of the labor market where workers search for jobs both on the job and off the job. Search is directed in the sense that each worker chooses to search for the offer that provides the optimal tradeoff between the probability of obtaining the offer and the gain from the job. There are both aggregate and match-specific shocks, and contracts are complete. We characterize the equilibrium analytically, and show that the equilibrium is unique and socially efficient. On the quantitative side, we calibrate the model to the US data to measure the effects of aggregate productivity fluctuations on the labor market. We find that productivity fluctuations account for 80% of the cyclical volatility in US unemployment. Moreover, productivity fluctuations generate the same matrix of correlations between unemployment and other labor market variables as in the US. In particular, the Beveridge curve is negatively sloped over business cycles, and the magnitude of the slope is the same as in the data. In light of these findings, we conclude that productivity shocks are one of the main forces driving labor market fluctuations over business cycles. Furthermore, we find that recessions have a cleansing effect on the economy.

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1. Introduction

1.1. Motivation

During the 1969 recession, the US unemployment rate increased from 3.5 to 6.1 percent. In part, this increase was caused by a 20 percent drop in the rate at which unemployed workers became employed (henceforth, the UE rate). In part, it was caused by a 30 percent increase in the rate at which employed workers became employed (henceforth, the EU rate). Similarly, during the 1960, 1973 and 1990 recessions, the unemployment rate increased because of fluctuations in both the UE and the EU transition rates.

During the 2001 recession, the vacancy rate declined by more than 40 percent. Most likely as a consequence of this decline, the rate at which unemployed workers became employed dropped by 20 percent. Moreover, the rate at which employed workers moved from one employer to another (henceforth, the EE rate) dropped by approximately 15 percent. While we do not have data on employer-to-employer transitions prior to 1994, related evidence[1] suggests that, also during earlier recessions, the decline in the vacancy rate caused a drop in the hiring rate among both the employed and the unemployed.

These observations suggest that, in order to understand the cyclical fluctuations of unemployment and vacancies, an economist needs a unified theory of the workers' transitions between unemployment, employment, and across different employers.

1.2. Summary

In this paper, we build a search theoretic model of the labor market in which the workers' transitions between employment, unemployment and across employers are endogenous. We calibrate the model to match the key features about workers' turnover in the postwar US. And, finally, we use the calibrated model to measure the effect that aggregate productivity shocks have on unemployment, vacancies and other labor market variables at the business cycle frequency.

In particular, we consider a labor market populated by ex-ante homogeneous workers—each endowed with one indivisible unit of labor—and ex-ante homogeneous firms—each operating a technology that turns labor into final goods. In this market, trade is the

[1] Using data from the Panel Survey on Income Dynamics, Barlevy (2002) constructs a time series for the rate at which employed workers quit their jobs. He finds that, during both the 1981 and 1990 recessions, the quit rate significantly dropped.

outcome of a search-and-matching process. During the first stage of this process, firms choose how many vacancies to create and how much to offer to the workers who fill them. And workers—both those who are still unemployed and those who are already employed—choose how much to demand for filling one of these vacancies. During the second stage of the process, some of the vacancies and workers who offer and seek the same terms of trade successfully match. And when they do, trade and production begin. We assume that the productivity of a match is the sum of an idiosyncratic and an aggregate component.

For this market, we prove the existence of an equilibrium with the property that the agents' value and policy functions as well as the vacancy/applicant function (i.e. the ratio between the number of vacancies and the number of workers offering and seeking the same terms of trade) depend on the current realization of the aggregate component of productivity, but not on the other aggregate state variables (namely, the distribution of employed workers across different matches and the unemployment rate). Moreover, we prove that this equilibrium is unique and is efficient.

In this equilibrium, the vacancy/applicant ratio is lower the more favorable the terms of trade for the worker. Firms are indifferent between creating different types of vacancies, because the vacancies that offer more generous terms of trade attract more applicants and are easier to fill. Workers, however, have strict preferences over different types of vacancies. Unemployed workers seek vacancies that offer relatively less generous terms of trade but are easier to find. Employed workers seek vacancies that are harder to find but pay more. And workers who are employed at better jobs seek vacancies that offer more generous terms of trade. When a positive shock to productivity hits the economy, the vacancy/applicant ratio goes up. Unemployed workers seek vacancies that are both more generous and easier to find. And employed workers seek vacancies that pay more and, depending on the quality of their current job, may be easier or harder to find.

When a worker and a firm successfully match, they enter an employment relationship which continues until the worker either moves to another employer or to unemployment. The second event occurs if the idiosyncratic component of productivity is so low that the firm's and worker's joint value of the match is lower than the worker's value of unemployment. This event is more likely the lower is the aggregate component of productivity.

We calibrate our model to match the pattern of workers' turnover in the US labor market. In particular, we calibrate the parameters that describe the search-and-matching process so that the model reproduces the average rates at which workers transit from employment to unemployment, from unemployment to employment and from one employer to the other. We calibrate the stochastic process of the idiosyncratic component of productivity so that the model reproduces the cross-sectional distribution of workers across tenure lengths. And we calibrate the stochastic process for the aggregate component of productivity so that the average productivity of labor has the same statistical properties in the model and in the data.

Using the calibrated model, we measure the effect of aggregate productivity shocks on the labor market. We find that productivity shocks account for approximately 50 percent of the cyclical fluctuations in the UE transition rate and for all of the cyclical fluctuations in the EU transition rate. As a result, productivity shocks alone can explain more than 80 percent of the cyclical volatility of unemployment. We find that productivity shocks generate large procyclical fluctuations in the number of vacancies opened for both employed and unemployed workers. Overall, productivity shocks alone can account for 30 percent of the cyclical volatility of vacancies, as well as for the strong negative correlation between vacancies and unemployment. In light of these findings, we conclude that productivity shocks may well be the fundamental source of business cycle fluctuations in the postwar US.

1.3. Related Literature

First, our paper contributes to the literature that uses search theoretic models to measure the role played by productivity shocks in driving the cyclical fluctuations of the labor market. Shimer (2005) calibrates the canonical search model of Pissarides (1985) and finds that aggregate productivity shocks account for less than 10 percent of the cyclical volatility of unemployment and for less than 20 percent of the cyclical volatility of vacancies. Since Pissarides (1985) is a constrained version of our model, we are able to provide a precise explanation for the difference between Shimer's findings and ours. First, we find that, by constraining all matches to be homogeneous, Pissarides' model introduces a downward distortion in the measurement of the volatility of aggregate productivity shocks. Second, by constraining all matches to be homogeneous, Pissarides' model introduces a downward distortion in the measurement of the volatility of the EU rate caused by aggregate productivity shocks. Finally, by constraining all new hires to come from unemployment, Pissarides' model introduces a downward distortion in the measurement of the elasticity of the matching function with respect to vacancies. All three of these distortions tend to reduce the measurement of the volatility of unemployment and vacancies caused by productivity shocks.

The literature has identified some alternative generalizations/modifications of Pissarides (1985) that have the effect of increasing the estimate of the volatility of unemployment and vacancies caused by aggregate productivity shocks. Hall (2005) shows that, if wages were sufficiently sticky, then all of the observed volatility of unemployment and vacancies would be caused by aggregate productivity shocks. Mortensen and Nagypál (2007) show that, if hiring and firing costs were added to Pissarides' model, both unemployment and vacancies would become more responsive to aggregate productivity shocks. Nagypál (2008) shows that, if hiring costs, asymmetric information, and search on the job were added to Pissarides' model, aggregate productivity shocks alone would account for most of the volatility of vacancies and unemployment. While frictions in the wage setting process, hiring costs and firing costs may well be important aspects of the US economy, they are also very difficult to measure and, for this reason, we decided to keep them out of our model.

Next, our paper contains a theoretical contribution to the literature that uses on-the-job search models to understand the workers' transitions between the states of employment, unemployment and across different jobs (Burdett 1978, Burdett and Mortensen 1998, Van den Berg and Ridder 1999, Postel-Vinay and Robin 2002, Burdett and Coles 2003, Mortensen 2003). The scope of this literature has been limited by the fact that existing models of search on the job are difficult to solve in an environment with aggregate shocks. This is because, in these models, the distribution of employed workers across different jobs is a state variable which non-trivially affects the agents' value and policy functions, as well as the vacancy/applicant ratio. In this paper, we develop a model of search on the job with aggregate fluctuations that can be solved analytically because the workers' distribution does not affect the agents' value and policy functions, or the vacancy/applicant ratio. As discussed in Shi (2006) and Menzio and Shi (2008), it is the assumption of directed search that makes our model tractable.

2. The Model

2.1. Physical Environment

The economy is populated by a continuum of workers with measure one and by a continuum of firms with positive measure. Each worker has the von Neumann-Morgenstern utility function $\sum_{t=0}^{\infty} \beta^t c_t$, where $c_t \in \mathbb{R}$ is the worker's consumption in period t and $\beta \in (0,1)$ is the discount rate. Each firm has the von Neumann-Morgenstern utility function $\sum_{t=0}^{\infty} \beta^t \pi_t$, where $\pi_t \in \mathbb{R}$ is the firm's profit in period t. In this economy, the labor market is organized in a continuum of submarkets indexed by $x \in \mathbb{R}$, where x denotes the value offered to a worker in that submarket (explained further below). In submarket x, the ratio between the number of jobs that are vacant and the number of workers who are searching is denoted by $\theta(x) \in \mathbb{R}_+$. We refer to $\theta(x)$ as the *tightness* of submarket x.[2]

Time is discrete and continues forever. At the beginning of each period, the state of the economy can be summarized by the triple $(y, u, g) \equiv \psi \in \Psi$. The first element of ψ denotes the aggregate component of labor productivity, $y \in Y = \{y_1, y_2, ...y_{N_y}\}$, where $N_y \geq 2$. The second element denotes the measure of workers who are unemployed, $u \in [0,1]$. The last element is a function $g : Z \to [0,1]$, with $g(z)$ denoting the measure of workers who are employed at a job with idiosyncratic productivity $z \in Z = \{z_1, z_2, ...z_{N_z}\}$, where $N_z \geq 2$.[3] Clearly, $u + \sum_i g(z_i) = 1$.

Each period is divided into four stages: separation, search, matching and production. During the first stage, an employed worker becomes unemployed with probability $\tau \in [\delta, 1]$, where τ is determined by the worker's labor contract. The lower bound on τ denotes the probability of exogenous job destruction, $\delta \in (0,1)$.

During the second stage, a worker gets the opportunity of searching for a job with a probability that depends on his recent employment history. In particular, if the worker was unemployed at the beginning of the period, he can search with probability $\lambda_u \in [0,1]$. If the worker was employed at the beginning of the period and did not lose his job during the separation stage, he can search with probability $\lambda_e \in [0,1]$. If the worker lost his job during the separation stage, he cannot search. Conditional on being able to search, the worker chooses which submarket to visit. Also, during the second stage, a firm chooses how many vacancies to create and where to locate them. The cost of maintaining a vacancy for one period is $k > 0$. Both workers and firms take the tightness $\theta(x)$ parametrically.[4]

During the third stage, the workers and the vacancies in submarket x come together

[2]In submarkets that are not visited by any workers, $\theta(x)$ is an out-of-equilibrium conjecture that helps determine equilibrium behavior.

[3]The reader should notice that the assumption that Y and Z are finite sets is not necessary for establsihing any of the theoretical results in this paper. We make this assumption only to simplify the notation.

[4]That is, workers and firms treat the tightness $\theta(x)$ just like households and firms treat prices in a Walrasian Equilibrium.

through a frictional matching process. In particular, a worker finds a vacant job with probability $p(\theta(x))$, where $p : \mathbb{R}_+ \to [0,1]$ is a twice continuously differentiable, strictly increasing, strictly concave function which satisfies the boundary conditions $p(0) = 0$, $p(\bar{\theta}) = 1$. Similarly, a vacancy finds a worker with probability $q(\theta(x))$, where $q : \mathbb{R}_+ \to [0,1]$ is a twice continuously differentiable, strictly decreasing function such that $q(\theta) = \theta^{-1}p(\theta)$, $q(0) = 1$, and $\lim_{\theta\to\infty} q(\theta) = 0$. The properties of the functions p and q are meant to capture the realistic feature that, the tighter is the submarket, the higher is the probability that a worker finds a vacancy and the lower is the probability that a vacancy finds a worker.

When a worker meets a firm in submarket x, he is offered an employment contract which gives him the lifetime utility x if he accepts it. If the worker rejects the firm's offer (an event that does not occur along the equilibrium path), he returns to his previous employment position. If the worker accepts the offer, he first leaves his previous employment position to enter his new employment relationship with the firm. Then, the worker and the firm draw the the idiosyncratic productivity $\tilde{z} \in Z$ of their match, where $\tilde{z}$ is a random variable with a density function $f : Z \to [0,1]$.

During the last stage, an unemployed worker consumes b units of output, which include home production and unemployment benefits. A worker employed at a job z produces $y + z$ units of output and consumes w of them, where w is specified by the worker's labor contract. At the end of the last stage, nature draws next period's aggregate productivity $\hat{y}$ from the probability distribution $\phi(\hat{y}|y)$, $\phi : Y \times Y \to [0,1]$.

2.2. Contractual Environment

The literature has considered a variety of assumptions about the contractual environment in models of search on the job. For example, Burdett and Coles (2003), Stevens (2004) and Shi (2006) assume that a labor contract is a wage/tenure profile. Burdett and Mortensen (1998), Delacroix and Shi (2006) and Shimer (2006) assume that a contract is a wage that remains constant throughout the employment relationship. Barlevy (2002), Ramey (2007) and Nagypál (2008) assume that a contract can only prescribe the current wage and is renegotiated in every period. In this paper, we depart from the existing literature, and assume that employment contracts are complete. That is, the contracts prescribe the wage, the separation strategy, and the worker's on-the-job search strategy as a function of the entire history of the match. While the assumption of complete contracts is strong, it is a useful a benchmark that should be studied before considering alternative assumptions.

To specify the contracts, let the history of a match be a vector $\{z; y^t\} \in Z \times Y^t$, where z is the match-specific component of productivity and $y^t = \{y_1, y_2, ... y_t\}$ is the sequence of realizations of the aggregate component of productivity since the inception of the match.[5] An employment contract $\underline{a} \in A^{N_z}$ is an allocation $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$. The first element of $\underline{a}$ denotes the wage as a function of the worker's tenure t and the history of the match $\{z; y^t\}$, where $w_t : Z \times Y^t \to \mathbb{R}$. The second element denotes the separation probability as a function of the tenure t and the history $\{z, y^{t+1}\}$, where $\tau_t : Z \times Y^{t+1} \to [\delta, 1]$. The last element denotes the submarket where the worker searches while on the job as a function of the tenure t and the history $\{z, y^{t+1}\}$, where $n_t : Z \times Y^{t+1} \to \mathbb{R}$.

In the remainder of the paper, we let $\underline{a}(z; y^t) \in A$ denote the allocation prescribed by the employment contract $\underline{a}$ after the history $\{z; y^t\}$ is realized. And we use the fact that $\underline{a}(z; y^t)$ is equal to $\{w_t(z; y^t), \tau_t(z; y^t, \hat{y}), n_t(z; y^t, \hat{y})\} \cup \underline{a}(z; y^t, \hat{y})$.

3. Conditions and Definition of Equilibrium

In this paper, we are interested in recursive equilibria in which the agents' values, optimal decisions, and the market tightness depend on the aggregate state of the economy $\psi = (y, u, g)$ only through y and not through the multi-dimensional distribution of workers across employment states. In such equilibria, we can write the tightness in submarket x as $\theta(x; y)$, instead of $\theta(x; \psi)$, when the aggregate component of productivity is y. Moreover, we can denote $U(y)$ as the lifetime utility of an unemployed worker when the aggregate component of productivity is y. Similarly, $W(z; y|a)$ denotes the lifetime utility of a worker who is employed at a job with idiosyncratic productivity z and whose contract prescribes the allocation a. $J(z; y|a)$ denotes the lifetime profits of the firm that employs him. The lifetime utilities U, W, and J are measured at the beginning of the production stage.

3.1. Worker's Value of Searching

Consider a worker who has received the opportunity to look for a job at the beginning of the search stage. If the worker visits submarket x, he succeeds in finding a job with

[5] In general, a complete contract should specify w, τ, and n as functions of the match-specific component of productivity z and the sequence of realizations of the aggregate state of the economy since the inception of the match, $\psi^t = \{\psi_1, \psi_2, ... \psi_t\}$. However, in this paper, we are interested in equilibria in which the tightness function $\theta(x)$ depends on the aggregate state of the economy $\psi = (y, u, g)$ only through y and not through the entire distribution of workers across employment states. In these equilibria, the history $\{z; y^t\}$ provides enough contingencies for a contract to be efficient.

probability $p(\theta(x;y))$, and he fails with probability $1 - p(\theta(x;y))$. If he succeeds, he enters the production stage in a new employment relationship which gives him the lifetime utility x. If he fails, he enters the production stage in the same employment position that he previously held, which gives him the lifetime utility υ. Therefore, conditional on visiting submarket x, the worker's lifetime utility at the beginning of the search stage is $\upsilon + p(\theta(x;y))\,(x - \upsilon)$. Conditional on choosing x optimally[6], the worker's lifetime utility is $\upsilon + D(\upsilon;y)$, where

$$D(\upsilon;y) = \max_x p(\theta(x;y))(x - \upsilon). \tag{R1}$$

Denote $m(\upsilon;y)$ as the solution for x of the maximization problem in (R1).

3.2. Worker's Value of Unemployment

Consider an unemployed worker at the beginning of the production stage. In the current period, the worker produces and consumes b units of output. In the next period, the worker enters the search stage without a job and has the opportunity to look for one with probability λ_u. Therefore, the worker's lifetime utility $U(y)$ is equal to

$$U(y) = b + \beta\mathbb{E}[U(\hat{y}) + \lambda_u D(U(\hat{y});\hat{y})]. \tag{R2}$$

Throughout this paper, $\mathbb{E}$ denotes the conditional expectation on $\hat{y}$, calculated with the distribution $\phi(\hat{y}|y)$.

3.3. Joint Value of a Match

Consider a matched pair of a firm and a worker at the beginning of the production stage. The history of their match is $\{z, y^t\}$. Let $a = \{w, \tau, n\} \cup \hat{a}$ denote the allocation prescribed by their employment contract after the history $\{z; y^t\}$ has realized.

In the current period, the worker consumes w units of output. During the next separation stage, the worker loses his job with probability τ, and keeps it with probability $1 - \tau$. In the first case, the worker enters the search stage unemployed and does not have the opportunity to look for a new job. In the second case, the worker enters the search stage employed and, with probability λ_e, he has the opportunity to look for an alternative

[6] This qualification is relevant. When the worker is unemployed, he chooses x to maximize his lifetime utility. However, when the worker is employed, he chooses x according to the prescriptions of his labor contract, rather than to maximize his lifetime utility.

job in submarket n. Therefore, the worker's lifetime utility $W(z;y|a)$ is equal to

$$\begin{aligned} W(z;y|a) = \quad & w + \beta\mathbb{E}\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]W(z;\hat{y}|\hat{a}(\hat{y}))\} + \\ & +\beta\mathbb{E}\left\{[1-\tau(\hat{y})]\lambda_e p(\theta(n(\hat{y});\hat{y}))\left[n(\hat{y}) - W(z;\hat{y}|\hat{a}(\hat{y}))\right]\right\}. \end{aligned} \tag{R3}$$

In the current period, the firm's profit is $y+z-w$. During the next separation stage, the firm loses the worker with probability τ. During the next matching stage, the firm loses the worker with probability $(1-\tau)\lambda_e p(\theta(n))$. The probability that the firm keeps the worker until the next production stage is $(1-\tau)\ (1-\lambda_e p(\theta(n)))$. Therefore, the firm's lifetime profits $J(z;y|a)$ are equal to

$$J(z;y|a) = y + z - w + \beta\mathbb{E}\left\{[1-\tau(\hat{y})]\left[1-\lambda_e p(\theta(n(\hat{y});\hat{y}))\right] J(z;\hat{y}|\hat{a}(\hat{y}))\right\}. \tag{R4}$$

Now, consider the hypothetical problem of choosing the allocation a in order to maximize the sum of the worker's lifetime utility and the firm's lifetime profits from the match. As we prove in the appendix, the maximized joint value of the match $V(z;y)$ is

$$\begin{aligned} V(z;y) = \max_{w,\tau,n} \quad & y + z + \beta\mathbb{E}\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]\, V(z;\hat{y})\} + \\ & +\beta\lambda_e\mathbb{E}\{[1-\tau(\hat{y})]\, p(\theta(n(\hat{y});\hat{y}))[n(\hat{y}) - V(z;\hat{y})]\}, \\ & w \in \mathbb{R}, \quad \tau : Y \to [\delta, 1], \quad n : Y \to \mathbb{R}. \end{aligned} \tag{R5}$$

From equation (R5), we can immediately derive the properties of the allocation $a^*(z;y) = \{w_t^*, \tau_t^*, n_t^*\}_{t=0}^{\infty}$ that maximizes the joint value of the match. At the separation stage, $a^*(z;y)$ specifies that the worker and the firm should voluntarily break up if and only if the sum of their values is greater when they are apart than when they are together. That is, $\tau_{t-1}^*(y^t) = 1$ iff $U(y_t)$ is greater than $V(z;y_t) + \lambda_e D(V(z;y_t), y_t)$, and $\tau_t^*(y_t) = \delta$ otherwise. At the search stage, the allocation specifies that the worker should visit the submarket that maximizes the product between the probability of finding a job and the worker's and firm's joint value from finding a job, i.e. $n_{t-1}^*(y^t) = m(V(z;y_t);y_t)$. Finally, since the wage is just a transfer from the firm to the worker and both parties are risk neutral, the allocation may specify any $\{w_t^*\}_{t=0}^{\infty}$. Therefore, the allocation $a^*(z;y)$ may attain any division of the joint value of the match $V(z;y)$ between the firm and the worker.

3.4. Firm's Value of a Meeting

When a firm meets a worker in submarket x, it chooses an employment contract that maximizes its expected profits subject to providing the worker with the lifetime utility x.

Formally, the firm solves the problem

$$\begin{array}{ll} \max_{\underline{a}\in A^{N_z}} & \sum_i J(z_i; y|\underline{a}(z_i))f(z_i), \\ \text{s.t.} & \sum_i W(z_i; y|\underline{a}(z_i))f(z_i) = x. \end{array} \tag{R6}$$

What is the solution to (R6)? First, consider a generic contract $\underline{a}$. Conditional on any realization z of the idiosyncratic component of productivity, the firm's profits $J(z; y|\underline{a}(z))$ cannot be greater than the difference between the maximized joint value of the match, $V(z; y)$, and the worker's lifetime utility, $W(z; y|\underline{a}(z))$. Therefore, if the contract $\underline{a}$ provides the worker with the expected lifetime utility x, the firm's expected profits cannot be greater than $\sum_i V(z_i; y)f(z_i) - x$. Next, consider the contract $\underline{a}^* = \{a^*(z_i; y)\}_i$. Conditional on any realization z of the idiosyncratic component of productivity, the firm's profits $J(z; y|a^*(z; y))$ are equal to the difference between the maximized joint value of the match, $V(z; y)$, and the worker's lifetime utility, $W(z; y|a^*(z; y))$. Therefore, for the appropriate selection of wages, the contract $\underline{a}^*$ provides the worker with the expected lifetime utility x and the firm with the expected profits $\sum_i V(z_i; y)f(z_i) - x$. These observations lead to the following proposition.

Proposition 3.1. *(Optimal Contract) (i) The firm's value from meeting a worker in submarket x is $\sum_i V(z_i; y)f(z_i) - x$. (ii) Any employment contract that solves the firm's problem (R6) prescribes the allocation:* (a) $n_{t-1}(z; y^t) = m(V(z; y_t); y_t)$, *for all* $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$; *(b)* $\tau_{t-1}(z; y^t) = d(z; y_t)$, *for all* $\{z; y^t\} \in Z \times Y^t$, $t = 1, 2, ...$, *where* $d(z; y) = 1$ *iff* $U(y) > V(z; y) + \lambda_e D(V(z; y); y)$ *and* $d^*(z; y) = \delta$ *otherwise.*
Proof. In Appendix B. ■

In the remainder of the paper, we are going to describe the prescriptions of the optimal employment contract with the policy functions $\{d(z; y), m(v; y)\}$, rather than with the sequence $\{\tau_t, n_t\}_{t=0}^{\infty}$.

3.5. Market Tightness

During the search stage, a firm chooses how many vacancies to create and where to locate them. The firm's benefit of creating a vacancy in submarket x is the product between the probability of meeting a worker, $q(\theta(x; y))$, and the value of meeting a worker, $\sum_i V(z_i; y)f(z_i) - x$. The firm's cost of creating a vacancy in submarket x is k. When the benefit is strictly smaller than the cost, the firm's optimal policy is to create no vacancies in x. When the benefit is strictly greater than the cost, the firm's optimal policy is to create infinitely many vacancies in x. And when the benefit and the cost are equal, the firm's profits are independent from the number of vacancies it creates in submarket x.

In any submarket that is visited by a positive number of workers, the tightness $\theta(x;y)$ is consistent with the firm's optimal creation strategy if and only if

$$q(\theta(x;y))\left[\sum_i V(z_i;y)f(z_i) - x\right] \leq k, \tag{R7}$$

and $\theta(x;y) \geq 0$, with complementary slackness. In any submarket that workers do not visit, the tightness $\theta(x;y)$ is consistent with the firm's optimal creation strategy if and only if $q(\theta(x;y)) \cdot [\sum_i V(z_i;y)f(z_i) - x]$ is smaller or equal than k. Following most of the literature on directed search (e.g. Acemoglu and Shimer 1999, Shi 2006, Menzio 2007), we restrict attention to equilibria in which the tightness $\theta(x;y)$ satisfies condition (R7) in all submarkets.[7]

3.6. Laws of Motion

From the optimal policy functions, we can compute the probability that a worker transits from one employment state to the other. First, consider a worker who is unemployed at the beginning of the period. Let $\theta_u(y)$ denote $\theta(m(U(y);y);y)$. Then, at the end of the period, the worker is still unemployed with probability $1 - \lambda_u p(\theta_u(y))$, and he is employed at job of type $\hat{z}$ with probability $\lambda_u p(\theta_u(y))f(\hat{z})$. Next, consider a worker who is employed at a job of type z at the beginning of the period. Let $\theta_z(z;y)$ denote $\theta(m(V(z;y);y);y)$. Then, at the end of the period, the worker is unemployed with probability $d(z;y)$. He is employed at a job of type $\hat{z} \neq z$ with probability $[1 - d(z;y)]\ \lambda_e p(\theta_z(z;y))f(\hat{z})$, and at a job of type z with probability $[1 - d(z;y)]\ \{1 - \lambda_e p(\theta_z(z;y))[1 - f(z)]\}$.

From these transition probabilities, we can compute the laws of motion for the measure of unemployed workers and for the measure of workers employed at each idiosyncratic productivity z. In particular, the measure of workers who are unemployed at the end of the period is:

$$\hat{u} = u(1 - \lambda_u p(\theta_u(y))) + \sum_i d(z_i;y)g(z_i). \tag{R8}$$

[7]This restriction is made without loss in generality. To see why, consider an equilibrium in which submarket x_0 is not visited by any workers and its tightness $\theta(x_0)$ is such that $\theta(x_0) > 0$ and $q(\theta(x_0))[\sum_i V(z_i;y)f(z_i) - x_0] < k$. Then, modify the equilibrium by replacing $\theta(x_0)$ with $\tilde{\theta}(x_0)$, where $\tilde{\theta}(x_0)$ is the tightness of submarket x_0 that satisfies condition (R7). In this modified equilibrium, the workers' search strategy is unchanged because $\tilde{\theta}(x_0)$ is smaller than $\theta(x_0)$. In this modified equilibrium, the firms' creation startegy is unchanged because $q(\tilde{\theta}(x_0))\,[\sum_i V(z_i;y)f(z_i) - x_0]$ is smaller than k.

Similarly, the measure of workers who, at the end of the period, are employed at a job with idiosyncratic productivity z is:

$$\hat{g}(z) = h(\psi)f(z) + (1 - d(z; y))(1 - \lambda_e p(\theta_z(z; y)))g(z). \tag{R9}$$

The function $h(\psi)$ denotes the measure of workers who are hired during the matching stage and is given as follows:

$$h(\psi) = u\lambda_u p(\theta_u(y)) + \textstyle\sum_i (1 - d(z_i; y))\lambda_e p(\theta_z(z_i; y))g(z_i).$$

3.7. Tractable Recursive Equilibrium

The previous paragraphs motivate the following definition of equilibrium.

Definition 1: *A Tractable Recursive Equilibrium (TRE) consists of a market tightness function $\theta^* : \mathbb{R} \times Y \to \mathbb{R}_+$; a search value function $D^* : \mathbb{R} \times Y \to \mathbb{R}$, and policy function $m^* : \mathbb{R} \times Y \to \mathbb{R}$; an unemployment value function $U^* : Y \to \mathbb{R}$; a match value function $V^* : Z \times Y \to \mathbb{R}$; a separation function $d^* : Z \times Y \to \mathbb{R}$; and the laws of motion $\hat{u}^* : \Psi \to [0,1]$, and $\hat{g}^* : Z \times \Psi \to [0,1]$ for unemployment and employment. These functions satisfy the following requirements:*

(i) *For all $x \in \mathbb{R}$ and all $\psi \in \Psi$, θ^* satisfies the functional equation (R7);*

(ii) *For all $V \in \mathbb{R}$ and all $\psi \in \Psi$, D^* satisfies the functional equation (R1), and m^* is the associated optimal policy function;*

(iii) *For all $\psi \in \Psi$, U^* satisfies the functional equation (R2);*

(iv) *For all $z \in Z$ and all $\psi \in \Psi$, V^* satisfies the functional equation (R6), and d^* is the associated optimal policy function;*

(v) *For all $\psi \in \Psi$, $\hat{u}^*$ and $\hat{g}^*$ satisfy the equations (R8) and (R9).*

4. Existence and Efficiency of an Equilibrium

In this section, we prove existence, uniqueness and efficiency of a Tractable Recursive Equilibrium. To this aim, we first formulate the problem of the social planner and characterize its solution. Next, we prove that, if a Tractable Recursive Equilibrium exists, then it generates the same allocation that solves the planner's problem. Moreover, we prove that a TRE can always be built from the solution to the planner's problem. We conclude the section by providing a qualitative characterization of the equilibrium in and out of steady state.

4.1. Social Planner's Problem

At the beginning of the period, the social planner observes the state of the economy $\psi = \{y, u, g\}$. At the separation stage, he chooses the destruction probability $d(z)$ for matches with idiosyncratic productivity z, $d : Z \to [\delta, 1]$. At the search stage, he chooses the tightness θ_u for the submarket where he sends unemployed workers to look for jobs, $\theta_u \in \mathbb{R}_+$, and the tightness $\theta_z(z)$ for the submarket where he sends workers employed on jobs of type z to look for better jobs, $\theta_z : Z \to \mathbb{R}_+$. The choices of d, θ_u and θ_z determine the distribution of workers across employment states at the production stage and, hence, at the beginning of next period. The social planner's objective is to maximize the sum of current and future aggregate consumption discounted at the rate β. Denote the planner's value function as $s^0(\psi)$. The planner's problem is

$$\begin{aligned} & s^0(\psi) = \max\nolimits_{d,\theta_u,\theta_z} F(d, \theta_u, \theta_z|\psi) + \beta \mathbb{E} s^0(\hat{\psi}) \\ \text{s.t.}\quad & \hat{u} = u\left[1 - \lambda_u p(\theta_u)\right] + \textstyle\sum_i d(z_i) g(z_i), \\ & \hat{g}(z) = h(\psi) f(z) + \left[1 - d(z)\right]\left[1 - \lambda_e p(\theta_z(z))\right] g(z), \\ & h(\psi) = \lambda_u p(\theta_u) u + \lambda_e \textstyle\sum_i \left[1 - d(z_i)\right] p(\theta_z(z_i)) g(z_i), \end{aligned} \tag{P1}$$

where F is the current period's aggregate consumption given by

$$F(d, \theta_u, \theta_z|\psi) = \hat{u}b + \textstyle\sum_i (y + z_i)\hat{g}(z_i) - k[\lambda_u u \theta_u + \lambda_e \sum_i (1 - d(z_i)) g(z_i) \theta_z(z_i)].$$

The planner's value function $s^0(\psi)$ is linear in both the measure u of workers who are unemployed and the measure $g(z)$ of workers who are employed at jobs with idiosyncratic productivity z. That is,

$$s^0(\psi) = s_u^0(y) u + \textstyle\sum_i s_z^0(z_i; y) g(z_i). \tag{P2}$$

The coefficient $s_u^0(y)$ can be interpreted as the difference between the present value of output produced by a worker who is currently unemployed and the present value of output invested in creating vacancies for him. Similarly, the coefficient $s_z^0(z; y)$ can be interpreted as the present value of net output produced by a worker who is currently employed at a job of type z. In line with basic economic intuition, the coefficient $s_z^0(z;y)$ is increasing in z. These properties of the planner's value function are established in the following proposition.

Proposition 4.1. *(Social Planner's Problem) (i) The value of the plan $s^0 : \Psi \to \mathbb{R}$ is the unique solution to the functional equation (P1). (ii) There exist functions $s_u^0 : Y \to \mathbb{R}$ and $s_z^0 : Z \times Y \to \mathbb{R}$ such that the value of the plan $s^0(y,u,g)$ is equal to $s_u^0(y)u + \sum_i s_z^0(z_i;y)g(z_i)$. (iii) The function $s_z^0(z_i;y)$ is non-decreasing in z.*
Proof. In Appendix C. ■

The planner's assignment of vacancies to the submarket with unemployed workers is optimal only if

$$k \geq p'(\theta_u)\{y - b + \beta\mathbb{E}[\textstyle\sum_i s_z^0(z_i;\hat{y})f(z_i) - s_u^0(\hat{y})]\} \tag{P3}$$

and $\theta_u \geq 0$, with complementary slackness. This condition is easy to understand. The left hand side of (P3) is the cost of assigning an extra vacancy to the submarket with unemployed workers. The right hand side of (P3) is the expected benefit from such an extra vacancy, given by the product of two terms. The first term, $p'(\theta_u)$, is the number of unemployed workers who find a job because of the extra vacancy. The second term is the difference between the present value of net output produced by an employed and an unemployed worker, measured at the production stage. Notice that, since the left hand side is independent from θ_u and the right hand side is strictly decreasing, the optimality condition (P3) admits a unique solution in each aggregate state ψ. Moreover, since (P3) depends on the aggregate state of the economy only through y, the optimal policy is a function $\theta_u^0 : Y \to \mathbb{R}_+$.

The planner's assignment of vacancies to the submarket with workers who are employed at jobs of type z is optimal only if

$$k \geq p'(\theta_z(z))\{-z + \beta\mathbb{E}[\textstyle\sum_i s_z^0(z_i;\hat{y})f(z_i) - s_z^0(z;\hat{y})]\} \tag{P4}$$

and $\theta_z(z)$, with complementary slackness. The interpretation of the optimality condition (P4) is similar to that of (P3), except that the extra vacancy is assigned to a submarket populated by workers who are employed at jobs with idiosyncratic productivity z rather than unemployed. As it is the case for (P3), the optimality condition (P4) admits a unique solution for $\theta_z(z)$ in each aggregate state ψ. Moreover, since (P4) depends on the aggregate state of the economy ψ only through y, the optimal policy is a function $\theta_z^0 : Z \times Y \to \mathbb{R}_+$.

The planner's choice of the destruction probability for matches with idiosyncratic productivity z is optimal if and only if $d(z) = 1$ whenever

$$\begin{aligned} b + \beta \mathbb{E} s_u^0(\hat{y}) > \quad & -\lambda_e k \theta_z^0(z;y) + \left[1 - \lambda_e p(\theta_z^0(z;y))\right]\left[y + z + \beta \mathbb{E} s_z^0(z;\hat{y})\right] + \\ & + \lambda_e p(\theta_z^0(z;y)) \left\{ y + \beta \mathbb{E}\left[\textstyle\sum_i s_z^0(z_i;\hat{y}) f(z_i)\right]\right\}, \end{aligned} \tag{P5}$$

and $d(z) = \delta$ otherwise. The interpretation of this condition is straightforward. The left hand side of (P5) is the present value of net output produced by a worker who is unemployed at the beginning of the production stage. The right hand side of (P5) is the present value of net output produced by a worker who is employed at a job with idiosyncratic productivity z at the beginning of the search stage. Clearly, the optimality condition (P5) admits only one solution for $d(z)$ in each aggregate state ψ. Moreover, since (P5) depends on the aggregate state of the economy ψ only y, the optimal policy is a function $d^0 : Z \times Y \to [\delta, 1]$.

Finally, the derivative of the social planner's value function with respect to the measure of unemployed workers is:

$$\begin{aligned} s_u^0(y) = \quad & -k \lambda_u \theta_u^0(y) + \left[1 - \lambda_u p(\theta_u^0(y))\right]\left[b + \beta \mathbb{E} s_u^0(\hat{y})\right] + \\ & + \lambda_u p(\theta_u^0(y)) \left\{ y + \beta \mathbb{E}\left[\textstyle\sum_i s_z^0(z_i;y_+) f(z_i)\right]\right\}. \end{aligned} \tag{P6}$$

Similarly, the derivative of the social planner's value function with respect to the measure of workers employed at jobs of type z is:

$$\begin{aligned} s_z^0(z;y) = \quad & d^0(z;y)\left[b + \beta \mathbb{E} s_u^0(\hat{y})\right] - \left[1 - d^0(z;y)\right] k \lambda_e \theta_z^0(z;y) + \\ & + \left[1 - d^0(z;y)\right]\left[1 - \lambda_e p(\theta_z^0(z;y))\right]\left[y + z + \beta \mathbb{E} s_z^0(z;\hat{y})\right] + \\ & + \left[1 - d^0(z;y)\right] \lambda_e p(\theta_z^0(z;y)) \left\{ y + \beta \mathbb{E}\left[\textstyle\sum_i s_z^0(z_i;\hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{P7}$$

4.2. Equilibrium Allocation

Denote with $\{D^*, m^*, U^*, V^*, d^*, \theta^*\}$ a Tractable Recursive Equilibrium. The market tightness function $\theta^*(x;y)$ is derived from the equilibrium condition (R7). In particular, let $\tilde{x}(y)$ denote the difference between the firm's and worker's joint value of a match and the cost of a vacancy, i.e. $\tilde{x}(y) \equiv \sum_i V^*(z_i;y) f(z_i) - k$. In all of the submarkets where workers are offered less than $\tilde{x}(y)$, the equilibrium tightness is strictly positive and such that the firm's benefit from opening a vacancy is equal to the cost. As the lifetime utility offered to the workers approaches $\tilde{x}(y)$, the equilibrium tightness converges towards zero. In all of the submarkets where workers are offered more than $\tilde{x}(y)$, $\theta^*(x;y)$ is equal to zero. Formally, the equilibrium market tightness is:

$$\theta^*(x;y) = \begin{cases} q^{-1}\left(k/(\sum_i V^*(z_i;y)f(z_i) - x)\right) & \text{if } x \leq \tilde{x}(y), \\ 0 & \text{if } x > \tilde{x}(y). \end{cases} \tag{E1}$$

The search policy function $m^*(\upsilon;y)$ satisfies the equilibrium condition (R1). That is, $m^*(\upsilon;y)$ maximizes the product between the worker's probability of finding a job, i.e. $p(\theta^*(x;y))$, and the worker's value of taking the job and leaving his previous employment position, i.e. $x - \upsilon$. Equation (E1) implies that the worker's probability of finding a job is zero in all submarkets $x > \tilde{x}(y)$. Equation (E1) also implies that, in all submarkets $x \leq \tilde{x}(y)$, the worker's value of a job is equal to the difference between the worker's and firm's joint value of a match and the firm's expected cost of creating a match, i.e. $x = \sum_i V^*(z_i;y)f(z_i) - k/q(\theta^*(x;y))$. Therefore, the search policy function is:

$$m^*(\upsilon;y) \in \arg\max_x \{-k\theta^*(x;y) + p(\theta^*(x;y))\left[\sum_i V^*(z_i;y)f(z_i) - \upsilon\right]\}. \tag{E2}$$

In equilibrium, whenever an unemployed worker has the opportunity to search, he visits submarket $m^*(U^*(y);y)$. Let $\theta_u^*(y)$ denote the tightness of this submarket. In equilibrium, whenever a worker employed at a job with idiosyncratic productivity z has the opportunity to search, he visits submarket $m^*(V^*(z;y);y)$. Let $\theta_z^*(z;y)$ denote the tightness of this submarket. From equation (E2), it follows that the tightness $\theta_u^*(y)$ satisfies the condition

$$k \geq p'(\theta_u^*(y))\left[\sum_i V^*(z_i;y)f(z_i) - U^*(y)\right] \tag{E3}$$

and $\theta_u^*(y) \geq 0$, with complementary slackness. Similarly, from equation (E2), it follows that the tightness $\theta^*(z;y)$ satisfies the condition

$$k \geq p'(\theta^*(x;y))\left[\sum_i V^*(z_i;y)f(z_i) - V^*(z;y)\right] \tag{E4}$$

and $\theta_z^*(z;y) \geq 0$, with complementary slackness.

In equilibrium, the lifetime utility of an unemployed worker is $U^*(y)$ at the beginning of the production stage. Let $s_u^*(y)$ denote the lifetime utility of an unemployed worker at the beginning of the separation stage, i.e. $s_u^*(y) = U^*(y) + \lambda_u D(U^*(y);y)$. In equilibrium, the worker's and firm's joint value of a match is $V^*(z;y)$ at the beginning of the production stage. Let $s_z^*(z;y)$ denote the worker's and firm's joint value of a match at the beginning of the separation stage, i.e. $s_z^*(z;y)$ equals the sum between $d^*(z;y)\cdot U^*(z;y)$ and $(1-d^*(z;y))[V^*(z;y)+\lambda_e D^*(V^*(z;y);y)]$. Then, the equilibrium condition (R2) implies that

$$\begin{aligned} s_u^*(y) = \; & -k\lambda_u\theta_u^*(y)+\left[1-\lambda_u p(\theta_u^*(y))\right]\left[b+\beta\mathbb{E}s_u^*(\hat{y})\right]+ \\ & +\lambda_u p(\theta_u^*(y))\left\{y+\beta\mathbb{E}\left[\textstyle\sum_i s_z^*(z_i;\hat{y})f(z_i)\right]\right\}. \end{aligned} \tag{E5}$$

And the equilibrium condition (R5) implies that

$$\begin{aligned} s_z^*(z;y) = \; & d^*(z;y)\left[b+\beta\mathbb{E}s_u^*(\hat{y})\right]-\left[1-d^*(z;y)\right]k\lambda_e\theta_z^*(z;y)+ \\ & +\left[1-d^*(z;y)\right]\left[1-\lambda_e p(\theta_z^*(z;y))\right]\left[y+z+\beta\mathbb{E}s_z^*(z;\hat{y})\right]+ \\ & +\left[1-d^*(z;y))\lambda_e p(\theta_z^*(z;y))\right]\left\{y+\beta\mathbb{E}\left[\textstyle\sum_i s_z^*(z_i;\hat{y})f(z_i)\right]\right\}. \end{aligned} \tag{E6}$$

where $d^*(z;y)$ is equal to 1 if

$$\begin{aligned} b+\beta\mathbb{E}s_u^*(\hat{y}) > \; & -\lambda_e k\theta_z^*(z;y)+\left[1-\lambda_e p(\theta_z^*(z;y))\right]\left[y+z+\beta\mathbb{E}s_z^*(z;\hat{y})\right]+ \\ & +\lambda_e p(\theta_z^*(z;y))\left\{y+\beta\mathbb{E}\left[\textstyle\sum_i s_z^*(z_i;\hat{y})f(z_i)\right]\right\}, \end{aligned} \tag{E7}$$

and $d^*(z;y)=\delta$, otherwise.

At this point, the reader may have recognized that the equilibrium objects $\{d^*,\theta_u^*,\theta_z^*,s_u^*,s_z^*\}$ satisfy the same system of equations that is satisfied by the solution to the social planner's problem $\{d^0,\theta_u^0,\theta_z^0,s_u^0,s_z^0\}$. This system of equations admits only one solution. Therefore, any Tractable Recursive Equilibrium is efficient. Moreover, the equations (E3)–(E7) are not only necessary for a Tractable Recursive Equilibrium, but they are also sufficient. Therefore, an equilibrium can always be constructed from the solution to the social planner's problem. We summarize these findings as the paper's main theoretical result.

Theorem 4.2. *(Existence, Uniqueness and Efficiency) (i) A Tractable Recursive Equilibrium exists. (ii) Let $\{D^*,m^*,U^*,V^*,d^*,\theta^*\}$ be a Tractable Recursive Equilibrium. Let $\theta_u^*(y)$ denote $\theta^*(m^*(U^*(y);y);y)$, and let $\theta_z^*(z;y)$ denote $\theta^*(m^*(V^*(z;y);y);y)$. Then, the equilibrium allocation $\{\theta_u^*,\theta_z^*,d^*\}$ is equal to the social planner's allocation $\{\theta_u^0,\theta_z^0,d^0\}$.*
Proof: In the Appendix D. ■

The efficiency of the equilibrium is an intuitive result. Complete contracts guarantee that, whenever an employed worker has to make a choice, he takes into account the effect of his decision on the profits of his current employer. Moreover, free entry of firms guarantees that, whenever a worker has to choose where to search for a new job, he implicitly takes into account the effect of his decision on the profits of his prospective employer.

A surprising result is the existence of an equilibrium in which the agents' value and policy functions and the market tightness function do not depend on the distribution of workers across employment states. Given the equivalence between the equilibrium allocation and the plan, we can provide some intuition for this result by looking at the social planner's problem.

For example, consider the planner's choice of θ_u. The cost of assigning θ_u vacancies to the submarket visited by unemployed workers is $k\theta_u$. This cost does not depend on the distribution of workers across employment states because the technology for creating vacancies is linear. The probability that an unemployed worker finds a match is $p(\theta_u)$. This probability does not depend on the number of workers who are unemployed because the matching process between vacancies and applicants features constant returns to scale. In addition, this probability does not depend on the number of workers who are in other employment states, because the latter workers visit different submarkets. Finally, the additional output produced by a worker who is employed rather than unemployed is independent from the workers' distribution because the production technology is linear in labor (both at home and in the market). Since the planner's objective function is independent from the distribution of workers across employment states, so are the optimal policy function $\theta_u^0(y)$ and the value function $s_u^0(y)$. The reader should notice that, for the previous argument to hold, it is critical that different workers search in different submarkets. That is, it is critical that search is directed.

4.3. Characterization of Equilibrium

Now, we are in the position to characterize the equilibrium of our model economy. Equation (E3) implies that the tightness of the submarket visited by an unemployed worker is an increasing function of the difference between the value of a new match, i.e. $\sum V^*(z_i; y) f(z_i)$, and the value of unemployment, i.e. $U^*(y)$. Equation (E4) implies that the tightness of the submarket visited by an employed worker is an increasing function of the difference between the value of a new match and the value of his current match. Since the value of a match is increasing in the idiosyncratic component of its productivity, $\theta_z^*(z; y)$ is a decreasing function of z.

Equation (E7) characterizes the workers' transitions from employment to unemployment. In particular, an employed worker becomes unemployed with probability 1 if the value of his match at the beginning of the separation stage is smaller than the value of unemployment. Otherwise, he becomes unemployed with probability δ. Since the value of a match is strictly increasing in the idiosyncratic component of productivity, there exists a $z^{eu}(y)$ such that $d^*(z; y) = 1$ for all $z < z^{eu}(y)$ and $d^*(z; y) = \delta$ for all.$z \geq z^{eu}(y)$.

Even though we are not able to characterize analytically the relationship between $\{d^*, \theta_u^*, \theta_z^*\}$ and y, we can easily compute it. For the parameter values in Table 2, the difference between the value of a match and the value of unemployment is increasing in the aggregate component of productivity. On the one hand, this implies that the tightness of the submarket visited by unemployed workers is an increasing function of y. On the other hand, this implies that the probability that a worker employed at a job of type z is a decreasing function of y.

For the parameter values in Table 2, the difference between the value of a new match and the value of a match with a relatively low idiosyncratic productivity is increasing in y. The difference between the value of a new match and a relatively high productivity match is decreasing in y. Therefore, the effect that a positive shock to aggregate productivity has on the tightness of the submarket visited by an employed worker depends on the quality of his job.

Given the functions $\{d^*, \theta_u^*, \theta_z^*\}$ and an initial state $\psi \in \Psi$, we can study the effect that a 1% increase in the aggregate component of productivity has on unemployment, vacancies, transition rates and other labor market outcomes. In Figure 3, we report the results of this study given that the parameter values are set as in Table 2 and the initial state of the economy is the non-stochastic steady state.

When the economy is hit by the shock, the rate at which unemployed workers become employed increases because they search tighter submarkets. And the rate at which employed workers become unemployed falls because $d^*(z; y)$ is a decreasing function of y. As a result, the unemployment rate declines.

When the economy is first hit by the shock, aggregate vacancies increase because both unemployed and (on average) employed workers search tighter submarkets, while the distribution of workers is the same as in steady state. Over time, the aggregate number of vacancies regresses towards its steady state value because workers progressively move from employment states in which they search tighter submarkets to states in which they search slacker ones (namely, from unemployment to employment, and from low productivity jobs to higher productivity ones).

Finally, notice that, when the economy is hit by the shock, the average idiosyncratic productivity of a job is subject to two opposing forces. On the one hand, the average idiosyncratic productivity tends to increase because workers who are employed at jobs with relatively low z search in tighter submarkets. On the other hand, the average idiosyncratic productivity tends to decrease because $z^{eu}(y)$ is lower. The second force dominates the first one. As a result, the average productivity of labor increases only by 0.6 percent in response to the shocks.

5. Calibration

We begin this section by describing the dataset that we are going to use to calibrate our model. This dataset includes all the information used by Shimer (2005) to calibrate the textbook search model of Pissarides (1985). However, since our model has more parameters than Pissarides', the dataset contains additional information about the job-to-job transition rate and the tenure distribution. In the second part of the section, we describe and motivate the calibration strategy. In particular, we explain why we can recover the distribution of idiosyncratic productivities from the tenure distribution. In the last part of the section, we report the results of the calibration.

5.1. Data

We measure quarterly productivity as the CPS output per worker in the non-farm business sector. And we measure unemployment as a 3-month average of the CPS monthly rate of unemployment in the civilian population. We construct the cyclical component of these two variables as the difference between the log of the raw data and an HP trend (with smoothing parameter 1600). Over the period between 1951(I) and 2006(II), the average of our measure of productivity is 82 (100 being productivity in 1992) and the average of our measure of unemployment is 5.6 percent. Over the same period, the cyclical components of productivity and unemployment move together. However, cyclical unemployment is more than 10 times as volatile as productivity. These and other statistics are reported in Table 1.

We measure the rate at which employed workers become unemployed (the EU rate) as well as the rate at which unemployed workers become employed (the UE rate) using the methodology developed by Shimer (2005). Specifically, we measure the EU rate in month t as $h_t^{eu} = u_{t+1}^s/(1 - u_t)$, where u_{t+1}^s is the CPS short-term unemployment rate[8] in month $t + 1$, and u_t is the CPS unemployment rate. We measure the UE rate in month t as $h_t^{ue} = 1 - (u_{t+1} - u_{t+1}^s)/u_t$. Then, we construct the quarterly transition rates by taking 3-month averages of h_t^{eu} and h_t^{ue}. Over the period between 1951(I) and 2006(II), the average EU rate is 2.6 percent, and the average UE rate is 45 percent. Over this period, the cyclical component of the EU rate is positively correlated with cyclical unemployment and it is approximately 60 percent as volatile. The cyclical component of the UE rate is negatively correlated with unemployment and it is approximately 65 percent as volatile.

The rate at which workers move from employer to employer is measured by Nagypál (2008) from the CPS microdata. Specifically, she measures the EE rate in month t as $h_t^{ee} = f_t^{ee}/e_t$, where f_t^{ee} is the number of workers who are employed at different firms in months t and $t + 1$, and e_t is the number of workers who are employed in month t. Over the period between 1994(I) and 2006(II), the average EE rate is 2.9 percent. Over the same period, the cyclical component of the EE rate is negatively correlated with cyclical unemployment and it is approximately 30 percent as volatile. Prior to 1994, Nagypál's measure of the EE rate cannot be constructed because the CPS did not collect data on job-to-job transitions.

We measure vacancies with the Conference Board Help-Wanted Index. Over the period 1951(I)-2006(II), the contemporaneous correlation between cyclical vacancies and cyclical unemployment is -.92. Over the same period, the standard deviation of cyclical vacancies is 10 percent higher than the standard deviation of cyclical unemployment.

Finally, in order to calibrate the probability distribution of the match-specific component of productivity, we use information about the duration of employment relationships in the US labor market. In particular, we use the measure of the distribution of workers across tenure lengths that Diebold, Neumark and Polsky (1997) have constructed from the 1987 CPS tenure supplement. This tenure distribution is plotted in Figure 1.

[8] The CPS defines the short-term unemployment rate as the ratio between the number of civilians who have been unemployed for 0 to 4 weeks and the civilian labor force. However, with the 1994 redesign of the CPS, there has been a change in the measurement of the duration of unemployment. As discussed in Elsby, Michaels and Solon (2007), the change in the measurement can be corrected by multiplying the official short-term unemployment by 1.15 in each month from February 1994 on.

5.2. Calibration Strategy

With the data described in the previous paragraphs, we need to calibrate the household's preferences $\{b, \beta\}$, the search technology $\{\lambda_u, \lambda_e, p, \delta\}$, and the production technology $\{k, Z, f, Y, \phi\}$. For the sake of simplicity, we restrict attention to job finding probability functions of the form $p(\theta) = \min\{1, \theta^\gamma\}$, $\gamma \in (0, 1)$. We also restrict the distribution of the idiosyncratic component of productivity to be a 1,000 point approximation of a Weibull distribution with mean μ_z scale σ_z, and shape α_z.[9] And we restrict the stochastic process for the aggregate component of productivity to be a 3-state Markov process with unconditional mean μ_y standard deviation σ_y, and autocorrelation ρ_y. Without loss of generality, we can normalize μ_y to 1 and μ_z to 0.

We choose one month as the length of a model period. We set β so that the annual interest rate in the model is 5 percent. We set the vacancy cost k, the home productivity b, and the search probability λ_e so that steady-state UE, EU and EE rates in the model are equal to the corresponding average values in the data (see Table 1). We set the search probability λ_u to 1 because it is difficult to identify it separately from k and λ_e.

Our strategy for calibrating the remaining parameters is less standard and deserves some discussion. In the model, the parameter γ determines the elasticity of the UE rate with respect to the tightness of the submarket visited by unemployed workers, θ_u. Moreover, since a disproportionate number of vacancies are created in this submarket, the parameter γ is positively correlated with the elasticity of the UE rate with respect to the ratio between *total* vacancies and unemployment. Therefore, even without data on θ_u, we are able to identify γ from the coefficient of $\log(v/u)$ in the regression of $\log h^{ue}$.

In the model, the parameters α_z and δ affect the shape of the hazard/tenure profile, i.e. the probability that a worker leaves his job as a function of tenure. A higher α_z reduces the skewness of the probability distribution of the match-specific component of productivity. In turn, this tends to reduce the hazard rate at short tenures (0 to 2 years) and to increase it at medium tenures (2 to 4 years). In contrast, a higher δ increases the hazard rate at all tenures, including long ones (more than 4 years). Therefore, we are able to identify

[9] The Weibull density function is:

$$f(z) = \frac{\alpha_z}{\sigma_z}\left(\frac{z-\mu_z}{\sigma_z}\right)^{\alpha_z - 1} \exp\left[-\left(\frac{z-\mu_z}{\sigma_z}\right)^{\alpha_z}\right].$$

both α_z and δ by minimizing the distance between the tenure distribution generated by the model and its empirical counterpart.

After re-calibrating the other parameters, an increase in the variance of the distribution of the idiosyncratic component of productivity leads to a drop in the ratio between the average productivity of labor at home and in the market, i.e. $b/(\mu_y+\sum_i z_i g(z_i))$. Therefore, we choose σ_z so that the model generates the same ratio of productivities that Hall and Milgrom (2008) estimate from US data (namely, 71 percent[10]). Finally, we choose σ_y and ρ_y so that the average productivity of labor has the same standard deviation and autocorrelation in the model and in the data.

5.3. Calibration Outcomes

Column a in Table 2 contains the results of our calibration. Most notably, we find that employed workers have the opportunity of searching the labor market nearly as often as unemployed workers ($\lambda_e = 0.81$, $\lambda_u = 1$). Yet, the rate at which employed workers move from one employer to the other is 20 times smaller than the rate at which unemployed workers become employed because the latter seek jobs that offer less generous terms of trade and are easier to find.

We also find that there is a great deal of uncertainty about the productivity of a new match. At the ninetieth percentile of the probability distribution $f(z)$, the productivity of a match is twice as large as at the tenth percentile. However, because the survival probability of a match is endogenous, not all of this uncertainty translates into dispersion in the cross-sectional productivity distribution $g(z)$. At the ninetieth percentile of $g(z)$, the productivity of a match is only 1.3 times as large as at the tenth percentile. This process of endogenous selection also creates a large wedge between the expected productivity of a new match and the average of the cross-sectional productivity distribution. In particular, the expected productivity of a new match, $\mu_y+\sum z_i f(z_i)$, is equal to 1, while the cross-sectional average productivity of a match, $\mu_y + \sum z_i g(z_i)$, is 1.35.

[10]If we were to target a higher ratio between home and market productivity (as advocated by Hagedorn and Manovskii, 2008), the model would generate an even larger response of vacancies and unemployment to aggregate productivity shocks.

6. Business Cycle Analysis

6.1. Aggregate Productivity Shocks

What is the effect of aggregate productivity shocks (henceforth, y-shocks) on the US labor market? In order to answer this question, we compute the Tractable Recursive Equilibrium of our calibrated model. Then, we draw a realization of the stochastic process for the aggregate component of productivity y, and we compute the time series of unemployment, vacancies and other labor market variables. Finally, we pass the log of these series through an HP-filter with smoothing parameter 1600.

Table 3 contains a statistical summary of our simulated data. The first lesson that we draw from this table is that y-shocks generate fluctuations in the EU transition rate that are negatively correlated with the fluctuations in the average productivity of labor and are approximately 8 times as large. In addition, y-shocks generate fluctuations in the UE transition rate that are positively correlated with average productivity fluctuations and are 3 times as large. As a result, unemployment moves in the opposite direction of average productivity and it is 10 times more volatile. The second lesson that we draw from Table 3 is that y-shocks generate vacancy fluctuations that are almost perfectly negatively correlated with unemployment fluctuations and 30 percent larger.

By comparing Tables 1 and 3, we find that aggregate productivity shocks alone generate more than 80 percent of the unemployment volatility that is observed in the US economy over the period 1951(I) - 2006(II). Moreover, aggregate productivity shocks generate the same correlation matrix between unemployment, vacancies and worker's transition rates that is observed in the postwar US. In light of these findings, we conclude that aggregate productivity shocks may well be the fundamental source of business cycle fluctuations in the US.

However, aggregate productivity shocks cannot be the only cause of the US business cycles. First of all, y-shocks alone generate a counterfactually strong correlation between average labor productivity and other labor market variables (e.g. unemployment, vacancies, etc.). Second, y-shocks generate only 40 percent of the observed volatility of vacancies. Finally, they generate too much unemployment volatility through fluctuations in the EU rate and too little of it through fluctuations in the UE rate.

6.2. Aggregate Productivity Shocks in the Canonical Search Model

The canonical search model, as formulated by Pissarides (1985, 2000) or Shimer (2005), is a version of our model in which matches are homogeneous and workers search only off the job. That is, the canonical search model is a version of our model in which the parameters λ_e and σ_z are constrained to be equal to zero. These constraints are rejected by the data, as shown in Section 5.3. Here, we want to find out whether (and why) these constraints distort the measurement of the effect that aggregate productivity shocks have on the US labor market.

To this aim, we first calibrate the constrained version of our model using the same targets that we used in Section 5.2, with the obvious exclusion of the EE transition rate and the tenure distribution. The results of this calibration are reported as column b in Table 2. Then, with these calibrated parameters, we compute the Tractable Recursive Equilibrium of the model. Finally, we draw a realization for the stochastic process of y and compute the time series for unemployment, vacancies and other labor market variables. The results of this simulation are reported in Table 4.

According to the constrained version of our model, y-shocks generate fluctuations in the EU transition rate that are negatively correlated with the fluctuations in the average productivity of labor and are 70 percent as large. Also, y-shocks do not generate any fluctuations in the UE transition rate. As a result, unemployment moves in the opposite direction of average productivity and it is about 60 percent as volatile. Comparing these and other findings from Table 4 with those in Table 3, we conclude that imposing the constraints $\lambda_e = \sigma_z = 0$ to our model (i.e. using the canonical search model) distorts downward the measures of the volatility of unemployment, vacancies and transition rate that is caused by aggregate productivity shocks.

It is easy to explain the difference between the measurements generated by the canonical model and ours. First, in our model, when a positive shock to the aggregate component of productivity hits the economy, the EU transition rate declines because workers and firms find it optimal to keep some of the low quality matches that previously they would have destroyed. In the canonical search model, a positive shock to aggregate productivity has no effect on the EU transition rate because all matches are identical.

Second, in our model, when a +1% shock to the aggregate component of productivity hits the economy, the average productivity of labor increases only by 0.6% because workers and firms become less selective about the quality of the matches that they want to keep. In the canonical model, a 1% increase in the aggregate component of productivity translates into a 1% increase in average productivity because all matches are identical. Since both models are calibrated to match the empirical volatility of average productivity, the magnitude of y-shocks is 40 percent smaller in the canonical model than in ours. In turn, smaller y-shocks generate smaller fluctuations in unemployment, vacancies and transition rates.

Third, in both models, vacancies v are equal to $u\theta_u + (1-u)\lambda_e\theta_e$, where θ_u is the tightness of the submarket visited by unemployed workers and θ_e is the average tightness of the submarkets visited by employed workers. In our model, when the economy is hit by a positive shock to productivity, both θ_u and θ_e increase and contribute to increase vacancies. In the canonical search model, when the economy is hit by a positive y-shock, θ_e does not contribute to increase vacancies because employed workers are not allowed to search.

Fourth, the effect of productivity shocks in the two models differs because the calibrated elasticity of the job-finding probability is different. That is, the two models have different values of the parameter γ in the job-finding probability function $p(\theta) = \min\{\theta^\gamma, 1\}$. In both models, the calibrated value of γ is such that the elasticity of the UE rate with respect to the vacancy/unemployment ratio is the same in the model as in the data, namely 0.22. Therefore, in both models, the calibrated value of γ is equal to $0.22 \cdot [\Delta \log(v/u)/\Delta \log \theta_u]$. In our model, because the number of vacancies created for employed workers moves together with θ_u, $\Delta \log(v/u)$ is greater than $\Delta \log \theta_u$. As a result, the calibrated value of γ is 0.65. In the canonical model, because workers are not allowed to search on the job, v/u is equal to θ_u and so γ is equal to 0.22. In turn, a smaller γ implies that the UE rate is less responsive to a given shock to the aggregate component of productivity.

7. Conclusions

In the first part of this paper, we have built a directed search model of the labor market in which the workers' transitions between employment, unemployment and across employers are endogenous. For this model, we have proved the existence, uniqueness and efficiency of a recursive equilibrium with the property that the distribution of workers across different jobs is a state variable which does not affect the agents' value and policy functions, or the tightness function. Because of these properties, we have been able to analytically characterize the equilibrium allocation in and out of steady state.

In the second paper of this paper, we have calibrated our model to match the features of workers' turnover in the US labor market over the period 1951(I)-2006(II). Then, we have used the calibrated model to measure the effect of aggregate productivity shocks on the volatility of unemployment and vacancies. We have found that aggregate productivity shocks alone account for approximately 50 percent of the cyclical fluctuations in the UE transition rate and for all of the cyclical fluctuations in the EU transition rate. As a result, productivity shocks alone can explain more than 80 percent of the cyclical volatility of unemployment. We have found that productivity shocks generate large procyclical fluctuations in the vacancy/worker ratio of the submarket visited by unemployed workers and in the average vacancy/worker ratio of the submarkets visited by employed workers. Overall, productivity shocks alone can account for 30 percent of the cyclical volatility of vacancies, as well as for the strong negative correlation between vacancies and unemployment.

A comparison of these findings with those derived using the textbook search model of Pissarides (1985) vindicates our initial conjecture. In order to properly measure the effect of productivity shocks on unemployment, an economist needs a model that endogenizes not only the rate at which unemployed workers become employed, but also the rate at which employed workers become unemployed. And in order to properly measure the effect of productivity shocks on vacancies, an economist needs a model that takes into account not only the hiring of unemployed workers, but also the hiring of employed workers.

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Appendix

A. Joint Value of a Match

The definition of $V(z;y)$ is

$$V(z;y) = \max_{a\in A}[W(z;y|a) + J(z;y|a)]. \tag{A1}$$

First, notice that the allocation $a = \{w,\tau,n\}\cup\hat{a}$ belongs to the set A if and only if $w\in\mathbb{R}$, $\tau : Y \to [\delta,1]$, $n : Y \to \mathbb{R}$, and $\hat{a} : Y \to A$. Second, notice that the worker's lifetime utility $W(z;y|a)$ is equal to the RHS of equation (R2) and the firm's lifetime profits $J(z;y|a)$ are equal to the RHS of equation (R3). In light of these observations, we can rewrite (A1) as

$$\begin{aligned} V(z;y) = \quad & \max_{w,\tau,n,\hat{a}} \; y + z + \beta\mathbb{E}\left\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]\lambda_e p(\theta(n(\hat{y});\hat{y}))n(\hat{y})\right\} + \\ & +\beta\mathbb{E}\left\{[1-\tau(\hat{y})][1-\lambda_e p(\theta(n(\hat{y});\hat{y}))]\left[J(z;\hat{y}|\hat{a}(\hat{y})) + W(z;\hat{y}|\hat{a}(\hat{y}))\right]\right\}, \\ & w\in\mathbb{R},\; \tau : Y \to [\delta,1],\; n : Y \to \mathbb{R},\; \hat{a} : Y \to A. \end{aligned} \tag{A2}$$

Now, notice that both the probability that the match survives during the separation stage, i.e. $1-\tau(\hat{y})$, and the probability that the match survives during the search stage, i.e. $1-\lambda_e p(\theta(n(\hat{y});\hat{y}))$, are non negative numbers. In light of this observation, we can rewrite (A2) as

$$\begin{aligned} V(z;y) = \quad & \max_{w,d,n} y + z + \beta\mathbb{E}\left\{\tau(\hat{y})U(\hat{y}) + [1-\tau(\hat{y})]\lambda_e p(\theta(n(\hat{y});\hat{y}))n(\hat{y})\right\} + \\ & +\beta\mathbb{E}\{[1-\tau(\hat{y})][1-\lambda_e p(\theta(n(\hat{y});\hat{y}))] \max_{\hat{a}\in A}\left[J(z;\hat{y}|\hat{a}) + W(z;\hat{y}|\hat{a})\right]\}, \\ & w\in\mathbb{R},\; \tau : Y \to [\delta,1],\; n : Y \to \mathbb{R}. \end{aligned} \tag{A3}$$

Finally, notice that the maximum of the sum between the worker's continuation utility $W(z;\hat{y}|\hat{a})$ and the firm's continuation profits $J(z;\hat{y}|\hat{a})$ is equal to $V(z;\hat{y})$. Therefore, (A3) is equal to equation (R5) in the main text. ■

B. Proof of Proposition 3.1

Let the contract $\underline{a}$ be a feasible choice for the firm's problem (R6). First, notice that, for any realization z_i of the idiosyncratic component of productivity, the contract $\underline{a}$ prescribes an allocation $\underline{a}(\mathrm{z}_i)$ which may not necessarily maximize the joint value of the match, i.e. $W(z_i;y|\underline{a}(z_i)) + J(z_i;y|\underline{a}(z_i))$ is smaller than or equal to $V(z_i;y)$. Second, notice that, since $\underline{a}$ is feasible, it provides the worker with the lifetime utility x, i.e. $\sum_i W(z_i;y|\underline{a}(z_i))f(z_i) = x$. In light of these observations, it follows that the contract $\underline{a}$ provides the firm with the following profits:

$$\begin{aligned} \textstyle\sum_i J(z_i;y|\underline{a}(z_i))f(z_i) \quad & \le \textstyle\sum_i V(z_i;y)f(z_i) - \sum_i W(z_i;y|\underline{a}(z_i))f(z_i) = \\ & = \textstyle\sum_i V(z_i;y)f(z_i) - x. \end{aligned} \tag{A4}$$

Let $\underline{a}^*$ denote the contract $\{w_t^*, \tau_t^*, n_t^*\}_{t=0}^{\infty}$ that has the following properties: (a) $\tau_{t-1}^*(z;y^t) = 1$ iff $U(y_t) > V(z;y_t) + \lambda_e D(V(z;y_t);y_t)$ and $\tau_{t-1}^*(z;y^t) = \delta$ otherwise, for all $\{z;y^t\} \in Z \times Y^t$, $t = 1,2,...$; (b) $n_{t-1}^*(z;y^t) = m(V(z;y_t);y_t)$, for all $\{z;y^t\} \in Z \times Y^t$, $t = 1,2,...$; (c) $w_t^*(z;y^t)$ is such that $\sum_i W(z_i;y|\underline{a}^*(z_i))f(z_i) = x$. First, notice that, for any realization z_i of the idiosyncratic component of productivity, the contract $\underline{a}^*$ prescribes an allocation $\underline{a}^*(\mathrm{z}_i)$ which maximizes the joint value of the match. Second, notice that $\underline{a}^*$ provides the worker with the lifetime utility x. In light of these two observations, it follows that the contract $\underline{a}^*$ provides the firm with the following profits:

$$\begin{aligned}\textstyle\sum_i J(z_i;y|\underline{a}^*(z_i))f(z_i) &= \textstyle\sum_i V(z_i;y)f(z_i) - \sum_i W(z_i;y|\underline{a}^*(z_i))f(z_i) = \\ &= \textstyle\sum_i V(z_i;y)f(z_i) - x.\end{aligned} \tag{A5}$$

The contract $\underline{a}^*$ is a feasible choice for the firm's problem (R6), and it provides the firm with more profits than any other feasible choice. Hence, it is optimal.

Finally, the reader can easily verify that, if a contract $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$ solves the firm's problem (R6), then it maximizes the joint value of the match. Hence, the contract $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$ prescribes that (a) $\tau_{t-1}(z;y^t) = 1$ iff $U(y_t) > V(z;y_t) + \lambda_e D(V(z;y_t);y_t)$ and $\tau_{t-1}(z;y^t) = \delta$ otherwise, for all $\{z;y^t\} \in Z \times Y^t$, $t = 1,2,...$; (b) $n_{t-1}(z;y^t) = m(V(z;y_t);y_t)$, for all $\{z;y^t\} \in Z \times Y^t$, $t = 1,2,...$ ■

C. Proof of Proposition 4.1

(i) Let Ψ denote the set $Y \times [0,1]^{N(z)+1}$. Let $C(\Psi)$ denote the set of bounded continuous functions $r : \Psi \to \mathbb{R}$, with the sup norm. Define the operator T on $C(\Psi)$ by

$$\begin{aligned} &(Tr)(\psi) = \max\nolimits_{d,\theta_u,\theta_z} F(d,\theta_u,\theta_d|\psi) + \beta\mathbb{E}\left[r(\hat{\psi})\right] \\ \text{s.t.}\quad &\hat{u} = u\left[1 - \lambda_u p(\theta_u)\right] + \textstyle\sum_i d(z_i)g(z_i), \\ &\hat{g}(z) = h(\psi)f(z) + \left[1 - d(z)\right]\left[1 - \lambda_e p(\theta_z(z))\right]g(z), \\ &d : Z \to [\delta, 1],\ \theta_u \in [0,\bar{\theta}],\ \theta_z : Z \to \left[0,\bar{\theta}\right]. \end{aligned} \tag{A6}$$

For each $r \in C(\Psi)$ and $\psi \in \Psi$, the problem in (A6) is to maximize a continuous function over a compact set. Hence the maximum is attained and the argmax is non-empty. Since both F and r are bounded, Tr is also bounded; and since F and r are continuous, it follows from the Theorem of the Maximum (see Stokey, Lucas and Prescott 1989, page 62) that Tr is also continuous. Hence, the operator T maps $C(\Psi)$ into itself.

Since the operator T satisfies the remaining hypotheses of Blackwell's sufficient conditions for a contraction (see Stokey, Lucas and Prescott 1989, page 54), it follows that T has a unique fixed point $\tilde{s} \in C(\Psi)$. And since $\lim_{t\to\infty} \beta^t \tilde{s}(\psi) = 0$ for all $\psi \in \Psi$, it follows that the fixed point $\tilde{s}$ is equal to the value of the plan s^0.

(ii) Let $L(\Psi)$ denote the set of bounded continuous functions $r : \Psi \to \mathbb{R}$ that are linear in the measure u of unemployed workers as well as in the measure $g(z)$ of workers employed at jobs with idiosyncratic productivity z, i.e.

$$r(\psi) = r_u(y)u + \sum_i r_z(z_i; y)g(z_i).$$

Given a function r in $L(\Psi)$, consider the problem (A6). For each $\psi \in \Psi$, the necessary condition for the optimality of θ_u is:

$$k \geq p'(\theta_u)\{y - b + \beta\mathbb{E}[\sum_i r_z(z_i; \hat{y})f(z_i) - r_u(\hat{y})]\} \tag{A7}$$

and $\theta_u \geq 0$, with complementary slackness. Since the function $p'(\theta)$ is strictly decreasing in θ, there is at most one θ_u that satisfies condition (A7). Hence the optimum is unique. Since (A7) depends on ψ only through y, the optimal policy is a function $\tilde{\theta}_u : Y \to [0, \bar{\theta}]$.

For each $\psi \in \Psi$, the necessary condition for the optimality of $\theta_z(z)$ is:

$$k \geq p'(\theta_z(z))\{-z + \beta\mathbb{E}[\sum_i r_z(z_i; \hat{y})f(z_i) - r_z(z; \hat{y})]\} \tag{A8}$$

and $\theta_z(z) \geq 0$, with complementary slackness. Since $p'(\theta)$ is strictly decreasing in θ, there is at most one $\theta_z(z)$ that satisfies condition (A8). Hence the optimum is unique. Since (A8) depends on ψ only through y, the optimal policy is a function $\tilde{\theta}_z : Z \times Y \to [0, \bar{\theta}]$.

For each $\psi \in \Psi$, the necessary and sufficient condition for the optimality of d is $d(z) = 1$ if

$$\begin{aligned} b + \beta\mathbb{E}[r_u(\hat{y})] > \quad & -\lambda_e k\theta_z(z) + [1 - \lambda_e p(\theta_z(z))]\,[y + z + \beta\mathbb{E}r_z(z; \hat{y})] + \\ & +\lambda_e p(\theta_z(z))\,\{y + \beta\mathbb{E}\,[\textstyle\sum_i r_z(z_i; \hat{y})f(z_i)]\}, \end{aligned} \tag{A9}$$

and $d(z) = \delta$ otherwise. Since (A9) does not depend on d, there is exactly one d that satisfies condition (A8). Since (A9) depends on ψ only through y, the optimal policy is a function $\tilde{d} : Z \times Y \to [\delta, 1]$.

Define the function $\tilde{r}_u : Y \to \mathbb{R}$ by

$$\begin{aligned} \tilde{r}_u(y) = \quad & -k\lambda_u\tilde{\theta}_u(y) + \left[1 - \lambda_u p(\tilde{\theta}_u(y))\right][b + \beta\mathbb{E}r_u(\hat{y})] + \\ & +\lambda_u p(\tilde{\theta}_u(y))\,\{y + \beta\mathbb{E}\,[\textstyle\sum_i r_z(z_i; \hat{y})f(z_i)]\}. \end{aligned} \tag{A10}$$

And define the function $\tilde{r}_z : Z \times Y \to \mathbb{R}$ by

$$\begin{aligned} \tilde{r}_z(z; y) = \quad & \tilde{d}(z; y)\,[b + \beta\mathbb{E}r_u(\hat{y})] - [1 - \tilde{d}(z; y)]k\lambda_e\tilde{\theta}_z(z; y) + \\ & +[1 - \tilde{d}(z; y)]\left[1 - \lambda_e p(\tilde{\theta}_z(z; y))\right][y + z + \beta\mathbb{E}r_z(z; \hat{y})] + \\ & +[1 - \tilde{d}(z; y)]\lambda_e p(\tilde{\theta}_z(z; y))\,\{y + \beta\mathbb{E}\,[\textstyle\sum_i r_z(z_i; \hat{y})f(z_i)]\}. \end{aligned} \tag{A11}$$

It is then immediate that

$$(Tr)(\psi) = \tilde{r}_u(y)u + \sum_i \tilde{r}_z(z_i; y)g(z_i).$$

Hence, the operator T maps $L(\Psi)$ into itself. Since $L(\Psi)$ is a closed subset of $C(\Psi)$, it follows that the fixed point s^0 of the operator T belongs to $L(\Psi)$ (see Stokey, Lucas and Prescott 1989, page 52).

(iii) Let $M(\Psi)$ denote the set of functions $r:\Psi\to\mathbb{R}$ such that $r\in L(\Psi)$ and $r_z: Z\times Y\to\mathbb{R}$ is non decreasing in z. Given a function $r\in M(\Psi)$, let $\tilde{r}$ denote Tr. As we proved in part (ii), the function $\tilde{r}$ belongs to the set $L(\Psi)$. Also as we proved in part (ii), the derivative $\tilde{r}_z(z;y)$ is equal to (A10). Using the optimality conditions (A7)–(A9), we can rewrite (A10) as

$$\begin{aligned}\tilde{r}_z(z,y) = \;& b+\beta\mathbb{E}r_u(y_+)+\max\nolimits_{d\in[\delta,1]}\{(1-d)[y+z-b+\beta\mathbb{E}[r_z(z,\hat{y})-r_u(\hat{y})]]\\ &+(1-d)\lambda_e\max\nolimits_{\theta\in\mathbb{R}_+}[-k\theta+p(\theta)[-z+\beta\mathbb{E}[\textstyle\sum_i r_z(z,\hat{y})f(z_i)-r_z(z,\hat{y})]]]\}.\end{aligned}$$

Since $r_z(z;y)$ is non decreasing in z, it follows that $\tilde{r}_z(z_2;y)\geq\tilde{r}_z(z_1;y)$ for all $z_2\geq z_1$. Hence, the operator T maps the set $M(\Psi)$ into itself. Since $M(\Psi)$ is a closed subset of $L(\Psi)$, it follows that the fixed point s^0 belongs to $M(\Psi)$ as well. ■

D. Proof of Theorem 4.2

(i) We want to prove that a Tractable Recursive Equilibrium exists. To this aim, we first construct a supposed equilibrium $\{D^*,m^*,U^*,V^*,d^*,\theta^*\}$ from the solution to the social planner's problem. Then, we verify that the putative equilibrium satisfies conditions (i)–(iv) in Definition 1.

In the supposed equilibrium, the worker's value from unemployment $U^*(y)$ is set equal to $b+\beta\mathbb{E}s^0_u(\hat{y})$, where s^0_u is the derivative of the social planner's value function s^0 with respect to the unemployment rate. The firm's and worker's joint value from a match $V^*(z;y)$ is set equal to $y+z+\beta\mathbb{E}s^0_z(z;\hat{y})$, where s^0_z is the derivative of the social planner's value function with respect to $g(z)$. The market tightness function $\theta^*(x;y)$ is set equal to $q^{-1}(k/(\sum_i V^*(z_i;y)f(z_i)-x))$ for all $x\leq\tilde{x}(y)$; and $\theta^*(x;y)$ is set equal to zero for all $x>\tilde{x}(y)$. Finally, the worker's search value function $D^*(\upsilon;y)$ and policy function $m^*(\upsilon;y)$ are set equal to the maximum and the maximizer of $p(\theta^*(x;y))\,(x-\upsilon)$.

By construction, the market tightness function θ^* satisfies the equilibrium condition (i). Also by construction, the worker's search value D^* and policy m^* satisfy the equilibrium condition (ii). As proved in the main text, whenever conditions (i) and (ii) are satisfied, we have that

$$m^*(\upsilon;y)\in\arg\max\nolimits_x\{-k\theta^*(x;y)+p(\theta^*(x;y))[\textstyle\sum_i V^*(z_i;y)f(z_i)-\upsilon]\},\tag{A12}$$

and $D^*(\upsilon;y)$ is the maximum of the problem in (A12). Hence the tightness $\theta^*_u(y)$ of the submarket visited by unemployed workers satisfies the optimality condition (E3); and the tightness $\theta^*_z(z;y)$ of the submarket visited by employed workers satisfies the optimality condition (E4). Since $U^*(y)$ is equal to $b+\beta\mathbb{E}s^0_u(\hat{y})$ and $V^*(z;y)$ is equal to $y+z+\beta\mathbb{E}s^0_z(z;\hat{y})$, the tightness $\theta^*_u(y)$ also satisfies the necessary condition (P3) for the optimality of the solution to the social planner's problem. Since (P3) admits only one solution, $\theta^*_u(y)$ is equal to $\theta^0_u(y)$. Similarly, we can prove that $\theta^*_z(z;y)$ is equal to $\theta^0_z(z;y)$ and that $d^*(z;y)$ is equal to $d^0(z;y)$.

Since $\theta_u^0(y)$ is equal to $\theta_u^*(y)$, the envelope condition (P6) can be written as

$$s_u^0(u) = U^*(y) + \lambda_u D^*(U^*(y); y). \tag{A13}$$

In turn, (A13) implies that $U^*(y)$ is equal to

$$U^*(y) = b + \beta \mathbb{E} s_u^0(\hat{y}) = b + \beta \mathbb{E}[U^*(\hat{y}) + \lambda_u D^*(U^*(\hat{y}); \hat{y}))]. \tag{A14}$$

Hence $U^*(y)$ satisfies the equilibrium condition (iii). Similarly, we can prove that the firm's and worker's joint value from a match $V^*(z; y)$ satisfies the equilibrium condition (iv).

(ii) We want to prove that any equilibrium is efficient. To this aim, let $\{D^*, m^*, U^*, V^*, d^*, \theta^*\}$ denote a Tractable Recursive Equilibrium. Let $s_u^*(y)$ denote the worker's value of unemployment at the beginning of the separation stage, i.e. $U^*(y) + \lambda_u D^*(U^*(y); y)$. Let $s_z^*(z; y)$ denote the firm's and worker's joint value of a match at the beginning of the separation stage, i.e. $V^*(z; y) + \lambda_e D^*(V^*(z; y); y)$. Let $\theta_u^*(y)$ denote the tightness of the submarket visited by unemployed workers, i.e. $\theta_u^*(y) = \theta^*(m^*(U^*(y); y); y)$. And let $\theta_z^*(z; y)$ denote the tightness of the submarket visited by workers who are employed at jobs with idiosyncratic productivity z, i.e. $\theta_z^*(z; y) = \theta^*(m^*(V^*(z; y); y); y)$.

Define the function $r : \Psi \to \mathbb{R}$ as $r_u(y)u + \sum r_z(z; y) g(z_i)$, where $r_u(y)$ is equal to $s_u^*(y)$ and $r_z(z; y)$ is equal to $s_z^*(z; y)$. Given the function r, consider the problem (A6). For each $(y, u, g) \in \Psi$, the optimal market tightness $\tilde{\theta}_u(y)$ satisfies the condition

$$k \geq p'(\tilde{\theta}_u(y))\{y - b + \beta \mathbb{E}[\textstyle\sum_i r_z(z_i; \hat{y}) f(z_i) - r_u(\hat{y})]\} \tag{A15}$$

and $\tilde{\theta}_u(y) \geq 0$, with complementary slackness. Since $r_z(z_i; \hat{y}) = s_z^*(z_i; \hat{y})$ and $r_u(\hat{y}) = s_u^*(\hat{y})$, $\tilde{\theta}_u(y)$ also satisfies condition (E4). Since (E4) admits only one solution, $\tilde{\theta}_u(y)$ is equal to $\theta_u^*(y)$. Similarly, we can prove that the optimal tightness $\tilde{\theta}_z(z; y)$ is equal to $\theta_z^*(z; y)$. And we can prove that the optimal job destruction probability $\tilde{d}(z; y)$ is equal to $d^*(z; y)$.

Define the function $\tilde{r} : \Psi \to \mathbb{R}$ as Tr. As we proved in Proposition 2, $\tilde{r}$ belongs to the set $L(\Psi)$. As we also proved in Proposition 2, the derivative $\tilde{r}_u(y)$ is equal to

$$\begin{aligned} \tilde{r}_u(y) = \quad & -k\lambda_u \tilde{\theta}_u(y) + \left[1 - \lambda_u p(\tilde{\theta}_u(y))\right] [b + \beta \mathbb{E} r_u(\hat{y})] + \\ & + \lambda_u p(\tilde{\theta}_u(y)) \left\{y + \beta \mathbb{E}\left[\textstyle\sum_i r_z(z_i; \hat{y}) f(z_i)\right]\right\}. \end{aligned} \tag{A16}$$

Since $r_z(z_i; \hat{y}) = s_z^*(z_i; \hat{y})$, $r_u(\hat{y}) = s_u^*(\hat{y})$ and $\tilde{\theta}_u(y) = \theta_u^*(y)$, the right hand side of (A16) is equal to the right hand side of (E5). Hence $\tilde{r}_u(y)$ is equal to $s_u^*(y)$. Similarly, we can prove that $\tilde{r}_z(z; y)$ is equal to $s_z^*(z_i; y)$. Taken together, these two observations imply that

$$(Tr)(\psi) = s_u^*(y)u + \textstyle\sum_i s_z^*(z_i; y) g(z_i) = r(\psi). \tag{A17}$$

Since it is a fixed point of the operator T, r is equal to the social planner's value function s^0. And the policy $\{\tilde{\theta}_u, \tilde{\theta}_z, \tilde{d}\} = \{\theta_u^*, \theta_z^*, d^*\}$ is equal to the solution to the social planner's problem $\{\theta_u^0, \theta_z^0, d^0\}$. ■

Table 1: U.S. Quarterly Data, 1951:I–2006:II

	u	v	h^{ue}	h^{eu}	h^{ee}	p
Average	.056	63.9	.452	.026	.029	84.2
Relative Std	12.2	13.5	7.56	7.03	4.15	1
Quarterly Acr	.873	.905	.820	.692	.595	.761
u	1	-.919	-.920	.777	-.631	-.250
v	—	1	.907	-.784	.661	.410
h^{ue}	—	—	1	-.677	.664	.258
h^{eu}	—	—	—	1	-.289	-.480
h^{ee}	—	—	—	—	1	.173
p	—	—	—	—	—	1

Source: Bureau of Labor Statistics.

Table 2: Calibration Outcomes

	Description	(a) Baseline	(b) P85	Target
β	discount rate	.996	.996	real interest rate
b	home productivity	.983	.710	EU rate
λ_u	off the job search prob.	1	1	normalization
λ_e	on the job search prob.	.807	—	EE rate
γ	elasticity of p wrt θ	.650	.220	reg. coef. of v/u on h^{ue}
k	vacancy cost	1.74	2.84	UE rate
δ	destruction prob.	.010	.027	tenure distribution
α_z	shape idios. prod.	3	—	tenure distribution
σ_z	scale idios. prod.	.838	—	home/mkt prod.
μ_z	average idios. prod.	0	—	normalization
σ_y	std. agg. prod.	1.52	1.02	std. average prod.
ρ_y	autocorr. agg. prod.	0.76	0.76	std. average prod.

Figure 1: Tenure Distribution

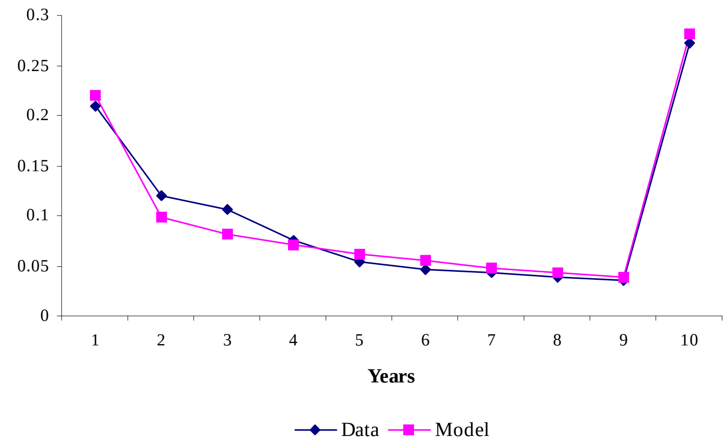

Figure 2: Job Distribution

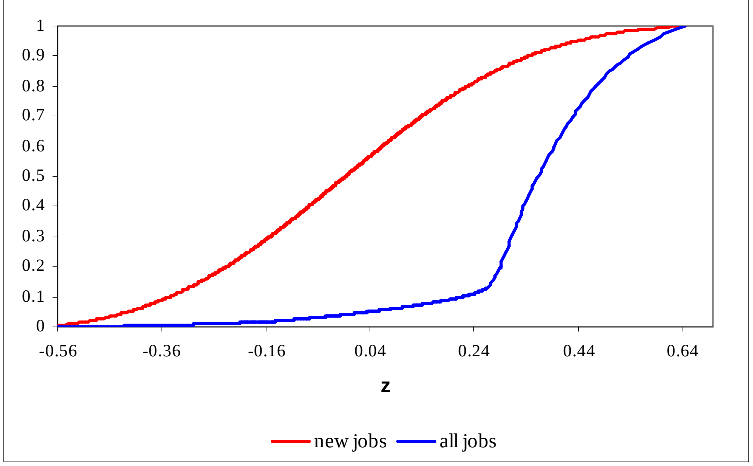

Table 3: Productivity Shocks

	u	v	h^{ue}	h^{eu}	h^{ee}	p
Relative Std	10.4	4.01	3.04	8.67	9.15	1
Quarterly Acr	.831	.645	.772	.755	.790	.774
u	1	-.801	-.969	.969	-.975	-.971
v	—	1	.904	-.887	.883	.896
h^{ue}	—	—	1	-.963	.984	.985
h^{eu}	—	—	—	1	-.957	-.974
h^{ee}	—	—	—	—	1	.988
p	—	—	—	—	—	1

Figure 3: Impulse Response Functions

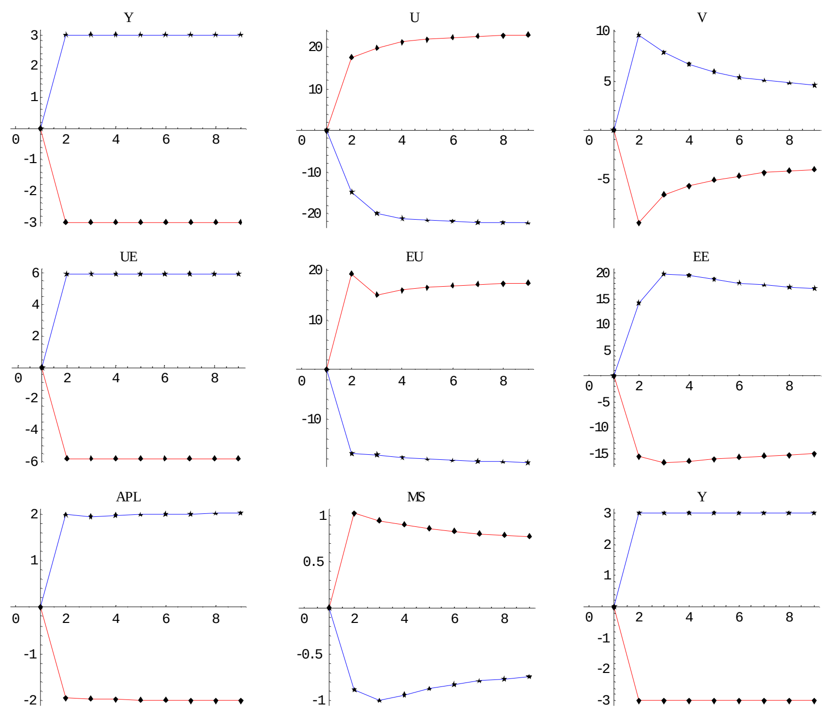

Table 4: Productivity Shocks in P85

	u	v	h^{ue}	h^{eu}	h^{ee}	p
Relative Std	.667	2.78	.742	0	—	1
Quarterly Acr	.826	.726	.770	1	—	.771
u	1	-.946	-.974	0	—	-.974
v	—	1	.994	0	—	.994
h^{ue}	—	—	1	0	—	.999
h^{eu}	—	—	—	1	—	0
h^{ee}	—	—	—	—	—	—
p	—	—	—	—	—	1

Figure 4: Impulse Response Functions in P85