

# A Solution to the Default Risk-Business Cycle Disconnect\*

Enrique G. Mendoza  
University of Maryland and NBER

Vivian Z. Yue  
New York University

June 2008

## Abstract

Models of business cycles in emerging economies explain the negative correlation between country spreads and output by modeling default risk as an exogenous interest rate on working capital. Sovereign default models explain the cyclical properties of country spreads by assuming an exogenous output cost of default with special features, and they underestimate debt-output ratios by a wide margin. We propose a solution to this default risk-business cycle disconnect based on a model of sovereign default with endogenous output dynamics. The model replicates observed V-shaped output dynamics around defaults, countercyclical sovereign spreads, and high debt ratios, and it also matches the variability of consumption and the countercyclical fluctuations of net exports. Three features of the model are key for these results: (1) working capital loans pay for imported inputs; (2) default triggers an efficiency loss as final goods producers switch to domestic inputs that are imperfect substitutes for imported inputs, and labor reallocates from final goods to intermediate goods production; and (3) default on the foreign obligations of firms and the government occurs simultaneously.

JEL Code: E32, E44, F32, F34

Key Words: Business Cycles, Sovereign Default, Sudden Stops, Emerging Economies

---

\*We thank Cristina Arellano, Andy Atkeson, Fernando Broner, Jonathan Eaton, Jonathan Heathcote, Olivier Jeanne, Pat Kehoe, Tim Kehoe, Narayana Kocherlakota, Guido Lorenzoni, Andy Neumeier, Fabrizio Perri, Victor Rios-Rull, Martin Uribe, Mark Wright, and Tom Sargent for helpful comments and suggestions. We also acknowledge comments by participants at seminars and conferences at Paris School of Economics, NYU, CUNY, Federal Reserve Banks of Kansas City and Minneapolis, SUNY-Albany, Duke University, Univ. of Texas-Austin, Ohio State University, 2007 SED Annual Meeting, 2007 LACEA Annual Meeting, the CREI-CEPR 2007 Conference on Sovereign Risk, the X Workshop in International Economics and Finance at Di Tella University, and the 2008 IEA World Congress.

# 1 Introduction

Three key empirical regularities characterize the relationship between sovereign debt and economic activity in emerging economies:

(1) *Output displays V-shaped dynamics around defaults.* Arellano (2007) reports that in recent default episodes GDP deviations from trend in the quarter in which default occurred were -14 percent in Argentina, -13 percent in Russia and -7 percent in Ecuador. Using quarterly data for 39 developing countries over the 1970-2005 period, Levy-Yeyati and Panizza (2006) show that the recessions associated with defaults tend to begin prior to the defaults and generally hit their trough when the defaults take place. Tomz and Wright (2007) study the history of defaults in industrial and developing countries over the period 1820-2004 and find that the frequency of defaults is at its maximum when output is at least 7 percent below trend. They also found, however, that some defaults occurred with less severe recessions, or when output is not below trend in annual data.

(2) *Interest rates on sovereign debt and GDP are negatively correlated.* Neumeyer and Perri (2005) report that the cyclical correlations between the two rates range from -0.38 to -0.7 in five emerging economies, with an average correlation of -0.55. Uribe and Yue (2006) report correlations for seven emerging economies ranging from zero to -0.8, with an average of -0.42.<sup>1</sup>

(3) *External debt as a share of GDP is high on average, and high when countries default.* Foreign debt was about a third of GDP on average over the 1998-2005 period for the group of emerging and developing countries as defined in IMF (2006). Within this group, the highly indebted poor countries had the highest average debt ratio at about 100 percent of GDP, followed by the Eastern European and Western Hemisphere countries, with averages of about 50 and 40 percent of GDP respectively. Reinhart et al. (2003) report that the external debt ratio during default episodes averaged 71 percent of GDP for all developing countries that defaulted at least once in the 1824-1999 period. The default episodes of recent years are in line with this estimate: Argentina defaulted in 2001 with a 64 percent debt ratio, and Ecuador and Russia defaulted in 1998 with debt ratios of 85 and 66 percent of GDP respectively.

These empirical regularities have proven difficult to explain. On one hand, quantitative business cycle models can account for the negative correlation between country interest rates and output *if* the interest rate on sovereign debt is introduced as the exogenous interest rate faced by a small open economy in which firms require working capital to pay the wages bill.<sup>2</sup> On the other hand, quantitative models of sovereign default based on the classic setup of Eaton and Gersovitz (1981) can generate countercyclical sovereign spreads *if* the sovereign country faces stochastic shocks to an exogenous output endowment and default

---

<sup>1</sup>Neumeyer and Perri used data for Argentina, Brazil, Korea, Mexico and the Philippines. Uribe and Yue added Ecuador, Peru and South Africa, but excluded Korea.

<sup>2</sup>See Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005).

entails exogenous output costs with special features.<sup>3</sup> The latter are also needed in order to obtain equilibria that feature non-trivial levels of debt at the observed default frequencies, but still the predicted mean debt ratios are under 10 percent of GDP (or when they are higher the models underestimate default probabilities by a wide margin).<sup>4</sup> Thus, there is a crucial disconnect between business cycle models and sovereign default models: the former lack an explanation of the default risk premia that drive their findings, while the latter lack an explanation of the business cycle dynamics that are critical for their results.

This paper offers a solution to the country risk-business cycle disconnect by proposing a model of sovereign default with endogenous output fluctuations. The model borrows from the sovereign default literature the classic Eaton-Gersovitz recursive formulation of strategic default in which a sovereign borrower makes optimal default choices by comparing the payoffs of repayment and default. In addition, the model borrows from the business cycle literature a transmission mechanism that links default risk with economic activity via the financing cost of working capital. We extend the two classes of models by developing a framework in which the equilibrium dynamics of output and default risk are determined jointly, and influence each other via the interaction between foreign lenders, the domestic sovereign borrower, and domestic firms. In particular, a fall in productivity in our setup increases the likelihood of default and hence sovereign spreads, and this in turn increases the firms' financing costs leading to a further fall in output, which in turn feeds back into default incentives and sovereign spreads.

We demonstrate via numerical analysis that the model can explain the three key empirical regularities of sovereign debt mentioned earlier: The model mimics the V-shaped pattern of output dynamics around defaults with large recessions that hit bottom during defaults, yields countercyclical interest rates on sovereign debt, and supports high debt-GDP ratios on average and in default episodes. These results are obtained requiring a small fraction of firms' purchases of imported inputs to be paid with working capital (only 10 percent). Moreover, the model matches key business cycle features like the variability of consumption and the countercyclical behavior of net exports.

These results hinge on three key features of the model: First, producers of final goods obtain working capital loans from abroad to finance purchases of imported intermediate goods. Second, default causes an efficiency loss because it forces final goods producers to switch to domestic inputs that are imperfect substitutes for imported inputs, and labor reallocates from production of final goods to production of intermediate goods. Third, the

---

<sup>3</sup>See, for example, Aguiar and Gopinath (2006), Arellano (2007), Yue (2006), and Bai and Zhang (2005).

<sup>4</sup>Arellano (2007) obtained a mean debt ratio of 6 percent of GDP assuming an output cost of default such that income is the maximum of actual output or 0.97 of average output while the economy is in financial autarky. Aguiar and Gopinath (2006) obtained a mean debt ratio of 27 percent assuming a cost of 2 percent of output per quarter, but the default frequency is only 0.02 percent (in their model without trend shocks and debt bailouts). Yue (2006) assumed the same output cost in a model with renegotiation calibrated to observed default frequencies, but obtained a mean debt ratio of 9.7 percent of output.

government can divert the firms' repayment of working capital loans when it defaults on its own debt, so that both agents default on their foreign obligations at the same time, and the interest rates they face are equal at equilibrium.

The transmission mechanism that connects country risk and business cycles in our model operates as follows: Final goods producers use labor and a CES composite of imported and domestic inputs as factors of production. Foreign and domestic inputs are imperfect substitutes. Imported inputs need to be paid in advance using working capital loans from abroad, while domestic inputs require labor to be produced. Under these assumptions, the optimal mix of imported and domestic inputs depends on the country interest rate (inclusive of default risk), which drives the financing cost of working capital, and on the state of total factor productivity (TFP). When the country has access to world financial markets, final goods producers use both imported and domestic inputs, and hence fluctuations in default risk affect the cost of working capital and induce fluctuations in factor demands and output. In contrast, when the country does not have access to world financial markets, final goods producers switch to use domestic inputs only because of the prohibitive financing cost of imported inputs. Hence, financial autarky imposes two distortions that reduce production efficiency: it forces final goods producers to operate using domestic inputs (which are imperfect substitutes for imported inputs), and induces labor reallocation away from final goods production, so that the supply of domestic inputs can match the increased demand.

When the economy defaults, both the government and firms are excluded from world credit markets for some time, with an exogenous probability of re-entry as is common in the recent quantitative studies of sovereign default. Since the probability of default depends on whether the country's value of default is higher than that of repayment, there is feedback between the economic fluctuations induced by changes in interest rate premia, default probabilities, and country risk. In particular, rising country risk in the periods leading to a default causes a decline in economic activity as the firms' financing costs increase. In turn, the expectation of lower output at higher levels of country risk alters repayment incentives for the sovereign, affecting the equilibrium determination of default risk premia.

The transmission mechanism linking country risk and business cycles generates an endogenous output cost of default that is larger in "better" states of nature (i.e., *increasing* in the state of TFP). This result follows from the loss in production efficiency when default takes place, because, as long as the elasticity of substitution between foreign and domestic inputs is higher than 1, the decline in GDP at the moment of default is higher the higher TFP was just before default. The assumption that the two inputs are imperfect substitutes is crucial for this result. If the inputs are perfect substitutes there is no output cost at default, and if their elasticity is lower than 1, the output cost does not rise with TFP (the costs are larger at lower elasticities for a given TFP level, but they are less sensitive to changes in TFP).

The elasticity of labor supply also influences the output costs of default. In particular, output costs of default are larger the higher this elasticity because default triggers a reduction in labor, thereby magnifying the efficiency loss caused by the exclusion from credit markets. However, output costs of default, and the efficiency loss that drives them, are still present even if labor supply is inelastic. Final goods producers still have to shift from imported to domestic inputs, and labor still reallocates from final goods to intermediate goods production.

The increasing output cost of default is a key feature of the model because it implies that the option to default brings more “state contingency” into the asset market, allowing the model to produce equilibria that support significantly higher mean debt ratios than those obtained in existing models of sovereign default. The increasing output cost of default also implies that output can fall sharply when the economy defaults, and that, because this output drop is driven by an efficiency loss, part of the output collapse will appear as a drop in the Solow residual (i.e. the fraction of aggregate GDP not accounted for by capital and labor). This is consistent with the data of emerging economies in crisis showing that a large fraction of the output collapse is attributed to the Solow residual (see Meza and Quintin (2006) and Mendoza (2007)). Moreover, Benjamin and Meza (2007) show that in Korea’s 1997 crisis, the productivity drop followed in part from a sectoral reallocation of labor.

Our treatment of the financing cost of working capital differs from the treatment in Neumeyer and Perri (2005) and Uribe and Yue (2006), both of which treat the interest rate on working capital as an exogenous variable set to match the interest rate on sovereign debt. In contrast, in our setup both interest rates are driven by endogenous sovereign risk. In addition, in the Neumeyer-Perri and Uribe-Yue models, working capital loans pay the wages bill in full, while in our model firms use working capital to pay only for a fraction of imported intermediate goods. This lower working capital requirement is desirable because, at standard labor income shares, working capital loans would need to be about 2/3rds of GDP to cover the wages bill, and this is difficult to reconcile with observed ratios of bank credit to the private sector as a share of output in emerging economies, which hover around 50 percent (including *all* credit to households and firms at all maturities, in addition to short-term revolving loans to firms).

The rest of the paper proceeds as follows: Section 2 presents the model. Section 3 explores the model’s quantitative implications for a benchmark calibration. Section 4 conducts sensitivity analysis. Section 4 concludes.

## **2 A Model of Sovereign Default and Business Cycles**

We study a dynamic stochastic general equilibrium model of sovereign default and business cycles. There are four groups of agents in the model, three in the “domestic” small open economy (households, firms, and the sovereign government) and one abroad (foreign lenders).

There are also two sets of producers in the domestic economy: a sector  $f$  of final goods producers and a sector  $m$  of intermediate goods producers.

## 2.1 Households

Households choose consumption and labor supply so as to maximize a standard time-separable utility function  $E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t - h(L_t)) \right]$ , where  $0 < \beta < 1$  is the discount factor, and  $c_t$  and  $L_t$  denote consumption and labor supplied in period  $t$  respectively.  $u(\cdot)$  is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. Following Greenwood, Hercowitz and Huffman (1988), we remove the wealth effect on labor supply by specifying period utility as a function of consumption net of the disutility of labor  $h(L_t)$ , where  $h(\cdot)$  is increasing, continuously differentiable and convex. This GHH formulation of preferences plays an important role in allowing international real business cycle models to explain observed business cycle facts, and it also simplifies the "supply side" of the model by removing intertemporal considerations from the labor supply choice.

Households take as given the wage rate  $w_t$ , profits paid by firms in the  $f$  and  $m$  sectors  $(\pi_t^f, \pi_t^m)$  and government transfers  $(T_t)$ . Households do not borrow directly from abroad, but they are still able to smooth consumption because the government borrows, pays transfers, and makes default decisions internalizing their utility function. This assumption implies that the households' optimization problem reduces to the following static problem:

$$\max_{c_t, L_t} E \left[ \sum \beta^t u(c_t - h(L_t)) \right] \quad (1)$$

$$s.t. \quad c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t \quad (2)$$

Since the GHH utility function implies that the marginal rate of substitution between consumption and labor is equal to the marginal disutility of labor (and independent of consumption), the optimality condition for labor supply is:

$$h'(L_t) = w_t \quad (3)$$

For purposes of the quantitative analysis, we will define the labor disutility function in isoelastic form  $h(L) = \frac{L^\omega}{\omega}$  with  $\omega > 1$ . Hence, the Frisch elasticity of labor supply will be given by  $1/(\omega - 1)$ . The period utility function takes the standard constant-relative-risk-aversion form  $u(c, L) = \frac{(c - \frac{L^\omega}{\omega})^{1-\sigma} - 1}{1-\sigma}$  with  $\sigma > 0$ .

## 2.2 Final Goods Producers

Firms in the  $f$  sector use two variable factors, labor  $L_t^f$  and intermediate goods  $M_t$ , and a time-invariant capital stock  $k$ . They face Markov TFP shocks  $\varepsilon_t$ , with transition probability

distribution function  $\mu(\varepsilon_t|\varepsilon_{t-1})$ . The production function is Cobb-Douglas:

$$y_t = \varepsilon_t \left( M \left( m_t^d, m_t^* \right) \right)^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} \quad (4)$$

with  $0 < \alpha_L, \alpha_M, \alpha_k < 1$  and  $\alpha_L + \alpha_M + \alpha_k = 1$ .

Intermediate goods are a CES composite of imported inputs  $m_t^*$  and inputs produced at home  $m_t^d$ :

$$M \left( m_t^d, m_t^* \right) = \left( \lambda \left( m_t^d \right)^\mu + (1 - \lambda) \left( m_t^* \right)^\mu \right)^{\frac{1}{\mu}} \quad (5)$$

where  $-\infty \leq \mu \leq 1$ ,  $0 \leq \lambda \leq 1$ . The elasticity of substitution between  $m_t^*$  and  $m_t^d$  is equal to  $|1/(\mu - 1)|$ . The two inputs are perfect substitutes when  $\mu = 1$ , and they are used in Leontieff fixed proportions as  $\mu$  goes to  $-\infty$ .

Imported inputs are sold in a competitive world market at the exogenous relative price  $p_m^*$ .<sup>5</sup> A fraction  $\theta$  of the cost of these inputs needs to be paid in advance using working capital loans  $\kappa_t$ , which are intraperiod loans repaid at the end of the period that are offered by foreign creditors at the interest rate  $r_t$ . This interest rate is linked to the sovereign interest rate at equilibrium, as shown in the next section. Working capital loans satisfy the standard payment-in-advance condition:

$$\frac{\kappa_t}{1 + r_t} \geq \theta p_m^* m_t^* \quad (6)$$

Profit-maximizing producers of final goods choose  $\kappa_t$  so that this condition holds with equality.

The profits of final goods producers are given by:

$$\pi_t^f = \varepsilon_t \left( M \left( m_t^d, m_t^* \right) \right)^{\alpha_M} \left( L_t^f \right)^{\alpha_L} k^{\alpha_k} - p_m^* (1 + \theta r_t) m_t^* - p_t^m m_t^d - w_t L_t^f \quad (7)$$

where  $p_t^m$  is the endogenous price of domestic intermediate goods. As noted earlier, domestic inputs do not require working capital financing. This assumption is just for simplicity, the key element for the analysis is that at high levels of country risk (including periods without access to foreign credit markets) the financing cost of foreign inputs is higher than that of domestic inputs.

Final goods producers choose factor demands so as to maximize profits taking  $w_t$ ,  $r_t$ ,  $p_m^*$ , and  $p_t^m$  as given. The first-order conditions of their optimization problem are:

$$\alpha_M \varepsilon_t k^{\alpha_k} \left( M \left( m_t^d, m_t^* \right) \right)^{\alpha_M - \mu} \left( L_t^f \right)^{\alpha_L} (1 - \lambda) \left( m_t^* \right)^{\mu - 1} = p_m^* (1 + \theta r_t) \quad (8)$$

$$\alpha_M \varepsilon_t k^{\alpha_k} \left( M \left( m_t^d, m_t^* \right) \right)^{\alpha_M - \mu} \left( L_t^f \right)^{\alpha_L} \lambda \left( m_t^d \right)^{\mu - 1} = p_t^m \quad (9)$$

$$\alpha_L \varepsilon_t k^{\alpha_k} M_t^{\alpha_M} \left( L_t^f \right)^{\alpha_L - 1} = w_t \quad (10)$$

These are standard optimality conditions equating marginal products of factors of pro-

---

<sup>5</sup>This price can also be modeled as a terms-of-trade shock with a given stochastic process.

duction (i.e. factor demands) to the corresponding marginal costs.

### 2.3 Intermediate Goods Producers

Domestic inputs do not require advance payment, but labor  $L_t^m$  is needed in order to produce them. Producers in the  $m$  sector operate with a production function given by  $A(L_t^m)^\gamma$ , with  $0 \leq \gamma \leq 1$  and  $A > 0$ .  $A$  represents both the role of a fixed factor and an invariant state of TFP. Given  $p_t^m$  and  $w_t$ , these producers choose  $L_t^m$  so as to solve the following profit maximization problem:

$$\max_{L_t^m} \pi_t^m = p_t^m A(L_t^m)^\gamma - w_t L_t^m \quad (11)$$

Their optimal labor demand satisfies this standard condition:

$$\gamma p_t^m A(L_t^m)^{\gamma-1} = w_t \quad (12)$$

### 2.4 Competitive Equilibrium of the Private Sector

**Definition 1** *A competitive equilibrium for the private sector of the economy is given by sequences of allocations  $\left[ c_t, L_t, L_t^f, L_t^m, m_t^d, m_t^*, \kappa_t \right]_{t=0}^\infty$  and prices  $\left[ w_t, p_t^m, \pi_t^f, \pi_t^m \right]_{t=0}^\infty$  such that:*

1. *The allocations  $\left[ c_t, L_t \right]_{t=0}^\infty$  solve the households' utility maximization problem.*
2. *The allocations  $\left[ L_t^f, m_t^d, m_t^*, \kappa_t \right]_{t=0}^\infty$  solve the profit maximization problem of sector  $f$  producers.*
3. *The allocations  $\left[ L_t^m \right]_{t=0}^\infty$  solve the profit maximization problem of sector  $m$  producers.*
4. *The market-clearing conditions for the labor market ( $L_t^f + L_t^m = L_t$ ) and the domestic intermediate goods market ( $A(L_t^m)^\gamma = m_t^d$ ) hold.*

In this economy, GDP at factor costs is given by  $w_t L_t^f + w_t L_t^m + \pi_t^f + \pi_t^m$ . Using this definition together with the definitions of profits and the optimality conditions of the  $f$  and  $m$  sectors, it follows that GDP can be expressed as  $gdp = (1 - \alpha_m)\varepsilon_t(M_t)^{\alpha_m}(L_t^f)^{\alpha_L}k^{\alpha_k} + p_t^m A(L_t^m)^\gamma$ . The first and second terms in the right-hand-side of this expression represent value added in the  $f$  and  $m$  sectors respectively (note that with the CES formulation of  $M_t$  it can be shown that  $\alpha_m \varepsilon_t(M_t)^{\alpha_m}(L_t^f)^{\alpha_L}k^{\alpha_k} = p_m^*(1 + \theta r_t)m_t^* + p_t^m m_t^d$ ). In matching the model with actual GDP data, however, we need to take into account that national accounts at constant prices are measured at a base year's prices, so we evaluate  $gdp$  keeping  $p_t^m$  constant at its long-run average. This adjustment does not affect our quantitative findings significantly.

A key constraint on the problem of the sovereign borrower making the default decision will be that the private-sector allocations must be a competitive equilibrium. Since the sovereign government's problem and the equilibrium of the credit market will be characterized in recursive form as functions defined in the state space domain, it is useful to also characterize



the above competitive equilibrium in terms of functions of state variables, and to distinguish private sector allocations in states in which the economy has credit market access from those in which it does not.

We start by expressing the private sector equilibrium when sector  $f$  firms have access to credit markets in terms of functions of  $r$  and  $\varepsilon$  that solve the following nonlinear system of equations:

$$\alpha_M \varepsilon k^{\alpha_k} \left( M(m^d, m^*) \right)^{\alpha_M - \mu} \left( L^f \right)^{\alpha_L} (1 - \lambda) (m^*)^{\mu - 1} = p_m^* (1 + \theta r) \quad (13)$$

$$\alpha_M \varepsilon k^{\alpha_k} \left( M(m^d, m^*) \right)^{\alpha_M - \mu} \left( L^f \right)^{\alpha_L} \lambda (m^d)^{\mu - 1} = p^m \quad (14)$$

$$\alpha_L \varepsilon k^{\alpha_k} \left( M(m^d, m^*) \right)^{\alpha_M} \left( L^f \right)^{\alpha_L - 1} = w \quad (15)$$

$$\gamma p^m A(L^m)^{\gamma - 1} = w \quad (16)$$

$$w = h'(L) \quad (17)$$

$$L^f + L^m = L \quad (18)$$

$$A(L^m)^\gamma = m^d \quad (19)$$

$$\kappa(r, \varepsilon) = \theta p_m^* m^* (1 + r) \quad (20)$$

We define the functions  $m^*(r, \varepsilon)$ ,  $m^d(r, \varepsilon)$ ,  $L^f(r, \varepsilon)$ ,  $L^m(r, \varepsilon)$ ,  $L(r, \varepsilon)$ ,  $p^m(r, \varepsilon)$ ,  $w(r, \varepsilon)$ , and  $\kappa(r, \varepsilon)$  to represent the solutions to the above equation system for a given pair  $(r, \varepsilon)$ . Notice in particular that equation (13) implies that increases in the interest rate increase the marginal cost of imported inputs, and thus the demand for  $m^*$  falls as  $r$  rises. Moreover, since as we show later the interest rate faced by firms is directly influenced by sovereign default risk, it follows that increases in country risk have "supply side effects." One of these effects is the direct negative effect of  $r$  on  $m^*$ , but there are also indirect general equilibrium effects that operate via the substitution of foreign for domestic inputs and the labor market. We study these effects in detail in the next subsection.

During periods of exclusion from world credit markets, sector  $f$  does not have access to foreign working capital financing, and hence it can only use domestic inputs. The equilibrium allocations for this scenario can be solved as the limiting case of the above nonlinear system as  $r \rightarrow \infty$ . However, for  $0 < \mu < 1$  (which we show later is the relevant range for obtaining an output cost of default increasing in TFP), it is possible to solve for this equilibrium in closed form. In this case, the private sector equilibrium reduces to the following functions of  $\varepsilon$ :

$$\tilde{L}(\varepsilon) = \left[ (\alpha_L + \gamma\alpha_m)\varepsilon k^{\alpha_k} \eta^{\frac{\alpha_m}{\mu}} A^{\alpha_m} (z_{L_m})^{\alpha_m \gamma} (z_{L_f})^{\alpha_L} \right]^{1/(\omega - \alpha_L - \alpha_m \gamma)} \quad (21)$$

$$\tilde{L}^f(\varepsilon) = z_{L_f} \tilde{L}(\varepsilon) \quad (22)$$

$$\tilde{L}^m(\varepsilon) = z_{L_m} \tilde{L}(\varepsilon) \quad (23)$$

$$\tilde{m}(\varepsilon) = A \left( \tilde{L}^m(\varepsilon) \right)^\gamma \quad (24)$$

$$\tilde{p}^m(\varepsilon) = \alpha_m \eta^{\frac{\alpha_m}{\mu}} \varepsilon (\tilde{m}(\varepsilon))^{\alpha_m - 1} \left( \tilde{L}^f(\varepsilon) \right)^{\alpha_L} k^{\alpha_k} \quad (25)$$

where  $z_{L_m} = \frac{\gamma\alpha_m}{\gamma\alpha_m + \alpha_L}$  and  $z_{L_f} = \frac{\alpha_L}{\gamma\alpha_m + \alpha_L}$ .

## 2.5 Endogenous Output Cost of Default

The effects of interest rate changes on the private sector equilibrium play a central role in our analysis because they drive output dynamics and the output cost of default, which are two key determinants of the default/repayment decision of the sovereign. The economic intuition behind these effects can be explained by studying how interest rate hikes affect the private sector equilibrium. Since these interest rate effects include direct and indirect general equilibrium effects that cannot be solved for analytically, we use a numerical example. For this example, we start from a baseline that uses the parameter values set for  $\alpha_m$ ,  $\alpha_L$ ,  $\alpha_k$ ,  $A$ ,  $\gamma$ ,  $\omega$ ,  $\lambda$  and  $\mu$  in the calibration exercise of Section 3.

Figure 1 shows six charts with the allocations of  $L$ ,  $L^f$ ,  $L^m$ ,  $M$ ,  $m^d$ , and  $m^*$  for values of  $r$  ranging from 0 to 80 percent. Each chart includes results for the baseline value of  $\mu$  (0.656), which corresponds to an elasticity of substitution between foreign and domestic inputs of 2.9, and two lower elasticities (1.5, which is the threshold below which  $m^d$  and  $m^*$  switch from gross substitutes to gross complements, and the Cobb-Douglas case of unitary elasticity of substitution). We also show results for the baseline value of  $\mu$  but assuming that labor supply is inelastic. To facilitate comparison across the charts, the allocations are plotted in ratios relative to the allocations when  $r = 0$ .

An increase in the rate of interest reduces the demand for  $m^*$  because of the direct effect by which the hike in  $r$  increases the price of imported inputs  $p_m^*(1 + \theta r)$ . At equilibrium, this also reduces the demand for total intermediate goods  $M$ . The magnitude of these two effects, for a given change in  $r$ , depends on whether  $m^*$  and  $m^d$  are gross substitutes or gross complements. If they are gross substitutes (i.e. if  $m^d$  increases as  $p_m^*(1 + \theta r)$  rises), then  $m^d$  rises as  $m^*$  falls, and  $M$  falls less than it would if the two inputs are gross complements. For the same reason, however,  $m^*$  falls more when the inputs are gross substitutes. Moreover, the increase in  $r$  also reduces both  $L$  and  $L^f$ , and again the reduction in  $L$  is smaller and the decline in  $L^f$  larger when foreign and domestic inputs are gross substitutes (i.e. the higher is

$\mu$ , or the higher is the elasticity of substitution between  $m^*$  and  $m^d$ ). In addition, if foreign and domestic inputs are gross substitutes,  $L^f$  and  $L^m$  also behave as gross substitutes, so that  $L^m$  rises as  $L^f$  falls.

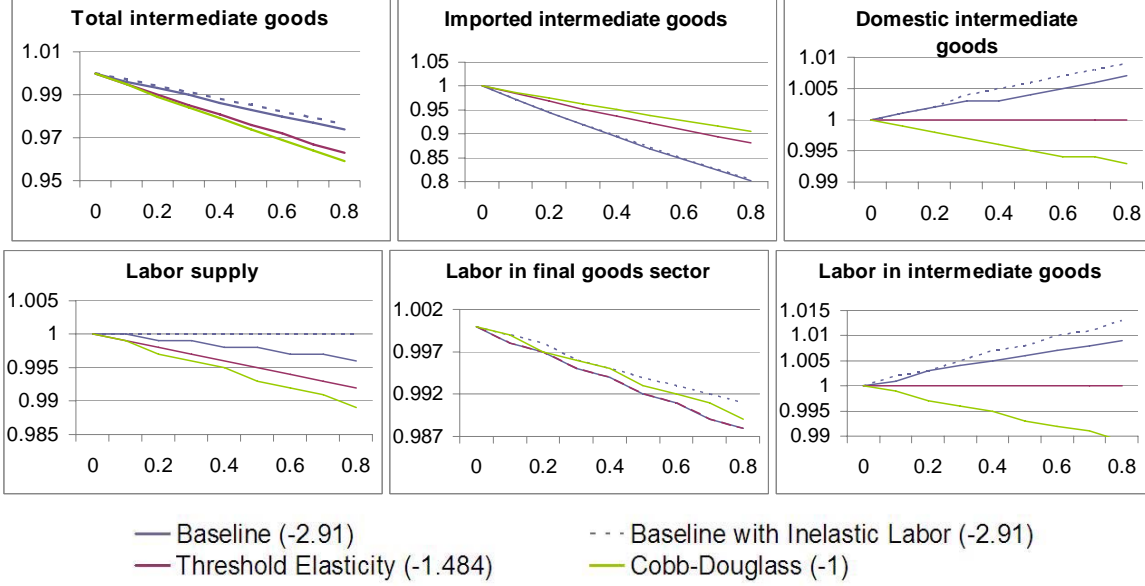


Figure 1: Effects of interest rate shocks on intermediate goods and labor allocations at different elasticities of substitution (ratios relative to allocations at zero interest rate)

The charts in Figure 1 also illustrate how the elasticity of labor supply affects the responses of factor allocations to interest rate changes. Keeping  $\mu$  at its baseline value, the effect on  $m^*$  is nearly unchanged if we make labor supply inelastic.  $M$  falls less with inelastic labor because  $m^d$  rises more, and this is possible because with inelastic labor supply  $L$  cannot fall in response to interest rate hikes, and this results in a larger increase in  $L^d$  and a smaller decline in  $L^f$ . Thus, even with inelastic labor supply, increases in  $r$  affect the efficiency of production by inducing a shift from foreign to domestic inputs, and by reallocating labor from production of final goods to production of intermediate goods.

The left panel in Figure 2 plots the output cost of default (i.e. the drop in output as  $r \rightarrow \infty$ ) as a function of TFP in final goods for different values of  $\mu$ . This Figure illustrates two important results: First, as long as  $0 < \mu < 1$ , the output cost of default is increasing in the size of the TFP shock that hits the economy when it defaults. Second, this relationship is steeper at higher elasticities of substitution between  $m^*$  and  $m^d$ . As the two inputs become gross complements the relation is nearly flat, for unitary elasticity of substitution the output cost is independent of TFP, and for elasticities less than unitary the output cost of default and TFP are inversely related (albeit only slightly). The right panel shows a third key result: While at higher elasticities of substitution the output cost of default is a steeper positive

function of TFP, the actual size of the output drop at default is smaller. At elasticities higher than 10 the numerical example shows virtually no output loss (and it can be shown analytically that if the two inputs are perfect substitutes there is no output cost of default).

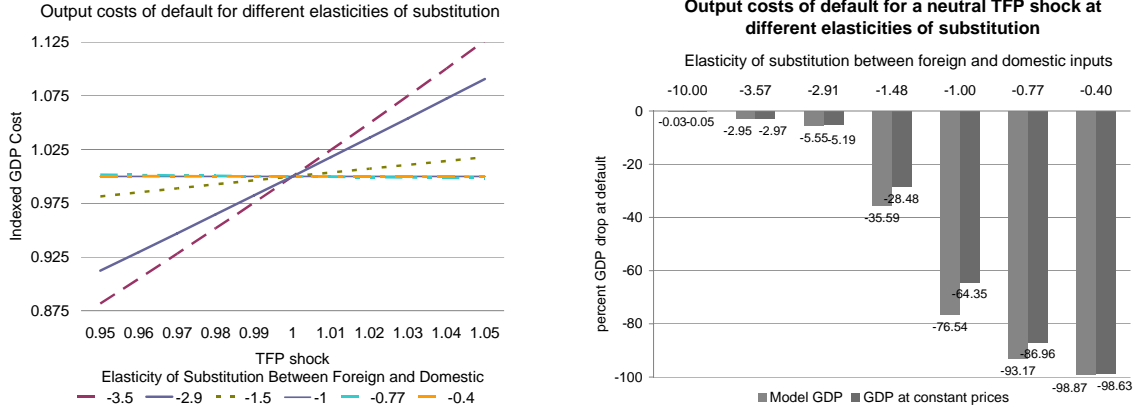


Figure 2: Output Costs of Default at Different Elasticity of Substitution Between Foreign and Domestic Intermediate Inputs

A similar analysis of the output costs of default for different values of  $\omega$  (instead of  $\mu$ ) shows that higher labor supply elasticities (i.e. lower  $\omega$ ) increase the output cost of default, converging to a cost of around 12 percent for infinitely elastic labor supply. The output cost of default is increasing in TFP for any value of  $\omega$ , but, in contrast with what we found for  $\mu$ , the slope of the relationship does not change with different labor supply elasticities. Interestingly, we also found that adjusting the productivity parameter in production of intermediate goods (A) has qualitatively similar effects as changing  $\omega$ .

The intuition behind the result that higher labor supply elasticity produces larger output costs of default can be explained by examining the labor market equilibrium using Figure 3. For simplicity, we plot labor demands and supply as linear functions. The labor demand functions are given by the marginal products in the left-hand-side of (10) and (12), and the labor supply is given by the marginal disutility of labor in the left-hand-side of (3). Since labor is homogenous across sectors, total labor demand is just the sum of the sectoral labor demands. The initial labor market equilibrium is at point A with wage  $w^*$ , total labor  $L^*$  with sectoral allocations  $Ld^*$  and  $Lf^*$ .

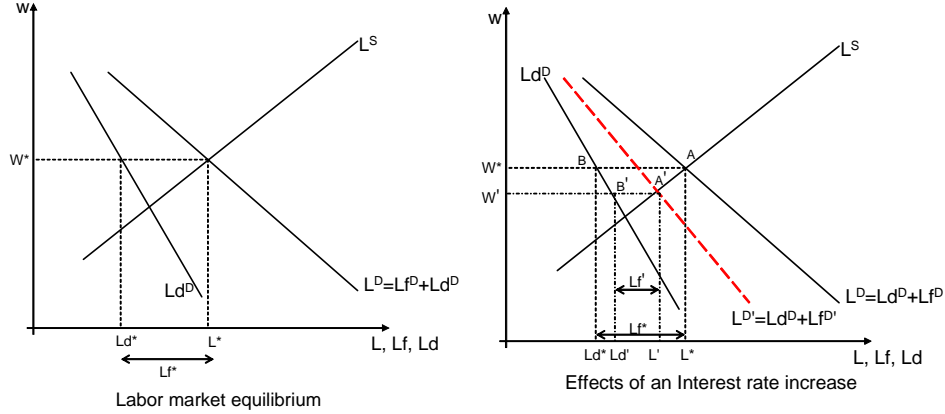


Figure 3: Labor Market Equilibrium

Consider now a positive interest rate shock that leads to a reduction in labor demand in final goods from  $Lf$  to  $Lf'$ . This occurs because higher  $r$  causes a reduction in  $M$ , as explained earlier, and the marginal product of  $Lf$  is a negative function of  $M$  (since the production function of final goods is Cobb-Douglas). As a result, total labor demand shifts from  $L^D$  to  $L^{D'}$ .<sup>6</sup> The new labor market equilibrium is at point  $A'$ . The wage is lower than before, and so are the total labor allocation and the labor allocated to final goods, while labor allocated to production of domestic inputs rises.

It follows from visual examination of Figure 3 that if labor is infinitely elastic (if  $L^S$  is an horizontal line at the level of  $w^*$ ), the interest rate hike leaves  $w$  unchanged instead of reducing it,  $L$  falls more,  $Ld$  is unchanged instead of rising, and  $Lf$  falls less.<sup>7</sup> Hence, the adverse effect on output is stronger. Similarly, going to the other extreme, if labor is inelastic (if  $L^S$  is a vertical line at the level of  $L^*$ ),  $L$  cannot change, but  $w$  falls more than in the scenario plotted in Figure 3,  $Ld$  rises more, and  $Lf$  falls more. Hence, the decline in output is smaller.

Figure 4 shows the output cost of default as an increasing function of TFP using the baseline values of  $\mu$  and  $\omega$ . The Figure plots GDP when the economy has access to credit markets ( $Y^*$ ) and when the economy defaults ( $Y^d$ ), and the fraction of GDP lost when default occurs,  $1 - (Y^d/Y^*)$ . The fact that the output cost of default increases with the size of the TFP shock implies that default is more painful at higher levels of TFP. This result plays a key role in enabling the model to support high debt levels together with observed default frequencies, because it makes the default option more attractive at lower states of

<sup>6</sup>In Figure 3, we hold constant  $p_m$  for simplicity. At equilibrium, the relative price of domestic inputs changes, and this alters the value of the marginal product of  $Ld$ , and hence labor demand by the  $m$  sector. The results of the numerical example do take this into account and still are roughly in line with the intuition derived from Figure 3.

<sup>7</sup>The last effect hinges on the fact that the gap between the demand for labor of the  $f$  sector and the total labor demand widens as the wage falls. This is a property of factor demands with Cobb-Douglas production.

productivity, and works as a desirable implicit hedging mechanism given the incompleteness of asset markets.

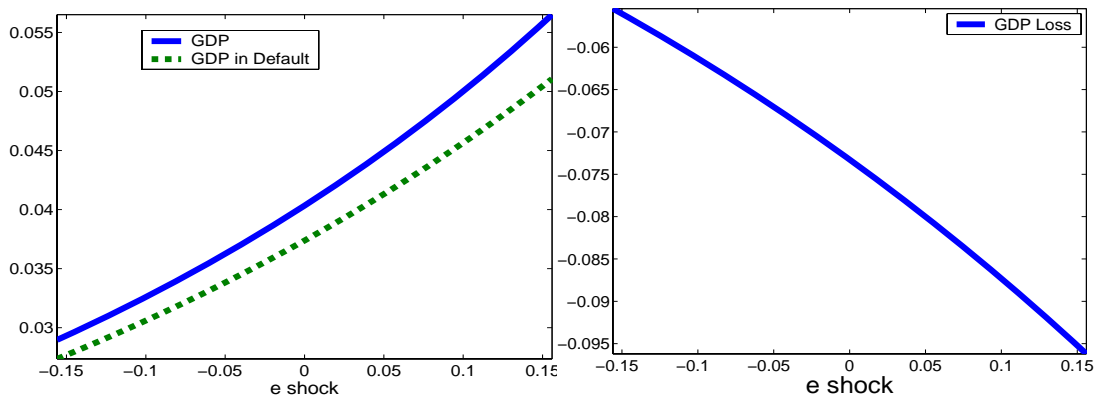


Figure 4: Output and the Output Cost of Default as Functions of TFP

This increasing output cost of default is in line with Arellano’s (2007) result showing that an exogenous default cost with similar features can allow the Eaton-Gersovitz model to support non-trivial levels of debt together with observed default frequencies. In particular, she proposed an exogenous default cost function such that below a threshold level of an output endowment default does not entail an output cost, but above that threshold default reduces the endowment to a state-invariant fraction of the long-run average of GDP. In this second range, the size of the output loss is increasing in the output realization at the time of default. Still, the mean debt ratio in her baseline calibration was only about 6 percent of GDP (assuming output at default is 3 percent below mean output). In contrast, we show later that our model with endogenous output cost of default yields a mean debt ratio about four times larger.

## 2.6 The Sovereign Government

The sovereign government trades with foreign lenders one-period, zero-coupon discount bonds, so markets of contingent claims are incomplete. The face value of these bonds specifies the amount to be repaid next period and is denoted as  $b_{t+1}$ . When the country purchases bonds  $b_{t+1} > 0$ , and when it borrows  $b_{t+1} < 0$ . The set of bond face values is  $B = [b_{\min}, b_{\max}] \subset R$ , where  $b_{\min} \leq 0 \leq b_{\max}$ . We set the lower bound  $b_{\min} < -\frac{\bar{y}}{r}$ , which is the largest debt that the country could repay with full commitment. The upper bound  $b_{\max}$  is the highest level of assets that the country may accumulate.<sup>8</sup>

The sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz model, we assume that when the country defaults it does not repay at date  $t$  and the punishment is

<sup>8</sup> $b_{\max}$  exists when the interest rates on a country’s saving are sufficiently small compared to the discount factor, which is satisfied in our paper since  $(1 + r^*)\beta < 1$ .

exclusion from the world credit market in the same period. The country re-enters the credit market with an exogenous probability  $\eta$ , and when it does it starts with a fresh record and zero debt.<sup>9</sup> Also as in the Eaton-Gersovitz setup, the country cannot hold positive international assets during the exclusion period, otherwise the model cannot support equilibria with debt.

We add to the Eaton-Gersovitz setup an explicit link between default risk and private financing costs. This is done by assuming that a defaulting sovereign can divert the repayment of the firms' working capital loans to foreign lenders. Hence, both firms and government default together. This is perhaps an extreme formulation of the link between private and public borrowing costs, but we provide later some evidence in favor of this view.

The sovereign government solves a problem akin to a Ramsey problem.<sup>10</sup> It chooses a debt policy (amounts and default or repayment) that maximizes the households' welfare subject to the constraints that: (a) the private sector allocations must be a competitive equilibrium; and (b) the government budget constraint must hold. The state variables are the bond position, working capital loans as of the end of last period, and the state of TFP, denoted by the triplet  $(b_t, \kappa_{t-1}, \varepsilon_t)$ . The price of sovereign bonds is given by the bond pricing function  $q_t(b_{t+1}, \varepsilon_t)$ . Since at equilibrium the default risk premium on sovereign debt will be the same as on working capital loans, it follows that the interest rate on working capital is a function of  $q_t(b_{t+1}, \varepsilon_t)$ . Hence, the recursive functions that represent the competitive equilibrium of the private sector when the economy has access to world credit markets can be rewritten as  $\kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ ,  $M(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ ,  $m^*(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ ,  $m^d(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ ,  $L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ ,  $L^m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ , and  $L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$ .

The recursive optimization problem of the government is summarized by the following value function:

$$V(b_t, \kappa_{t-1}, \varepsilon_t) = \begin{cases} \max \{v^{nd}(b_t, \varepsilon_t), v^d(\kappa_{t-1}, \varepsilon_t)\} & \text{for } b_t < 0 \\ v^{nd}(b_t, \varepsilon_t) & \text{for } b_t \geq 0 \end{cases} \quad (26)$$

If the country has access to the world credit market at date  $t$ , the value function is the maximum of the value of continuing in the credit relationship with foreign lenders (i.e., repayment or "no default"),  $v^{nd}(b_t, \varepsilon_t)$ , and the value of default,  $v^d(\kappa_{t-1}, \varepsilon_t)$ . If the economy holds a non-negative bond position, the value function is simply the continuation value because in this case the economy is using the credit market to save, receiving a return equal to the world's risk free rate  $r^*$ .

---

<sup>9</sup>We abstract from debt renegotiation. See Yue (2006) for a quantitative analysis of sovereign default with renegotiation in which the length of financial exclusion is endogenous.

<sup>10</sup>See Cuadra and Saprizza (2007) for an analysis of optimal fiscal policy as a Ramsey problem in the presence of sovereign default in an endowment economy.

The continuation value  $v^{nd}(b_t, \varepsilon_t)$  is defined as follows:

$$v^{nd}(b_t, \varepsilon_t) = \max_{c_t, b_{t+1}} \left\{ \begin{array}{l} u(c_t - h(L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t))) \\ + \beta E[V(b_{t+1}, \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), \varepsilon_{t+1}))] \end{array} \right\} \quad (27)$$

subject to

$$c_t + q_t(b_{t+1}, \varepsilon_t) b_{t+1} - b_t \leq \varepsilon_t f \left( M(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), k) - m^*(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) p_m^* \left( 1 + \frac{\theta}{q_t(b_{t+1}, \varepsilon_t)} - \theta \right) \right) \quad (28)$$

where  $f(\cdot) = M^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k}$ . The constraint of this problem is the resource constraint of the economy at a competitive equilibrium. The left-hand-side is the sum of consumption and net exports, and the right-hand-side is GDP. This constraint is obtained by combining the households' budget constraint (2) with the government budget constraint,  $T_t = b_t - q_t(b_{t+1}, \varepsilon_t) b_{t+1}$ , and recalling that total factor payments  $w_t^f L_t^f + w_t^m L_t^m + \pi_t^f + \pi_t^m$  equal GDP.

The resource constraint captures three important features of the model: First, the government internalizes how interest rates affect the competitive equilibrium allocations of output and factor demands. Second, the households cannot borrow from abroad, but the government internalizes their desire to smooth consumption and transfers to them an amount equal to the negative of the balance of trade (i.e. it gives the private sector the flow of resources it needs to finance the gap between GDP and consumption). Third, the working capital loans  $\kappa_{t-1}$  and  $\kappa_t$  do not enter explicitly in the continuation value or in the resource constraint. Still, we need to keep track of the state variable  $\kappa_t$  because the amount of working capital loans taken by final goods producers at date  $t$  affects the sovereign's incentive to default at  $t+1$ , as explained below.

The value of default  $v^d(\kappa_{t-1}, \varepsilon_t)$  is:

$$v^d(\kappa_{t-1}, \varepsilon_t) = \max_{c_t} \left\{ \begin{array}{l} u(c_t - h(\tilde{L}(\varepsilon_t))) \\ + \beta (1 - \eta) E v^d(0, \varepsilon_{t+1}) + \beta \eta E V(0, 0, \varepsilon_{t+1}) \end{array} \right\} \quad (29)$$

subject to:

$$c_t = \varepsilon_t f \left( \lambda^{\frac{1}{\mu}} \tilde{m}(\varepsilon), \tilde{L}^f(\varepsilon), k \right) + \kappa_{t-1} \quad (30)$$

Note that  $v^d(\kappa_{t-1}, \varepsilon_t)$  takes into account the fact that in case of default at date  $t$ , the country has no access to financial markets this period, and hence the country consumes the total income given by the resource constraint in the default scenario. In this case, since firms



cannot borrow to finance purchases of imported inputs,  $\tilde{m}(\varepsilon)$ ,  $\tilde{L}(\varepsilon)$  and  $\tilde{L}^f(\varepsilon)$  are competitive equilibrium allocations that correspond to the case when the  $f$  sector operates with domestic inputs only (notice in this case, if  $0 < \mu < 1$ ,  $M = \lambda^{\frac{1}{\mu}} \tilde{m}(\varepsilon)$ ). Moreover, because the defaulting government diverts the repayment of last period's working capital loans, total household income includes government transfers equal to the appropriated repayment for the amount  $\kappa_{t-1}$  (i.e., on the date of default, the government budget constraint is  $T_t = \kappa_{t-1}$ ). The value of default at  $t$  also takes into account that at  $t+1$  the economy may re-enter world capital markets with probability  $\eta$  and associated value  $V(0, 0, \varepsilon_{t+1})$ , or remain in financial autarky with probability  $1 - \eta$  and associated value  $v^d(0, \varepsilon_{t+1})$ .

For a debt position  $b_t < 0$  and given a level of working capital  $\kappa_{t-1}$ , default is optimal for the set of realizations of the TFP shock for which  $v^d(\kappa_{t-1}, \varepsilon_t)$  is at least as high as  $v^{nd}(b_t, \varepsilon_t)$ :

$$D(b_t, \kappa_{t-1}) = \left\{ \varepsilon_t : v^{nd}(b_t, \varepsilon_t) \leq v^d(\kappa_{t-1}, \varepsilon_t) \right\} \quad (31)$$

It is critical to note that this default set has a different specification than in the typical Eaton-Gersovitz model of sovereign default (see Arellano (2007)), because the state of working capital affects the gap between the values of default and repayment. This results in a two-dimensional default set that depends on  $b_t$  and  $\kappa_{t-1}$ , instead of just  $b_t$ .

Despite the fact that the default set depends on  $\kappa_{t-1}$ , the probability of default remains a function of  $b_{t+1}$  and  $\varepsilon_t$  only. This is because the  $f$  sector's optimality conditions imply that the next period's working capital loan  $\kappa_t$  depends on  $\varepsilon_t$  and the interest rate, which is a function of  $b_{t+1}$  and  $\varepsilon_t$ . Thus the probability of default at  $t+1$  perceived as of date  $t$  for a country with a productivity  $\varepsilon_t$  and debt  $b_{t+1}$ ,  $p_t(b_{t+1}, \varepsilon_t)$ , can be induced from the default set, the decision rule for working capital, and the transition probability function of productivity shocks  $\mu(\varepsilon_{t+1}|\varepsilon_t)$  as follows:

$$p_t(b_{t+1}, \varepsilon_t) = \int_{D(b_{t+1}, \kappa_t)} d\mu(\varepsilon_{t+1}|\varepsilon_t) \quad (32)$$

$$\text{where } \kappa_t = \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) \quad (33)$$

The economy is considered to be in financial autarky when it has been in default for at least one period and remains without access to world credit markets as of date  $t$ . As noted above, the economy can exit this exclusion stage at date  $t+1$  with probability  $\eta$ . We assume that during the exclusion stage the economy cannot build up its own stock of savings to supply working capital loans to firms, which could be used to purchase imported inputs.<sup>11</sup> This assumption ensures that, as long as the economy remains in financial autarky, the optimization problem of the sovereign is the same as the problem in the default period

---

<sup>11</sup>Alternatively, we could assume that the default punishment includes exclusion from world capital markets and from the world market of intermediate goods.

but evaluated at  $\kappa_{t-1} = 0$  (i.e.  $v^d(\varepsilon_t, 0)$ ).

The model preserves a standard feature of the Eaton-Gersovitz model: Given  $\varepsilon_t$ , the value of defaulting is independent of the level of debt, while the value of not defaulting increases with  $b_{t+1}$ , and consequently the default set and the equilibrium default probability grow with the country's debt. The following theorem formalizes this result:

**Theorem 1** *Given a productivity shock  $\varepsilon$  and level of working capital loan  $\kappa$ , for  $b^0 < b^1 \leq 0$ , if default is optimal for  $b^1$ , then default is also optimal for  $b^0$ . That is  $D(b^1, \kappa) \subseteq D(b^0, \kappa)$ . The country agent's probability of default in equilibrium satisfies  $p^*(b^0, \varepsilon) \geq p^*(b^1, \varepsilon)$ .*

**Proof.** See Appendix. ■

## 2.7 Foreign Lenders

International creditors are risk-neutral and have complete information. They invest in sovereign bonds and in private working capital loans. Foreign lenders behave competitively and face an opportunity cost of funds equal to the world risk-free interest rate. Competition implies that they expect zero profits at equilibrium, and that the returns on sovereign debt and the world's risk-free asset are fully arbitrated:

$$q_t(b_{t+1}, \varepsilon_t) = \begin{cases} \frac{1}{1+r^*} & \text{if } b_{t+1} \geq 0 \\ \frac{[1-p_t(b_{t+1}, \varepsilon_t)]}{1+r^*} & \text{if } b_{t+1} < 0 \end{cases} \quad (34)$$

This condition implies that at equilibrium bond prices depend on the risk of default. For a high level of debt, the default probability is higher. Therefore, equilibrium bond prices decrease with indebtedness. This result, formalized in Theorem 2 below, is consistent with the empirical evidence documented by Edwards (1984).

**Theorem 2** *Given a productivity shock  $\varepsilon$  and level of working capital loan  $\kappa$ , for  $b^0 < b^1 \leq 0$ , the equilibrium bond price satisfies  $q^*(b^0, \varepsilon) \leq q^*(b^1, \varepsilon)$*

**Proof.** See Appendix. ■

The returns on sovereign bonds and working capital loans are also fully arbitrated. Because the sovereign government diverts the repayment of working capital loans when it defaults, foreign lenders assign the same risk of default to private working capital loans as to sovereign debt, and hence the no-arbitrage condition between sovereign lending and working capital loans implies:

$$r_t(b_{t+1}, \varepsilon_t) = \frac{1}{q_t(b_{t+1}, \varepsilon_t)} - 1, \text{ if } b_{t+1} < 0 \text{ and } \kappa_t > 0 \quad (35)$$

## 2.8 Country Risk & Private Interest Rates: Some Empirical Evidence

The result that the interest rates on sovereign debt and private working capital are the same raises a key empirical question: Are sovereign interest rates and the rates of interest faced by private firms closely related in emerging economies?

Providing a complete answer to this question is beyond the scope of this paper, but we do provide empirical evidence suggesting that corporate and sovereign interest rates do move together. To study this issue, we constructed country estimates of firms' financing costs that aggregate measures derived from firm-level data. We constructed a measure of firm-level effective interest rates as the ratio of a firm's total debt service divided by its total debt obligations using the *Worldscope* database, which provides the main lines of balance-sheet and cash-flow statements of publicly listed corporations. We then constructed the corresponding aggregate country measure as the median across firms. Table 1 reports these estimates of corporate interest rates together with the standard EMBI+ measure of interest rates on sovereign debt and the correlations between the two.

Table 1: Sovereign and Corporate Interest Rates

Country	Sovereign Interest Rates	Median Firm Interest Rates	Correlation
Argentina	13.32	10.66	0.87
Brazil	12.67	24.60	0.14
Chile	5.81	7.95	0.72
China	6.11	5.89	0.52
Colombia	9.48	19.27	0.86
Egypt	5.94	8.62	0.58
Malaysia	5.16	6.56	0.96
Mexico	9.40	11.84	0.74
Morocco	9.78	13.66	0.32
Pakistan	9.71	12.13	0.84
Peru	9.23	11.42	0.72
Philippines	8.78	9.27	0.34
Poland	7.10	24.27	0.62
Russia	15.69	11.86	-0.21
South Africa	5.34	15.19	0.68
Thailand	6.15	7.30	0.94
Turkey	9.80	29.26	0.88
Venezuela	14.05	19.64	0.16

Table 1 shows that the two interest rates are positively correlated in most countries, with a median correlation of 0.7, and in some countries the relationship is very strong (see

Figure 5).<sup>12</sup> The Table also shows that the effective financing cost of firms is generally *higher* than the sovereign interest rates. This fact indicates that the common conjecture that firms (particularly the large corporations covered in our data) may pay lower rates than governments with default risk is incorrect.

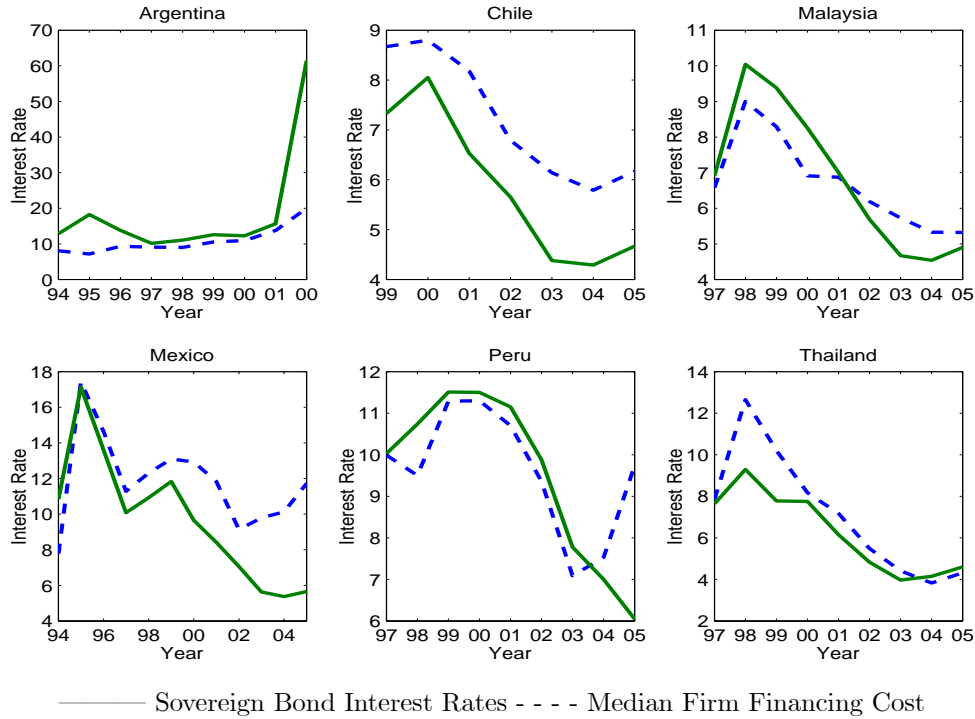


Figure 5: Sovereign Bond Interest Rates and Median Firm Financing Costs

Arteta and Hale (2007) provide further evidence on the strong effects of sovereign debt on the terms of private-sector debt contracts in emerging economies. In particular, they show strong, systematic negative effects on private corporate bond issuance during and after default episodes.

There is also evidence suggesting that our assumption that the government can divert the repayment of the firms' foreign obligations is realistic. In particular, it is not uncommon for the government to take over the foreign obligations of the corporate sector in actual default episodes. The following quote by the IMF historian explains how this was done in Mexico's 1982-83 default, and notes that arrangements of this type have been commonly used since then: *"A simmering concern among Mexico's commercial bank creditors was the handling of private sector debts, a substantial portion of which was in arrears...the banks and some official agencies had pressured the Mexican government to assume these debts...Known as the FICORCA scheme, this program provided for firms to pay dollar-denominated commercial*

<sup>12</sup> Arellano and Kocherlakota (2007) document a positive correlation between private domestic lending rates and sovereign spreads using the domestic lending-deposit spread data from the Global Financial Data.

debts in pesos to the central bank. The creditor was required to reschedule the debts over several years, and the central bank would then guarantee to pay the creditor in dollars. Between March and November 1983, close to \$12 billion in private sector debts were rescheduled under this program... FICORCA then became the prototype for similar schemes elsewhere.” (Boughton (2001), Ch. 9, pp. 360-361)

## 2.9 Recursive equilibrium

**Definition 2** *The model’s recursive equilibrium is given by (i) a decision rule  $b_{t+1}(b_t, \kappa_{t-1}, \varepsilon_t)$  for the sovereign government with associated value function  $V(b_t, \kappa_{t-1}, \varepsilon_t)$ , consumption and transfers rules  $c(b_t, \kappa_{t-1}, \varepsilon_t)$  and  $T(b_t, \kappa_{t-1}, \varepsilon_t)$ , default set  $D(b_t, \kappa_{t-1})$  and default probabilities  $p^*(b_{t+1}, \varepsilon_t)$ ; and (ii) an equilibrium pricing function for sovereign bonds  $q^*(b_{t+1}, \varepsilon_t)$  such that:*

1. *Given  $q^*(b_{t+1}, \varepsilon_t)$ , the decision rule  $b_{t+1}(b_t, \kappa_{t-1}, \varepsilon_t)$  solves the recursive maximization problem of the sovereign government (26).*
2. *The consumption plan  $c(b_t, \kappa_{t-1}, \varepsilon_t)$  satisfies the resource constraint of the economy*
3. *The transfers policy  $T(b_t, \kappa_{t-1}, \varepsilon_t)$  satisfies the government budget constraint.*
4. *Given  $D(b_t, \kappa_{t-1})$  and  $p^*(b_{t+1}, \varepsilon_t)$ , the bond pricing function  $q^*(b_{t+1}, \varepsilon_t)$  satisfies the arbitrage condition of foreign lenders (34).*

Condition 1 requires that the sovereign government’s default and saving/borrowing decisions be optimal given the interest rates on sovereign debt. Condition 2 requires that the private consumption allocations implied by these optimal borrowing and default choices be both feasible and consistent with a competitive equilibrium (recall that the resource constraint of the sovereign’s optimization problem considers only private-sector allocations that are competitive equilibria). Condition 3 requires that the decision rule for government transfers shifts the appropriate amount of resources between the government and the private sector (i.e. an amount equivalent to net exports when the country has access to world credit markets, or the diverted repayment of working capital loans when a default occurs, or zero when the economy is in financial autarky beyond the date of default). Notice also that given conditions 2 and 3, the consumption plan satisfies the households’ budget constraint. Finally, Condition 4 requires the equilibrium bond prices that determine country risk premia to be consistent with optimal lender behavior.

A solution for the above recursive equilibrium includes solutions for the private sector equilibrium allocations with and without credit market access. A solution for equilibrium interest rates on working capital as a function of  $b_{t+1}$  and  $\varepsilon_t$  follows from (35). Expressions for equilibrium wages, profits and the price of domestic inputs as functions of  $r_t$  and  $\varepsilon_t$  follow then from the firms’ optimality conditions and the definitions of profits described earlier.

### 3 Quantitative analysis

#### 3.1 Calibration

We study the quantitative implications of the model by conducting numerical simulations setting the model to a quarterly frequency and using the following benchmark calibration. The risk aversion parameter  $\sigma$  is set to 2 and the quarterly world risk-free interest rate  $r^*$  is set to 1 percent, which are standard values in quantitative business cycle and sovereign default studies. The curvature of labor disutility in the utility function is set to  $\omega = 1.455$ , which implies a Frisch wage elasticity of labor supply of  $1/(\omega - 1) = 2.2$ . This is the value typically used in RBC models of the small open economy (e.g. Mendoza (1991) and Neumeyer and Perri (2005))), and is based on estimates for the U.S. quoted by Greenwood, Hercowitz and Huffman (1988). The probability of re-entry after default is 0.125, which implies that the country stays in exclusion for 2 years after default on average, in line with the finding of Gelos et al. (2003).

The share of intermediate goods in gross output  $\alpha_M$  is set to 0.43. This value is the average ratio of intermediate goods consumption to gross production calculated using annual data for Argentina for the period 1993-2005 from the United Nation's *UNData*.<sup>13</sup> Given the value of  $\alpha_M$ , we set  $\alpha_k = 0.17$  so that the capital income share *in value added* of the  $f$  sector ( $\alpha_k/(1 - \alpha_m)$ ) matches the standard 30 percent ( $0.16/(1 - 0.43) = 0.3$ ). These factor shares imply a labor share in gross output of final goods of  $\alpha_L = 1 - \alpha_m - \alpha_k = 0.40$ , which yields a labor share *in value added*  $\alpha_L/(1 - \alpha_m) = 0.7$  that matches the standard 70 percent. The labor share in intermediate goods production  $\gamma$  is also set to 0.7, since this is also the share of labor in value added in the  $m$  sector.

Productivity shocks in final goods production follow an AR(1) process:

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \epsilon_t \quad (36)$$

with  $\epsilon_t \stackrel{iid}{\sim} N(0, \sigma_\epsilon^2)$ . Data limitations prevent us from estimating directly this process using actual TFP data, so we set  $\sigma_\epsilon^2$  and  $\rho_\varepsilon$  (together with other parameters to be discussed below) using the simulated method of moments (SMM). The target moments used to set  $\sigma_\epsilon^2$  and  $\rho_\varepsilon$  are the variability and persistence of GDP, which we calibrate to quarterly data for Argentina. This facilitates comparisons with the literature on quantitative models of default, which largely focuses on data for Argentina. We use seasonally-adjusted quarterly real GDP from the Ministry of Finance (MECON) for the period 1980Q1 to 2005Q4. The standard deviation and first-order autocorrelation of the cyclical component of H-P filtered GDP are 4.7 percent and 0.79 respectively. Given these targets, the process of productivity shocks

---

<sup>13</sup>Mendoza (2007) reports a very similar share for Mexico, and Gopinath, Itskhoki, and Rigobon (2007) show shares in the 40-45 percent range for several countries.

derived using SMM features  $\rho_\varepsilon = 0.90$  and  $\sigma_\varepsilon = 2.14$  percent. Note that, as we explained earlier, GDP in the model is measured as  $(1 - \alpha_m)\varepsilon M^{\alpha_m} L_f^{\alpha_l} \bar{k}^{\alpha_k} + p_m m^d$ , but in actual GDP data at constant prices the price of intermediate goods is fixed at the base year’s price. Hence, we keep  $p_m$  fixed at the value implied by the private sector equilibrium at a 1 percent interest rate and neutral TFP shock to compute a measure of the model’s GDP comparable with the data.

Table 2: Benchmark Model Calibration

<i>Calibrated Parameters</i>		<i>Value</i>	<i>Target statistics or source</i>	
CRRA risk aversion	$\sigma$	2	Standard value	
Risk-free interest rate	$r^*$	1%	Standard value	
Capital share in final goods gross output	$\alpha_k$	0.17	Standard GDP capital share (0.3)	
Intermediate share in final gross output	$\alpha_m$	0.43	GDP Intermediate share (0.43)	
Labor share in final goods gross output	$\alpha_L$	0.40	Standard GDP labor share (0.7)	
Labor share in GDP of int. goods	$\gamma$	0.7	Standard GDP labor share (0.7)	
Labor elasticity para.	$\omega$	1.455	Frisch wage elasticity (2.2)	
Re-entry Probability	$\eta$	0.125	Length of exclusion (2 years)	
<i>Parameters set with SMM</i>		<i>Value</i>	<i>Targets from Argentina’s data</i>	
Productivity persistence	$\rho_\varepsilon$	0.90	GDP persistence	0.79
Productivity innovations std. dev.	$\sigma_\varepsilon$	2.14%	GDP std. dev.	4.70%
Intermediate goods TFP	$A$	0.20	Output drop in default	13%
Weight para in M aggregation	$\lambda$	0.61	Nominal ratio of imported m	(18%)
Substitution para in M aggregate	$\mu$	0.69	Real ratio of imported m	(16%)
Time discount factor	$\beta$	0.87	Default frequency	0.69%
Working capital friction	$\theta$	0.10	Trade balance volatility	2.88%

The additional parameters calibrated using SMM are the CES parameters of the intermediate goods aggregator  $\mu$  and  $\lambda$ , the productivity coefficient in production of domestic inputs  $A$ , the subjective discount factor  $\beta$ , and the share of imported inputs paid for with working capital  $\theta$ . These parameters are targeted to match (respectively) the average nominal and real ratios of imported intermediate goods to domestic intermediate goods, the fraction of output loss at default, the frequency of default, and the volatility of the trade balance-GDP ratio.<sup>14</sup> Given serious limitations of the national accounts data for Argentina, the target statistics for the average expenditure ratios of imported to domestic inputs are computed using Mexican data for the period 1988-2004. We assume that Argentina has similar ratios. The average ratios of imported to domestic inputs at current and constant prices are 18 and 15.7 percent respectively. The default frequency is 0.69 percent because Argentina has defaulted five times

<sup>14</sup>Note that  $A$  can be used to target the output drop at default because, as mentioned in Section 2, changes in  $A$  have similar effects as changes in  $\omega$ . In particular, lower values of  $A$  yield larger output drops at default.

on its external debt since 1824 (the average default frequency is 2.78 percent annually or 0.69 percent quarterly). The average output loss at default for Argentina is 13 percent based on the cyclical position of the country’s quarterly GDP around the December 2001 debt crisis. The standard deviation of the quarterly trade balance-to-GDP ratio for Argentina is 2.88 percent.

Table 2 shows the parameters of the benchmark calibration. The SMM estimate of  $\mu$  is 0.69 and the estimate of  $\lambda$  is 0.61, so the elasticity of substitution between  $m^*$  and  $m^d$  is 3.22 and there is a small bias in favor of domestic inputs. The subjective discount factor is 0.87, which is in the range of the values used in existing studies on sovereign default.<sup>15</sup> The estimate for  $A$  is 0.2. Finally, the estimate for  $\theta$  implies that firms pay only 1/10 of the cost of imported inputs in advance.

### 3.2 Results of the Benchmark Simulation

This subsection examines the model’s ability to account for the three key empirical regularities of sovereign debt highlighted in the Introduction: V-shaped output dynamics with deep recessions that hit bottom at times of default, countercyclical country interest rates, and high debt ratios. To explore this issue, we feed the TFP process to the model and conduct 2000 simulations, each with 500 periods and truncating the first 100 observations.

The quantitative predictions of the model approximate well the three stylized facts of sovereign debt, as well as two key business cycle regularities: the cyclical variability of consumption and the negative correlation of net exports with GDP. Table 3 compares the moments produced by the model with moments from Argentine data. The bond spreads data are quarterly EMBI+ spreads on Argentine foreign currency denominated bonds from 1994Q2 to 2001Q4, taken from J.P. Morgan’s EMBI+ dataset.

The model is consistent with the data in predicting a positive correlation between spreads and net exports, and negative correlations of spreads and net exports with GDP. The model accounts for the negative correlation between spreads and GDP because sovereign bonds have higher default risk in bad states. Several quantitative models of sovereign debt (e.g. Arellano (2007), Aguiar and Gopinath (2005), Yue (2006)) and business cycle models of emerging economies (e.g. Neumeyer and Perri (2005), Uribe and Yue (2006)) also produce countercyclical spreads, but the former treat output as an exogenous endowment and in the latter country risk is exogenous. In contrast, our model matches the negative correlation between GDP and spreads in a setting in which both output and country risk are endogenous, and influence each other because of the relationship between country risk and working capital financing. The countercyclical net exports follow from the fact that, when the country is in a bad state, it faces higher interest rates and tends to borrow less. The country’s trade balance

---

<sup>15</sup>The values of  $\beta$  used by Aguiar and Gopinath (2006), Arellano (2007), and Yue (2006) range from 0.8 to 0.953.



thus increases because of the lower borrowing, leading to a negative correlation between net exports and output.

Table 3: Model Simulation and Statistics in the Data

Statistics	Data	Model
Corr. between Bond Spreads and GDP	-0.62	-0.19
Corr. between Bond Spreads and Trade Balance	0.68	0.18
Corr. between Trade Balance and GDP	-0.58	-0.24
Consumption Std. Dev./Output Std. Dev.	1.44	1.31
Average Debt/GDP	35%	23.11%
Bond Spreads Std. Dev.	0.78%	0.71%
Average Bond Spreads	1.86%	0.58%
Corr. between $M$ and Spreads	-	-0.22
Corr. between $M$ and GDP	-	0.70
Corr. between total labor and Spreads	-	-0.21
Corr. between total labor and GDP	-	0.72

Consumption variability exceeds output variability in Argentina, and this is a common feature across emerging economies. The model is able to mimic this stylized fact because the ability to share risk with foreign lenders is negatively affected by the higher interest rates induced by increased default probabilities. The sovereign borrows less when the economy faces an adverse productivity shock, and thus households adjust consumption by more than in the absence of default risk. On the other hand, because agents are impatient, the benevolent government borrows more to increase private consumption when the productivity shock is good. Hence, the variability of consumption rises.

The model produces a debt-to-GDP ratio of 23 percent on average. This high debt ratio is mainly the result of the large output drop that occurs when the country defaults, and the fact that the size of the drop is increasing in the state of TFP. Although a 23 percent debt ratio is still below Argentina's 35 percent average debt-output ratio (based on data from the World Bank's WFD dataset for the 1980-2004 period), it is several orders of magnitude larger than the debt ratios typically obtained in quantitative models of sovereign default with exogenous output costs already targeted to improve the models' quantitative performance. For instance, Yue's (2006) model with renegotiation and an exogenous 2 percent output cost at default yields an average debt ratio of 9.7 percent. Arellano (2007) obtains a mean debt ratio of 6 percent of GDP assuming that output when the economy defaults equals the maximum of actual output or 97 percent of average output.<sup>16</sup>

The model also matches closely the volatility of the Argentine bond spreads observed in the data. Yet the average bond spread is lower than in the data. Because we assume a zero

<sup>16</sup>As mentioned earlier, Aguiar and Gopinath (2006) obtained a higher mean debt ratio ( 27 percent of GDP) assuming a cost of 2 percent of output, but with a default frequency of only 0.02 percent.

recovery rate on defaulted debt and risk-neutral creditors, bond spreads are linked one-to-one with default probabilities (see 34). Since the quarterly default frequency is 0.7 percent (as in the data), the model can only generate an average bond spread around this magnitude, which is about one third of the average spreads observed in the data.

Table 3 also shows that there is a negative correlation between sovereign spreads and intermediate goods due to the working capital requirement. As discussed in Section 2, the model also generates a negative correlation between labor and spreads, while at the same time both intermediate goods and labor are procyclical.

### 3.3 Dynamics of Output Around Default Episodes

We illustrate the model’s ability to match V-shaped dynamics of output around default episodes by applying event study techniques to simulated time series data. Figure 6 plots the model’s average path of output around default events together with the data for Argentina’s HP detrended GDP around the 2001 default (1999Q1 to 2004Q2). The event window covers 12 quarters before and 10 quarters after debt defaults, with the default events normalized to date 0. We plot the average for output in the model at each date  $t = -12, \dots, 10$  around default events in the 2000 simulations. Hence, the simulated GDP line represents the average behavior of output around defaults in the stationary distribution of the model. Since Argentina’s data is for a single default event, while the model’s output dynamics correspond to the model’s stochastic stationary state, we add dashed lines with one-standard-error bands around the simulation averages. Note that the relative magnitudes of the recession and recovery match the data quite well. The output dynamics for Argentina before and after the 2001 debt crisis are mostly within the one-standard-error bands of the model simulations.

Figure 6 shows that the model produces a substantial output drop when the country defaults, equivalent to about 13 percent of the pre-default output level. Defaults are always triggered by adverse productivity shocks, but these shocks do not need to be unusually large. The standard deviation of the calibrated TFP process ( $\sigma_\varepsilon$ ) is 4.91 percent. By contrast, the average decline in TFP in default events (i.e. at  $t = 0$  in Figure 6) is 5 percent, which is about the size of the TFP standard deviation. This suggests that the model embodies a business cycle transmission mechanism that can amplify significantly the real effects of TFP shocks when these shocks trigger default.

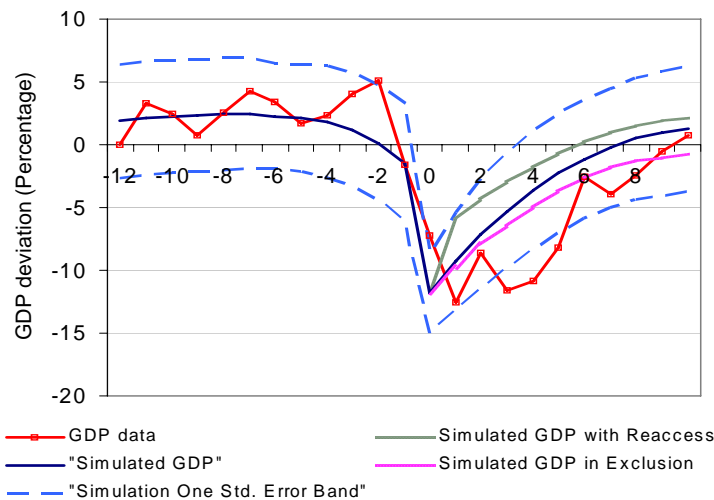


Figure 6: Output around Default Events

The model displays a V-shaped recovery during the financial exclusion period. This recovery is driven by two effects. First, since the TFP shock is mean-reverting, TFP is likely to improve after default (for example, on average in the simulations TFP rises by 1 percent at  $t = 1$ ). Therefore, even though the country remains in financial autarky on average from dates 1 to 10, the economy recovers because TFP improves. The second effect driving the recovery is the surge in output that occurs when the country re-enters credit markets (as final goods producers switch back to a more efficient mix of imported and domestic inputs).

The two effects that induce the post-default recovery are illustrated in Figure 6 by the lines that show the simulated paths of GDP with continued exclusion for 10 quarters after default and with immediate re-entry one period after default. In the first scenario, the recovery reflects only the effect of the mean reversion of the TFP shock. GDP remains below that in the simulation average, and it is also lower than in the data starting in the 6th quarter after default. In contrast, the second scenario with immediate re-entry to international debt markets shows a big rebound in GDP at  $t = 1$  because of the efficiency gain obtained as final goods producers switch back to imported inputs. The simulation average lies below this immediate re-entry line because in the model the re-entry to credit markets is stochastic with 12.5 percent probability. Since re-entry has a relatively low probability, the model simulation for average GDP weighs more the TFP recovery effect than the credit market re-entry effect.

These V-shaped dynamics are qualitatively consistent with the data of many emerging markets that suffered Sudden Stops. Calvo, Izquierdo and Talvi (2006) conducted a detailed cross-country empirical analysis of the recovery of emerging economies from Sudden Stops, and found that most recoveries are not associated with improvements in credit market access. In our model as well, recovery occurs (on average) even though the economy continues to be excluded from world credit markets.

The model’s output dynamics also suggest that the model can account for the seemingly dominant role of productivity shocks in explaining output collapses during financial crises in emerging markets. In particular, this can be the result of the efficiency loss caused by the sectoral reallocation of labor and the change in the mix of aggregate intermediate goods when the economy defaults. To demonstrate this point, we use the model’s simulated data on aggregate factor payments, GDP, and labor to compute Solow residuals in the standard way: We assume an aggregate Cobb-Douglas production function for economy-wide GDP,  $gdp_t = s_t(L_t)^a k^{1-a}$ , and compute the Solow residual  $s$  using the model’s data for  $L$  and  $gdp$ , setting  $a$  to the model’s average of the ratio of total wage payments to GDP,  $w_t L_t / gdp_t$ , which is about 0.7. By construction, however, the “true” TFP driving the model is  $\varepsilon_t$  in the production function of final goods.

Figure 7 compares the quarter-on-quarter average growth rates of the Solow residual, true TFP and GDP around default events in the baseline model simulations. Clearly, there is little difference between the Solow residual and true TFP except when the economy defaults. In default events, however, the Solow residual overestimates the true adverse TFP shock by a large margin (on average,  $s$  falls by nearly twice as much as  $\varepsilon$  when the economy defaults).

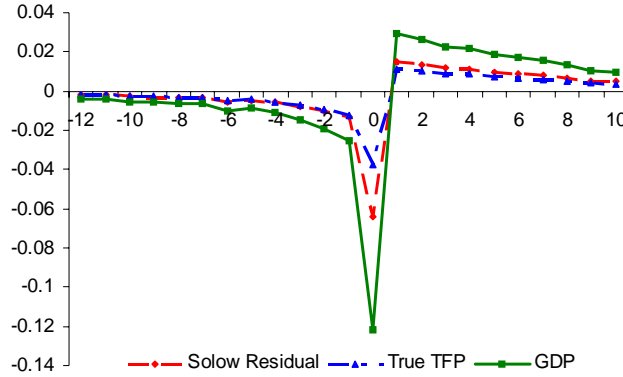


Figure 7: Growth Rates of GDP, True TFP and Solow Residual around Default

Moreover, a standard decomposition of the contributions of changes in TFP and factors of production to changes in GDP shows that the contribution of true TFP to the output collapse at default is about 31 percent. In contrast, the contribution of the Solow residual is nearly 53 percent, which would suggest misleadingly that the contribution of TFP shocks is 1.72 times larger than it actually is. The large difference between the two is due to the fact that the Solow residual treats the efficiency loss caused by the sectoral reallocation of labor and the lower use of intermediate goods as a reduction in TFP in final goods production.

As another way to illustrate the amplification effect, we also compute the level of output when the country receives a 4.9% TFP shock and yet does not default. If default did not happen, the same level of TFP shock which triggers default would only generate 7.15% output drop, which is substantially lower compared with the average 13% of output decline due to

default in the simulation. The ratio between the two output drops is 1.82. This amplification coefficient can measure how defaults in the model exacerbate the drop in production in the model.

The model also matches nicely the dynamics of sovereign bond spreads before a debt crisis. The left panel of Figure 8 presents event windows showing the mean of simulated output and bond spreads up to 12 quarters before default events in the stationary distribution of the model. This plot clearly illustrates the negative correlation between output and bond spreads before a debt crisis. In particular, the spread increases as the country approaches a debt crisis. The average *quarterly* spread increases from 0.6 percent at  $t = -12$  to almost 2.5 percent in the quarter before default. At the same time, HP detrended output starts to decline three quarters before default and suffers a sharp drop when default occurs. These features match relatively well the Argentine experience. The right-side panel of Figure 8 shows the HP detrended real GDP and EMBI+ sovereign bond spreads for Argentina from 1994Q1 to 2001 Q4. The data show a relatively stable sovereign spread before 2000 and a sharp increase in 2001, and Argentina also experienced a relatively steady output performance and then a very deep recession starting in 2001.

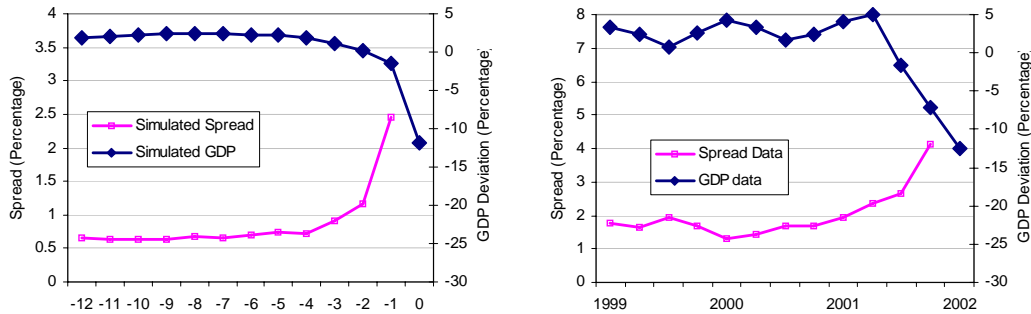


Figure 8: Dynamics of Output and Sovereign Spreads before a Debt Crisis

Figure 9 shows the event windows for the average of the model simulations of consumption, current account and debt, as well as labor, intermediate goods, and sectoral labor allocations (along with the corresponding one-standard-error bands). Consumption drops sharply when the government defaults and in the period that follows, and then it recovers following the V-shaped dynamics of GDP. The debt-output ratio is over 23 percent on average before default, and it increases to about 32 percent in the period just before default. The external accounts also experience sharp adjustments around default episodes. In particular, the model generates a sharp reversal in the current account. The country runs a small current account deficit on average, but default, and the loss of credit market access that it entails, produce a large jump of about 30 percentage points of GDP in the current account. Labor and intermediate goods also fall sharply when the economy defaults. Moreover, since default triggers a shift

from imported to domestic inputs in final goods production, labor is reallocated from the  $f$  sector to the  $m$  sector.

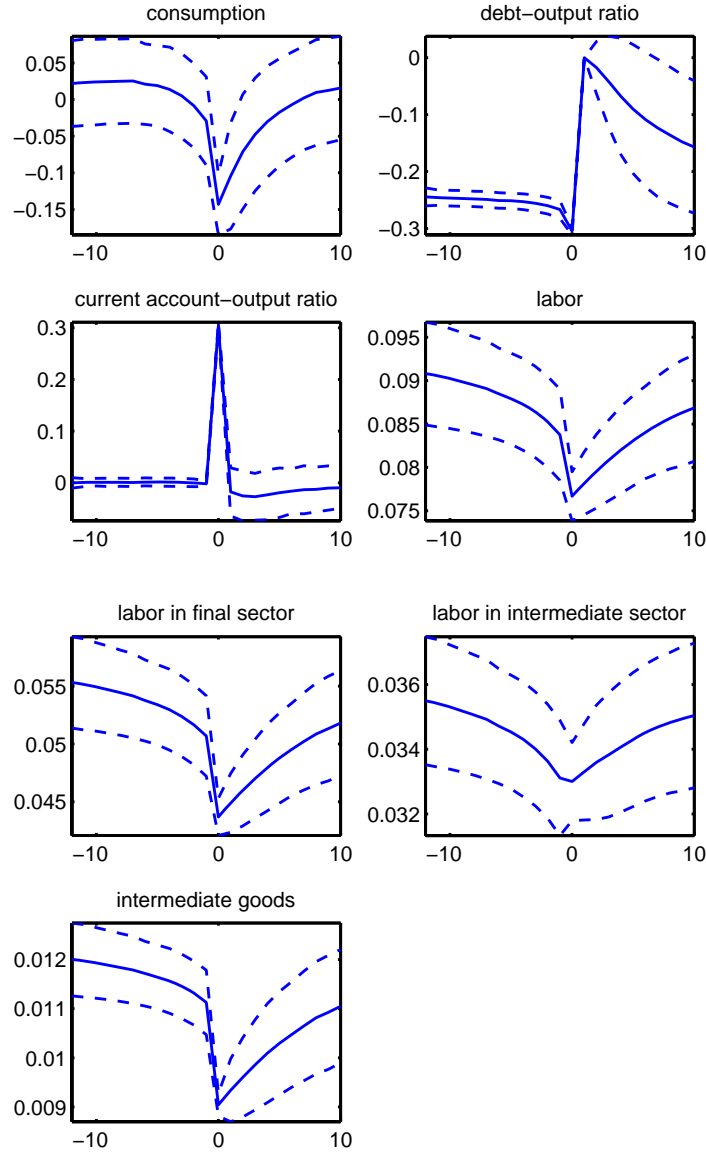


Figure 9: Macro Dynamics around Default Episodes

The sharp declines in GDP, consumption, labor and intermediate goods, together with the large reversal in the current account, indicate that the model yields predictions consistent with the Sudden Stops observed in emerging economies. In most of the Sudden Stops literature, however, the current account reversal is modeled as an exogenous shock, whereas in this model both the current account reversal and the economic collapse are endogenous. Moreover, this endogenous Sudden Stop is driven by default risk determined by an optimal recursive contract, instead of the ad-hoc collateral constraints emphasized in other models of endogenous Sudden

Stops (see Mendoza (2007)).

### 3.4 Key Features of the Equilibrium with Default

How does the interaction between endogenous output fluctuations and endogenous default risk affect the quantitative performance of the model? We answer this question by studying the behavior of the value function when the country has access to world financial markets, the bond pricing function, the saving decision rule, and output. Figure 10 shows these equilibrium functions for different TFP shocks and working capital levels as a function of the country's bond position.

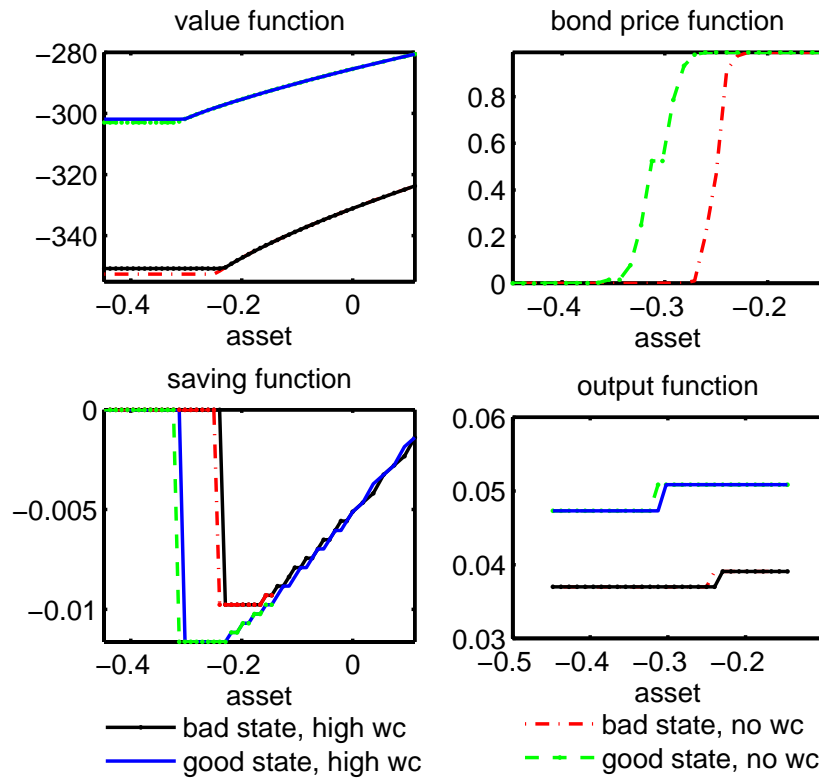


Figure 10: Value Function, Bond Price Function, and Decision Rules

The first panel of Figure 10 shows that the value function increases with the bond position for the range of bond positions higher than the value at which default is certain. For positions smaller than this value, the value function becomes independent of  $b$  because the country is in financial autarky. As the country's debt increases (i.e.  $b$  falls) the value of default can exceed the value of not defaulting. The country is more likely to default if TFP is low because the default option is more attractive. This is because it is more painful to repay the debt in a bad state, while at the same time default does not lead to a high output loss compared to the case with a good TFP shock. The value function also differs as we vary the amount of

working capital  $\kappa$ . The value of default increases slightly with working capital because the government transfers the repayment of working capital loans to households when it defaults.

The second panel of Figure 10 shows that bond prices increase with  $b$  (i.e. decrease with the debt position), reflecting the standard result that default risk premia are higher at higher levels of debt. Moreover, bond prices are higher when the country experiences a good TFP shock, and higher bond prices imply lower default risk premia, lower default probabilities and lower country interest rates. Working capital financing becomes less costly as a result, leading final goods producers to increase demand for foreign inputs and produce more. This feedback from country interest rates to output also affects the country's incentives to default, reinforcing the reduction in default risk. The opposite is true when the country experiences a negative TFP shock, and this is an important result because it implies that, for any given level of debt before the country is in financial autarky, default is more likely when TFP is low than when it is high (recall that  $b$  and  $\varepsilon$  are the state variables that determine  $q$ ).

The lower-left panel of Figure 10 shows that the country borrows less (i.e. chooses a higher  $b'$ ) when it experiences a low TFP shock. This property of the bond decision rule is reflected in the countercyclical trade balance and the positive correlation between the trade balance and sovereign spreads documented earlier.

Finally, the lower-right panel of Figure 10 shows that the relationship between output and  $b$  follows “almost” a two-step function. The lower step corresponds to the range of high debt in which firms operate with domestic inputs because the country defaults. The higher step pertains to the range of debt positions when the country has access to world credit markets and firms use both imported and domestic inputs. Output in this region depends on  $b$  (so the output plots are not truly two step functions), but the Figure would need a finer scale for the relationship to be visible. In this region, output fluctuates with country risk because the demand for  $m^*$  is directly affected by  $r$ . When the country borrows more, default risk increases and this raises the financing cost of working capital. In response, firms cut demand for imported inputs and output falls. The plot also shows that the size of the output drop at default is larger with the good productivity shock because the cost of default is increasing in TFP, as explained earlier.

## 4 Sensitivity Analysis

In this Section we conduct a sensitivity analysis to study how the model's quantitative predictions change under alternative assumptions about the working capital requirement  $\theta$ , the parameters of the CES composite of intermediate goods  $\mu$  and  $\lambda$ , and the labor supply curvature parameter  $\omega$ .



## 4.1 Working Capital

The second column of Table 4 shows the results of a simulation of a variant of the model without working capital ( $\theta = 0$ ). In this case, the output cost of default is invariant to productivity shocks, as in some of the existing quantitative studies of sovereign debt that assume that income is an exogenous endowment (e.g. Aguiar and Gopinath (2006), Yue (2006)). To keep the results comparable, we introduce an exogenous output cost of default into this variant of our model calibrated so as to keep matching the output cost of default of 13 percent observed in the data. The other parameters are kept unchanged. The third column of the Table reproduces the results for the benchmark model.

Table 4: Changes in the Working Capital Constraint

Statistics	No WC	Benchmark	Larger WC
		$\theta = 0.1$	$\theta = 0.15$
Output loss	13.0%	13.2%	9.1%
GDP std. dev.	4.76%	4.69%	5.11%
Default probability	0.12%	0.60%	4.19%
Debt/GDP	0.15%	23.11%	17.82%
Bond spreads std. dev.	0.16%	0.71%	2.98%
Average Bond Spreads	0.12%	0.58%	3.09%
Corr. between Spreads and GDP	-0.04	-0.19	-0.06
Corr. between Spreads and TB	-0.21	0.18	0.08
Corr. between TB and GDP	-0.28	-0.24	-0.03
Nominal ratio of imported m	17.9%	17.8%	12.3%
Real ratio of imported m	16.0%	15.6%	13.6%
Trade Balance std. dev.	0.01%	2.18%	2.54%

The model without working capital performs much worse than the benchmark in terms of its ability to match the important features of the data that the benchmark model approximated well. The frequency of defaults falls from 0.6 percent to 0.12 percent. The mean debt ratio declines by nearly 23 percentage points of GDP, and the average and standard deviation of country spreads fall by 55 basis points. These results follow from two important differences in the model without working capital relative to the benchmark: First, the cost of default becomes independent of the state of nature, and second, bond spreads no longer have a direct impact on production. As a result, debt is not as good a hedging mechanism as in the benchmark model, making default more painful ex ante in the model without working capital, and thus reducing the average debt ratio. Moreover, the model without working capital cannot produce the V-shaped output dynamics that the benchmark model generated (see Figure 6), because it maintains the disconnect between country risk and business cycles.

In addition, the GDP correlation of sovereign spreads increases sharply to -0.04, and the correlation between spreads and net exports changes from 0.18 to -0.21.

How sensitive are the model's results to the value of the working capital requirement beyond the extreme case of  $\theta = 0$ ? To answer this question, we report in the last column of Table 4 results for  $\theta = 0.15$ , instead of 0.1 as in the benchmark case. The higher working capital requirement reduces the mean debt ratio by 6 percentage points of GDP and generates a smaller output cost of default. In contrast, the variability of GDP, the probability of default, and the mean and standard deviation of spreads all increase sharply as  $\theta$  rises.

These changes reflect the fact that the higher  $\theta$  has opposing effects on default incentives and production plans. On one hand, final goods producers like to use domestic inputs more, since higher  $\theta$  increases the effective price of imported inputs, and thus changes in sovereign interest rates have a larger impact on production. This amplifies the response of output to productivity shocks, making output more volatile. This result is complementary to the finding in Uribe and Yue (2006) showing that the impact of output on country interest rates magnifies business cycle volatility, and the result in Neumeyer and Perri (2005) showing that working capital loans that charge sovereign interest rates also amplify business cycle volatility. On the other hand, default leads to a lower output cost of default on average because the TFP shock that triggers default is smaller with  $\theta = 0.15$  than in the benchmark case with  $\theta = 0.1$ . Thus, the output levels before and after default are closer, generating a smaller output loss. At the same time, this lower output cost of default and the higher volatility in GDP make the sovereign exercise the default option more often, increasing the default probability and the volatility of bond spreads, and reducing the mean debt/GDP ratio. The quantitative effects of tightening the working capital constraint on the debt/GDP ratio and the default frequency are particularly large, and we get these results even though average sovereign spreads, and hence the average interest rate on working capital, do not deviate sharply from the one-percent risk free rate.<sup>17</sup>

## 4.2 Imperfect Substitution between Foreign and Domestic Inputs

The second column of Table 5 reports simulation results reducing  $\mu$  from 0.69 in the benchmark case to  $\mu = 0.5$ . This reduces the elasticity of substitution between  $m$  and  $m^*$  from 3.22 to 2. The third column reproduces the results for the benchmark calibration. The fourth column reports results lowering the CES weight parameter from the benchmark value of 0.61 to  $\lambda = 0.5$ . All of the other parameters are the same as in the benchmark calibration.

---

<sup>17</sup>Neumeyer and Perri (2005) and Uribe and Yue (2006) use average interest rates around 7 percent and set  $\theta = 1$ , and they find that the working capital constraint is important for business cycle dynamics. Oviedo (2005) also showed that obtaining significant effects of working capital in the small open economy RBC model requires high values of  $r^*$  and  $\theta$ .

Table 5: Changes in Parameters of CES Aggregator of Intermediate Goods

Statistics	Lower Elasticity $\mu = 0.5$	Benchmark $\mu = 0.69$ $\lambda = 0.61$	Lower Weight $\lambda = 0.5$
Output loss	25.1%	13.2%	16.9%
GDP std. dev.	4.95%	4.69%	5.43%
Default probability	0.02%	0.60%	0.19%
Debt/GDP	168.59%	23.11%	92.36%
Bond spreads std. dev.	0.07%	0.71%	2.28%
Average Bond Spreads	0.02%	0.58%	0.12%
Corr. between Spreads and GDP	-0.03	-0.19	-0.33
Corr. between Spreads and TB	0.01	0.18	0.09
Corr. between TB and GDP	-0.29	-0.24	-0.37
Nominal ratio of imported m	34.3%	17.8%	58.2%
Real ratio of imported m	29.2%	15.6%	46.2%

The results reported in Table 5 show that the values of  $\mu$  and  $\lambda$  affect significantly the magnitude of the output loss at default, as would be expected given the analysis of Section 2. With a lower elasticity of substitution or a lower weight on domestic inputs, imported inputs are more important in final goods production.<sup>18</sup> When the elasticity of substitution is lower, domestic inputs are poorer substitutes for imported inputs, and hence the efficiency loss of the  $f$  sector when the economy defaults is bigger. Similarly, if domestic (imported) inputs have a lower (higher) weight in the CES aggregator  $M$ , default is more painful when access to foreign inputs is lost. Accordingly, the output costs of default in the scenarios with lower  $\mu$  and lower  $\lambda$  reach about 25 and 17 percent respectively, compared with 13.3 percent in the benchmark. Because of this higher output costs of default, the probabilities of default are lower (0.02 and 0.19 percent respectively) and the mean debt ratios are much higher (169 and 92 percent respectively).

Changes in  $\mu$  and  $\lambda$  also affect business cycle comovements. Lower  $\mu$  and lower  $\lambda$  produce higher output variability. The standard deviations of GDP increase to 4.9 and 5.4 percent respectively, compared with 4.7 in the benchmark. The correlations of GDP with spreads and net exports remain negative, as in the benchmark case, but they are significantly more negative with  $\lambda = 0.5$ .

<sup>18</sup>This is also reflected in the nominal and real expenditure ratios of imported to domestic inputs. These ratios are much higher with lower  $\mu$  or lower  $\lambda$  than in the benchmark case.

### 4.3 Labor Supply Elasticity

Table 6 presents the results of a simulation rising the curvature parameter  $\omega$  from 1.45 in the benchmark to  $\omega = 2.0$ . Hence the labor supply elasticity falls from 2.2 to 1 (i.e. unitary elasticity).

Table 6: Changes in Elasticity of Labor Supply

Statistics	Lower Elasticity	Benchmark
	$\omega = 2.0$	$\omega = 1.45$
Output loss	11.5%	13.2%
GDP std. dev.	3.45%	4.69%
Default probability	0.14%	0.60%
Debt/GDP	22.87%	23.11%
Bond spreads std. dev.	0.18%	0.71%
Average Bond Spreads	0.15%	0.58%
Corr. between Spreads and GDP	-0.11	-0.19
Corr. between Spreads and TB	0.01	0.18
Corr. between TB and GDP	-0.34	-0.24
Nominal ratio of imported m	17.8%	17.8%
Real ratio of imported m	15.6%	15.6%

As demonstrated in Section 2, a lower labor supply elasticity generates a lower output cost of default, without altering the slope of the relationship between default costs and TFP. Hence, the model with a less elastic labor supply can support less debt. At the same time, however, output variability declines, since labor variability is lower with the lower elasticity. As a result, the default probability does not necessarily increase, despite the lower default penalty. This is because the two effects push in opposite directions: The lower output cost tends to increase the default frequency, but the lower output variability tends to reduce it. The results in the second column of Table 6 show that in the simulation with  $\omega = 2.0$ , the net effect yields a lower default frequency than in the benchmark case. The bond spreads are lower and less volatile as well. The correlation between GDP and spreads rises, while net exports become more countercyclical. Also, in line with the results shown in Section 2, the expenditure ratios of imported to domestic inputs are not affected by changes in the labor supply elasticity.

## 5 Conclusions

This paper proposed a model of strategic sovereign default with endogenous output dynamics and examined its quantitative predictions. In the model, producers of final goods choose an

optimal mix of imported and domestic inputs. Purchases of foreign inputs require foreign working capital financing. Purchases of domestic inputs do not require credit, but these inputs are imperfect substitutes for foreign inputs, and producing them requires reallocation of labor away from final goods production. As result, default causes an efficiency loss by forcing final goods producers to operate using only domestic inputs, and inducing labor to reallocate from the final goods sector to the sector producing domestic inputs. Lenders charge the same default risk premium on working capital loans as on sovereign debt because the sovereign diverts the repayment of working capital loans when the country defaults. This is in line with empirical evidence showing that corporate and sovereign interest rates are strongly correlated, and that in sovereign defaults since the 1980s Debt Crisis we often observe governments taking over the foreign obligations of private firms.

The model is consistent with three key stylized facts of sovereign debt: (1) the V-shaped dynamics of output around default episodes, (2) the negative correlation between interest rates on sovereign debt and output, and (3) high debt-output ratios on average and when defaults take place. The model also replicates the observed countercyclical dynamics of net exports, the positive correlation between country spreads and GDP, and the variability of private consumption, and it is calibrated to be consistent with observed default frequencies.

The model produces an endogenous output cost of default that is increasing in the state of productivity. This result follows from the fact that the financing cost of working capital when default occurs rises too much for firms to find it profitable to use imported inputs, and hence they optimally switch to domestic inputs and suffer the corresponding efficiency loss. In turn, this efficiency loss is larger the higher TFP was before the switch. This increasing endogenous output cost of default is consistent with the shape of exogenous output costs that Arellano (2007) identified as necessary in order to obtain default incentives that trigger default in bad states of nature, at non-negligible debt ratios and at realistic spreads (or default frequencies). However, the endogenous feedback between production and default in our model produces a mean debt ratio four times larger than in Arellano's endowment economy model.

Our results also show that the model can provide an explanation for the seemingly large contribution of productivity shocks to output collapses during financial crises. In particular, we showed that a standard Solow residual overestimates significantly the contribution of true TFP to the collapse of output when the economy defaults, because it masks the efficiency loss due to costly labor reallocation and reduced usage of intermediate goods as a decline in TFP.

Three features of the model are critical for the results: imported inputs require working capital, the government diverts the firms' working capital repayment when it defaults, and default induces an efficiency loss in production, because it forces final goods producers to operate using only domestic inputs, which are imperfect substitutes for imported inputs and require reallocation of labor away from final goods to be produced. Without the first two

features, output would not respond to changes in country risk. On the other hand, the model would also fail if we rely “too much” on those two features: If exclusion from world credit markets implies that firms cannot buy foreign inputs and there are no domestic inputs available, the output collapse and the associated cost of default would be unrealistically large (infinitely large if 100 percent of the cost of imported inputs requires payment in advance). In reality, firms in emerging economies facing financial crisis substitute foreign inputs with high financing costs for domestic inputs that can be employed at permissible financial terms, and/or look for alternative forms of credit, including inter-enterprise credit and internal financing using retained earnings or redirecting capital expenditures. The efficiency loss is also critical. Without it the working capital channel would not produce a sharp and sudden drop in output during periods of financial turmoil.

Our findings suggest that the model we proposed can provide a solution to the disconnect between sovereign debt models (which rely on exogenous output dynamics with particular properties to explain the stylized facts of sovereign debt) and models of emerging markets’ business cycles (which assume an exogenous financing cost of working capital set to match the interest rate on sovereign debt). We acknowledge, however, that the linkages between sovereign default and private sector borrowers, and the mechanisms by which default induces economy-wide efficiency losses, should be the subject of further research. For instance, the studies by Cuadra and Saprizza (2008) and D’Erasmo (2008) show that political uncertainty may help endowment economy models in the Eaton-Gersovitz class to generate higher debt ratios at observed default frequencies. This suggests that introducing a mechanism to link political uncertainty with private sector decisions in a model with sovereign risk can be a promising line of research. Similarly, the findings of Bi (2008a and 2008b) on debt dilution effects and dynamic renegotiation in endowment economy models suggest that adding these features to default models with endogenous output dynamics can also be important. Finally, results obtained by Arellano (2007), Lizarazo (2005) and Volkan (2008) suggest that adding risk-averse foreign lenders can also contribute to produce higher debt ratios and break the one-to-one link between spreads and default probabilities, so that bond spreads include an additional risk premium and can get closer to the data.

## References

- [1] Aguiar, Mark and Gita Gopinath, 2006, "Defaultable Debt, Interest Rates and the Current Account," , Journal of International Economics.
- [2] Arellano, Cristina, 2007, "Default Risk and Income Fluctuations in Emerging Economies," American Economic Review, forthcoming.
- [3] Arellano, Cristina and Narayana Kocherlakota, 2007, "Internal Debt Crises and Sovereign Defaults," Manuscript, University of Minnesota.
- [4] Arteta, Carlos and Galina Hale, 2007, "Sovereign Debt Crises and Credit to the Private Sector," Journal of International Economics, Vol 74 (1), 53-69.
- [5] Bai, Yan and Jing Zhang, 2006, "Financial Globalization and International Risk Sharing," Manuscript, University of Michigan and Arizona State University.
- [6] Bi, Ran, 2008a, "Debt Dilution and the Maturity Structure of Sovereign Bonds," Ph.D. Dissertation, Department of Economics, University of Maryland.
- [7] Bi, Ran, 2008b, "Beneficial Delays in Debt Restructuring Negotiations," Ph.D. Dissertation, Department of Economics, University of Maryland.
- [8] Benjamin, David M. and Felipe Meza, 2007. "Total Factor Productivity and Labor Reallocation: the Case of the Korean 1997 Crisis," mimeo, Universidad Carlos III de Madrid.
- [9] Boughton, James M., 2001, "Silent Revolution: The International Monetary Fund 1979-1989," Ch. 9, IMF Publication.
- [10] Calvo, Guillermo A., Alejandro Izquierdo and Ernesto Talvi, 2006, "Phoenix Miracles in Emerging Markets: Recovering Without Credit form Systematic Financial Crises," NBER Working Paper 12101.
- [11] Cuadra, Gabriel and Horacio Sapriza, 2007, "Fiscal Policy and Default Risk in Emerging Markets," Bank of Mexico Working Paper.
- [12] Cuadra, Gabriel and Horacio Sapriza, 2008, "Sovereign Default, Interest Rates and Political Uncertainty in Emerging markets," Bank of Mexico Working Paper.
- [13] D'Erasmus, Pablo, 2008, "Government Reputation and Debt Repayment in Emerging Economies," Ph.D. Dissertation, Department of Economics, University of Texas-Austin.

- [14] Eaton, Jonathan and Mark Gersovitz, 1981, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, Vol 48(2), 289-309.
- [15] Edwards, Sebastian, 1984, "LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976-80," *American Economic Review*, Vol 74 (4), 726-734.
- [16] Finn, Mary G., 1995, "Variance Properties of Solow's Productivity Residual and Their Cyclical Implications," *Journal of Economics Dynamics and Control*, 19, 1249-1281.
- [17] Gelos, Gaston R., Ratna Sahay, Guido Sandleris, 2003, "Sovereign Borrowing by Developing Countries: What Determines Market Access?" IMF Working Paper.
- [18] Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon, 2007, "Currency Choice and Exchange Rate Pass-through," Manuscript, Harvard University.
- [19] Greenwood, Jeremy, Gregory W. Huffman and Zvi Hercowitz, 1988, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Association*, vol. 78(3), 402-417.
- [20] Hall, Robert E., 2007, "Sources and Mechanisms of Cyclical Fluctuations in the Labor Market," Manuscript, Stanford University.
- [21] Levy-Yeyati, Eduardo and Ugo Panizza, 2006, "The Elusive Cost of Sovereign Default," CIF Working Paper No. 11/2006.
- [22] Lizarazo, Sandra Valentina, 2005, "Default Risk and Risk Averse International Investors," mimeo, Department of Economics, ITAM, Mexico.
- [23] Mendoza, Enrique G., 1991. "Real Business Cycles in a Small Open Economy". *American Economic Review* Vol 81(4), 797-818.
- [24] Mendoza, Enrique G., 2007, "Endogenous Sudden Stops in a Business Cycle Model with Collateral Constraints: A Fisherian Deflation of Tobin's Q," NBER Working Paper No. W12564.
- [25] Meza, Felipe and Erwan Quintin, 2006, "Financial Crises and Total Factor Productivity," mimeo, Universidad Carlos III de Madrid.
- [26] Neumeyer, Pablo. A. and Fabrizio Perri, 2005. "Business Cycles in Emerging Economies: The Role of Interest Rates," *Journal of Monetary Economics*, Vol 52, Issue 2, 345-380.
- [27] Oviedo, Marcelo P., 2005, "World Interest Rate, Business Cycles, and Financial Intermediation in Small Open Economies," Manuscript, Iowa State University.



- [28] Reinhart Carmen. M., Kenneth S. Rogoff and M. A. Savastano, 2003, “Debt Intolerance,” NBER Working Paper No. 9908.
- [29] Tomz, Michael and Mark Wright, 2007, “Do Countries Default in ‘Bad Times’?”, mimeo, Department of Economics, UCLA
- [30] Uribe, Martin and Vivian Zhanwei Yue, 2006. “Country Spreads and Emerging Countries: Who Drives Whom?” *Journal of International Economics*, vol 69, 6-36.
- [31] Volkan, Engin, 2008, “Sovereign Default Risk, Risk-Averse Investors and Financial Contagion,” PhD Dissertation, Department of Economics, University of Southern California.
- [32] Yue, Vivian Zhanwei, 2006. “Sovereign Default and Debt Renegotiation,” Manuscript, New York University.

## Appendix

### PROOF of THEOREM 1

Given a productivity shock  $\varepsilon$  and level of working capital loan  $\kappa$ , the utility from defaulting  $v^d(\kappa, \varepsilon')$  is independent of  $b$ . We can also show that the utility from not defaulting  $v^{nd}(b, \varepsilon')$  is increasing in  $b_{t+1}$ . Therefore, if  $V(b^1, \kappa, \varepsilon') = v^d(\kappa, \varepsilon')$ , then it must be the case that  $V(b^0, \kappa, \varepsilon') = v^d(\kappa, \varepsilon')$ . Hence, any  $\varepsilon'$  that belongs in  $D(b^1, \varepsilon)$  must also belong in  $D(b^0, \varepsilon)$ .

Let  $d^*(b, \varepsilon')$  be the equilibrium default decision rule. The equilibrium default probability is then given by

$$p(b, \varepsilon) = \int d^*(b, \varepsilon') d\mu(\varepsilon'|\varepsilon)$$

From  $D(b^1, \varepsilon') \subseteq D(b^0, \varepsilon')$ , if  $d^*(b^1, \varepsilon') = 1$ , then  $d^*(b^0, \varepsilon') = 1$ . Therefore,

$$p(b^0, \varepsilon) \geq p(b^1, \varepsilon)$$

### PROOF of THEOREM 2

From Theorem 1, given a productivity shock  $\varepsilon$  and level of working capital loan  $\kappa$ , for  $b^0 < b^1 \leq 0$ ,  $p^*(b^0, \varepsilon) \geq p^*(b^1, \varepsilon)$ . The equilibrium bond price is given by

$$q(b', \varepsilon) = \frac{1 - p(b', \varepsilon)}{1 + r}$$

Hence, using Theorem 1, we obtain that:

$$q(b^0, \varepsilon) \leq q(b^1, \varepsilon)$$