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Matching and Inequality in the World Economy*

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Abstract

This paper develops a tractable general equilibrium model with a continuum of workers and sectors. In this environment, changes in relative factor supply or demand affect matching between workers and sectors. Changes in matching, in turn, affect workers' relative productivities and wages. Because of complementarities in production, this simple mechanism leads to sharp predictions about the full distribution of earnings for a wide range of comparative statics exercises, including North-South and North-North trade integration, skill-biased technological change, and offshoring.

1 Introduction

The past several decades have been marked by revolutionary technological innovation, greater economic integration, and dramatic changes in wage inequality. Though much has been said about the impact of technology and globalization on the relative wage of skilled and unskilled workers—see e.g. Katz and Murphy (1992), Berman, Bound, and Griliches (1994), Lawrence and Slaughter (1993), Feenstra and Hanson (1996b), Feenstra and Hanson (1999), Goldberg and Pavcnik (2007), and Verhoogen (2008)—recent empirical evidence suggests that large changes in inequality also have occurred within these two broad categories. For example, Autor, Katz, and Kearney (2008) document that the 90-50 wage gap has increased in the

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United States over the last fifteen years, while the 50-10 wage gap has recently stagnated or even narrowed. At the very high-end of the income distribution, Piketty and Saez (2003) report that large changes in inequality over the last thirty years in the United States are driven by a sharp increase of the top 0.1% income share. Similar patterns are reported by Atkinson (2006), Saez and Veall (2005), Banerjee and Piketty (2005) and Piketty and Qian (2008) in the United Kingdom, Canada, India, and China, respectively.

The objective of the present paper is to offer a unifying yet highly tractable framework that can deliver sharp predictions about the full distribution of earnings for a wide range of comparative statics exercises, including North-South and North-North trade integration, skill-biased technological change, and offshoring. To do so, we develop a simple general equilibrium model in which a continuum of workers, with different skills, are matched to a continuum of sectors, with different skill intensities. Sectors may either refer to industries or occupations in practice. On the supply side, we assume that workers are perfect substitutes within each sector, but that high-skill workers are relatively more productive in sectors with high-skill intensities. This first assumption implies positive assortative matching: in equilibrium, high-skill workers will only be found in high-skill-intensity sectors. On the demand side, we assume that goods are perfect complements. This second assumption guarantees that prices are purely determined on the supply side. Thus, we can first use market-clearing conditions to solve for the matching of workers to sectors, and then use profit-maximization conditions to determine the prices that support this assignment as an equilibrium outcome.

The mechanism emphasized by our theory is simple. First, changes in relative factor supply or demand affect the matching between workers and sectors. Second, changes in matching affect workers' relative productivities, and in turn, their relative wages. Using this idea, we first investigate the implications of a change in "skill abundance." Formally, we say that a distribution of workers V is skill abundant relative to a distribution V' if, for any pair of skill levels, V has relatively more high-skill workers than V'. In a closed economy, we show that a move from V to V' induces skill downgrading: each sector now employs workers with lower skills. The basic intuition is straightforward: as the relative supply of high-skill workers goes down, market clearing conditions require all workers to move into sectors with higher skill intensities. Because of the exact same complementarities between workers and sectors that yields positive assortative matching, this reallocation increases the marginal return of the high-skill workers relatively more, thereby leading to a pervasive rise in inequality. When a more and a less skill-abundant country open up to trade, i.e. North-

South trade integration, the previous logic offers continuum-by-continuum generalizations of the classic two-by-two Heckscher-Ohlin results.

Our second class of comparative statics exercises considers the implications of a change in "skill diversity." In the spirit of Grossman and Maggi (2000), we say that a distribution V is more diverse than a distribution V' if V' is skill-abundant for low levels of skill while V is skill-abundant for high levels. This definition captures the idea that there are relatively more workers with extreme skill levels (either high or low) in V. As Grossman and Maggi (2000) first emphasized, this notion of diversity is important because it allows us to think about the implications of North-North trade, i.e. trade between countries with similar average skill levels. While this accounts for the vast majority of world trade, the standard Heckscher-Ohlin model has nothing to say about its implications for inequality. By contrast, our model predicts that North-North trade leads to a polarization of inequality in the most diverse country: among the least skilled workers, those with lower skills get relatively richer, whereas the converse is true among the most skilled workers.

For our last class of comparative statics exercises, we turn to the implications of skill biased technological change (SBTC) and offshoring. We model SBTC as an exogenous increase in the relative demand for goods with high skill intensities. Because of market clearing conditions, this raises the relative demand for high-skill workers, leading to skill downgrading, and therefore, a pervasive rise in inequality. In a North-South environment, such a change further yields an increase in inequality between countries. Intuitively, high-skill workers gain relatively more from SBTC, and the skill-abundant country has relatively more of them. Finally, our framework provides sharp predictions regarding the implications of offshoring between a Northern and a Southern country, i.e. the ability of Northern firms to hire Southern workers using the North's superior technology. In our model, offshoring acts like an increase in the size of the Southern country, which makes the world distribution of workers less skill abundant. This, in turn, induces skill downgrading and a pervasive rise in inequality in both countries.

There is a rich theoretical literature designed to explain the consequences of either international trade, offshoring, or technological change on the skill premium; see e.g. Feenstra and Hanson (1996a), Acemoglu (1998), Acemoglu (2003), Zhu and Trefler (2005), and Grossman and Rossi-Hansberg (2008). Because the previous papers only consider skilled and unskilled workers, they have no implications for changes in inequality that occur within these two broad categories. Our general equilibrium model with a continuum of workers enables us to analyze such changes. In this respect, our paper is closely related to the work of Grossman and Maggi (2000) and Grossman (2004). Both papers develop a two-sector trade model with a continuum of workers and team production to study the pattern and consequences of North-North trade. At the cost of ruling out team production, we derive sharper predictions on the implications of North-North trade for inequality. We also provide a unified theoretical framework to analyze not only North-North trade, but also North-South trade, offshoring, and technological change.[1]

Our paper also is related Antras, Garicano, and Rossi-Hansberg (2006). They develop a one sector hierarchy model with a continuum of agents to investigate the implications of offshoring and technological change on inequality. Compared to their model, we abstract from the assignment of agents to management or production, which allows us to derive stronger, albeit narrower, predictions on the consequences of offshoring for inequality. More importantly, our model differs in that it incorporates multiple sectors, and hence, a rationale for North-South and North-North trade.[2]

Finally, the supply side of our model shares the same basic structure as Ohnsorge and Trefler (2007) and Costinot (2007). However, both papers abstract from the demand side, and therefore, cannot study changes in inequality. This is the main focus of our paper.

The rest of our paper is organized as follows. Section 2 presents the basic environment. Section 3 characterizes the unique competitive equilibrium in a closed economy. Section 4 analyzes the effects of changes in factor supply and factor demand on inequality. Section 5 turns to the implications of North-South and North-North trade. Section 6 studies the consequences of SBTC and offshoring. Section 7 offers concluding remarks.

2 The Basic Environment

Standard trade models usually comprise a small number of goods and factors. By contrast, we consider an economy with a continuum of goods or sectors, indexed by their skill intensity $\sigma \in \Sigma \equiv [\underline{\sigma}, \overline{\sigma}]$, and a continuum of workers, indexed by their skill $s \in S \equiv [\underline{s}, \overline{s}]$. We denote by $V(s) > 0$ the inelastic supply of workers with skill s.

[1] Yeaple (2005) offers another interesting, though less closely related paper on trade and matching. He analyzes the consequences of trade liberalization between two identical countries for technology adoption and inequality. However, unlike our paper, he abstracts from North-South trade and offshoring.

[2] Other one-sector matching models designed to explain changes in inequality, though less closely related to ours, include Kremer and Maskin (1996), Kremer and Maskin (2003), Garicano and Rossi-Hansberg (2006), Tervio (2008), and Gabaix and Landier (2008).

2.1 Preferences

Goods are perfect complements.[3] All consumers have the same Leontief utility function

$$U = \min_{\sigma \in \Sigma} \left[C(\sigma) / B(\sigma) \right], \tag{1}$$

where $C(\sigma) \geq 0$ is the consumption of good σ and $B(\sigma) > 0$ is an exogenous taste parameter. We discuss the role of this assumption in detail in Section 3.

2.2 Technology

Factors of production are perfect substitutes within each sector, but vary in the their productivity, $A(s, \sigma) > 0$. Aggregate output in sector σ is given by

$$Y(\sigma) = \int_{s \in S} A(s, \sigma) L(s, \sigma) ds, \tag{2}$$

where $L(s, \sigma) \geq 0$ is the number of workers with skill s in sector σ. We assume that $A(s, \sigma)$ is strictly increasing in s, twice differentiable, and strictly log-supermodular:

$$\frac{A(s, \sigma)}{A(s', \sigma)} > \frac{A(s, \sigma')}{A(s', \sigma')}, \text{ for all } s > s' \text{ and } \sigma > \sigma'. \tag{3}$$

In other words, high skill workers are more productive than low skill workers, but relatively more so in the sectors with high skill intensities. Throughout this paper, whenever we say "complementarities in production", we formally mean the log-supermodularity of A with respect to s and σ.

2.3 Market Structure

There is a large number of identical price-taking firms in each sector. Total profits in sector σ are given by

$$\Pi(\sigma) = \int_{s \in S} \left[p(\sigma) A(s, \sigma) - w(s) \right] L(s, \sigma) ds, \tag{4}$$

where $p(\sigma) > 0$ and $w(s) > 0$ are the price of good σ and the wage of a worker with skill s, respectively. Without loss of generality, we normalize prices such that $\int_{\underline{\sigma}}^{\overline{\sigma}} p(\sigma) B(\sigma) d\sigma = 1$.

[3] As mentioned in the introduction, one may also interpret "goods" in our model as occupations or tasks in practice. Under this interpretation, tasks are combined in order to produce final goods rather than utils.

For technical reasons, we also assume that B and V are continuous functions.

3 The Closed Economy

3.1 Definition of a Competitive Equilibrium

In a competitive equilibrium, consumers maximize their utility, firms maximize their profits, and good and labor markets clear. Because of Leontief preferences, utility maximization requires

$$C(\sigma) = B(\sigma) \cdot \left[\int_{s \in S} w(s) V(s) \, ds \right], \text{ for all } \sigma \in \Sigma. \tag{5}$$

Since there are constant returns to scale, profit maximization requires

$$\begin{aligned} &p(\sigma) A(s,\sigma) - w(s) \leq 0, \text{ for all } s \in S; \\ &p(\sigma) A(s,\sigma) - w(s) = 0, \text{ for all } s \in S \text{ such that } L(s,\sigma) > 0. \end{aligned} \tag{6}$$

Finally, good and labor market clearing require

$$C(\sigma) = \int_{s \in S} A(s,\sigma) L(s,\sigma) \, ds, \text{ for all } \sigma \in \Sigma; \tag{7}$$

$$V(s) = \int_{\sigma \in \Sigma} L(s,\sigma) \, d\sigma, \text{ for all } s \in S. \tag{8}$$

In rest of this paper, we formally define a competitive equilibrium as follows.

Definition 1 *A competitive equilibrium is a set of functions* (C, L, p, w) *such that Conditions* (5)-(8) *hold.*

3.2 Properties of a Competitive Equilibrium

Given our assumptions on worker productivity, $A(s,\sigma)$, the profit-maximization condition (6) imposes strong restrictions on competitive equilibria.

Lemma 1 *In a competitive equilibrium, the two following properties must be satisfied:* (i) w *is strictly increasing in* s*; and* (ii) *there exists an* increasing *bijection* $M : S \to \Sigma$ *such that* $L(s,\sigma) > 0$ *if and only if* $M(s) = \sigma$.

Lemma 1 reflects our assumptions on the supply-side of the economy. Property (i) derives from the fact that A is strictly increasing in s. Since high-skill workers are more productive in all sectors, they must command a higher wage; otherwise profit-maximizing firms would never use low-skill workers. Property (*ii*) derives from the fact that factors of production are perfect substitutes within each sector and that A is strictly log-supermodular. Perfect substitutability, on the one hand, implies the existence of a matching function M summarizing the allocation of workers to sectors. Because of the linearity of the aggregate production function, if a worker of skill s is allocated to sector σ, they all are. Log-supermodularity, on the other hand, implies the monotonicity of this matching function. Since high-skill workers are relatively more productive in sectors with high-skill intensity, firms in these sectors are willing to bid relatively more for these workers. In a competitive equilibrium, this induces positive assortative matching of high-s workers to high-σ sectors.

Lemma 1 considerably simplifies the analysis. To characterize a competitive equilibrium, we only need to determine the matching of workers to sectors, M, and the wage schedule, w, that sustain M as an equilibrium outcome. Once M and w have been determined, L, p, and C can be computed by simple substitutions. The rest of our analysis crucially relies on the following lemma:

Lemma 2 *In a competitive equilibrium, the matching function and wage schedule satisfy:*

$$\frac{dM}{ds} = \frac{A\left[s, M\left(s\right)\right] V\left(s\right)}{\left[\int_{s' \in S} w\left(s'\right) V\left(s'\right) ds'\right] B\left[M\left(s\right)\right]}, \tag{9}$$

$$\frac{dw}{ds} = \frac{w\left(s\right) A_s\left[s, M\left(s\right)\right]}{A\left[s, M\left(s\right)\right]}, \tag{10}$$

with boundary conditions $M\left(\underline{s}\right) = \underline{\sigma}$ *and* $M\left(\overline{s}\right) = \overline{\sigma}$.

To go from Lemma 1 to Lemma 2, we use our assumptions on the demand-side of the economy. The first differential equation summarizes how, because of market clearing, factor supply and factor demand determine the matching function. The second differential equation summarizes how, because of profit-maximization, the matching function determines the wage schedule. The Leontief utility function guarantees the block-recursiveness of the model. By imposing this strong restriction on the demand side, we guarantee that prices—up to a constant, $\int_{s' \in S} w\left(s'\right) V\left(s'\right) ds'$—are entirely determined on the supply side of the economy. This feature creates a tight connection between changes in matching and changes in inequality, which we exploit in the rest of this paper.[4] Using Lemma 2, we can show that:

[4] This feature also plays a crucial role in many one-sector matching models; see e.g. Sattinger (1993), Garicano and Rossi-Hansberg (2006), Antras, Garicano, and Rossi-Hansberg (2006). At a general level, one

Theorem 1 *A competitive equilibrium exists and is unique.*

To construct a competitive equilibrium, we start by constructing a solution of Equation (9). To do so, we impose the initial condition that $M(\underline{s}) = \underline{\sigma}$, and set aggregate income, $\int_{s' \in S} w(s') V(s') ds'$, to the unique level that satisfies the terminal condition, $M(\overline{s}) = \overline{\sigma}$. We then show that there exists a unique value of $w(\underline{s})$ such that the solution of Equation (10) is consistent with the previous level of aggregate income. All other equilibrium variables are computed by simple substitutions.

4 Comparative Statics in the Closed Economy

Armed with the knowledge that Equations (9) and (10) characterize a unique competitive equilibrium, we now investigate how exogenous changes in factor supply, V, and factor demand, B, affect our two key endogenous variables: the matching function, M, and the wage schedule, w. In each case, we first determine how exogenous changes affect the matching of workers to sectors using Equation (9). Given the impact on the matching function, we then consult Equation (10) to draw conclusions about its implications for inequality.

4.1 Change in Factor Supply (I): Skill Abundance

We first consider a change in factor supply from V to V' such that:

$$\frac{V(s)}{V(s')} > \frac{V'(s)}{V'(s')}, \text{ for all } s > s'. \tag{11}$$

Property (11) corresponds to the strict monotone likelihood ratio property; see Milgrom (1981). It states that, for any pair of distinct skill levels, there are relatively more high-skill workers in the economy characterized by V. This is illustrated in Figure 1 (a). Property (11) is the natural generalization, to a continuum of factors, of the notion of skill abundance in the two-factor Heckscher-Ohlin model. It will be the basis of our theory of North-South trade. In the rest of this paper, we say that:

Definition 2 *V is skill-abundant relative to V', denoted $V \succ_a V'$, if Property* (11) *holds.*

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can view the one-sector assumption as an implicit, but equally strong restriction on the demand side of the economy.

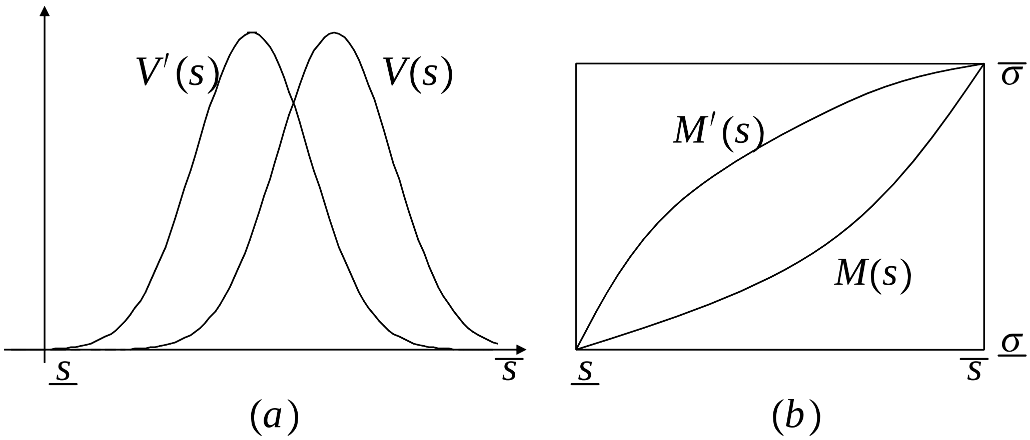

Figure 1: Changes in skill abundance and matching

We first analyze the impact of a change in skill abundance on matching. Let M and M' be the matching functions associated with V and V', respectively. Using Equation (9), we can show that:

Lemma 3 *Suppose $V \succ_a V'$. Then $M(s) < M'(s)$, for all $\underline{s} < s < \overline{s}$.*

From a worker standpoint, moving from V to V' implies *sector upgrading*: each type of worker is employed in a sector with higher skill intensity under V'. From a sector standpoint, this means *skill downgrading*: each sector now employs workers with lower skills. This is illustrated in Figure 1 (b). At a broad level, the intuition behind Lemma 3 is very simple. As the relative supply of the high-skill workers goes down, market clearing conditions require all sectors to employ lower-skill workers. So, the M schedule should shift up. To develop a more precise understanding of Lemma 3, suppose that $M(s) \geq M'(s)$ for some $s \in (\underline{s}, \overline{s})$. Then there must be two skill levels, $s_1 < s_2$, such that M crosses M' from below at s_1 and from above at s_2. This is precisely what Property (11) precludes.

Now let us consider the associated impact of a change in skill abundance on wages. Let w and w' be the wage schedules associated with V and V', respectively. Combining Lemma 3, Equation (10), and the log-supermodularity of A, we obtain

$$\frac{d \ln w}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s} < \frac{\partial \ln A[s, M'(s)]}{\partial s} = \frac{d \ln w'}{ds}.$$

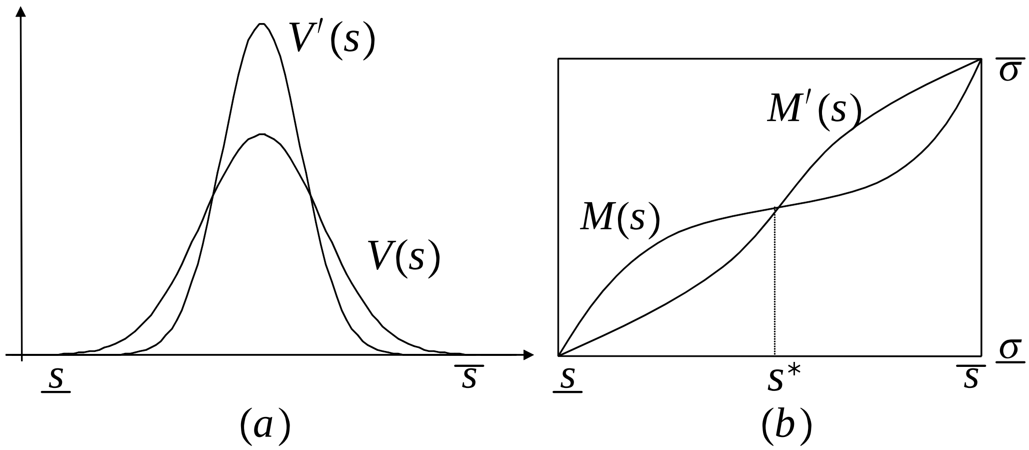

Figure 2: Changes in skill diversity and matching

Integrating the above inequality implies

$$\frac{w(s)}{w(s')} < \frac{w'(s)}{w'(s')}, \text{ for all } s > s'. \tag{12}$$

Moving from V to V' leads to a *pervasive rise in inequality*: for any pair of workers, the relative wage of the worker with a higher skill level—who is relatively less abundant under V'—goes up. In our model, a decrease in the relative supply of the high-skill workers triggers a reallocation of all workers towards the skill intensive sectors. Since A is log-supermodular, this increases the marginal return of the high-skill workers relatively more.

4.2 Change in Factor Supply (II): Skill Diversity

We now consider the case where V and V' are each symmetric functions around their common mean and satisfy:

$$\begin{aligned} V' &\succ_a V, \text{ for all } s < \widehat{s}; \\ V &\succ_a V', \text{ for all } s \geq \widehat{s}. \end{aligned} \tag{13}$$

with $\widehat{s} \in (\underline{s}, \overline{s})$. Property (13) captures the idea that there are relatively more workers with extreme skill levels (either high or low) under V than V'. This is illustrated in Figure 2 (a). Property (13) will be the basis of our theory of North-North trade. In the rest of this paper, we say that:

Definition 3 *V is more diverse than V', denoted $V \succ_d V'$, if Property* (13) *holds.*

Definition 3 is a stronger notion of diversity than in Grossman and Maggi (2000). Whereas they impose first-order stochastic dominance on either side of $\widehat{s}$, we impose likelihood ratio dominance. In order to make predictions in an environment with more than two goods, this stronger form of dominance cannot be dispensed with; see Costinot (2007).[5]

As before, let M and M' be the matching functions associated with V and V', respectively. Using Equation (9), we now prove that:

Lemma 4 *Suppose $V \succ_d V'$. Then there exists a unique skill level $s^* \in (\underline{s}, \overline{s})$ such that $M(s) > M'(s)$, for all $\underline{s} < s < s^*$, and $M(s) < M'(s)$, for all $s^* < s < \overline{s}$.*

Moving from V to V' implies *sector downgrading* for low-skill workers, $\underline{s} < s < s^*$; and *sector upgrading* for high-skill workers, $s^* < s < \overline{s}$. This is illustrated in Figure 2 (b). Like in the case of a change in skill abundance, these two results derive from our market clearing conditions. If $V \succ_d V'$, the relative supply of high-skill workers increases over the range $\underline{s} < s < \widehat{s}$. Thus, more sectors should employ these workers. The converse is true over the range $\widehat{s} < s < \overline{s}$.

Now let us turn to the associated wage schedules, w and w'. Combining Lemma 4, Equation (10), and the log-supermodularity of A, we obtain

$$\begin{array}{l} \frac{d \ln w}{ds} > \frac{d \ln w'}{ds}, \text{ for all } \underline{s} < s < s^*; \\ \frac{d \ln w}{ds} < \frac{d \ln w'}{ds}, \text{ for all } s^* < s < \overline{s}. \end{array}$$

Integrating this series of inequalities gives

$$\begin{array}{l} \frac{w(s)}{w(s')} > \frac{w'(s)}{w'(s')}, \text{ for all } \underline{s} \leq s' < s \leq s^*; \\ \frac{w(s)}{w(s')} < \frac{w'(s)}{w'(s')}, \text{ for all } s^* \leq s' < s' \leq \overline{s}. \end{array} \tag{14}$$

Within each group of workers, low- or high-skill, changes in skill diversity amount to changes in skill-abundance. For any pair of workers whose abilities are no greater or no less than s^*, the relative wage of the worker whose skill becomes relatively less abundant goes up.

4.3 Change in Factor Demand

In the two subsections above, we focused on exogenous changes in factor supply. In this subsection we consider an exogenous change in factor demand, modelled as a shift in the B

[5] By contrast, we could relax further the notion of symmetry imposed on V and V'. For example, having $V(\underline{s}) = V(\overline{s})$ and $V'(\underline{s}) = V'(\overline{s})$ would be sufficient for our results to hold. In our model, the only role of symmetry is to guarantee non-degenerate results.

schedule, from B to B', such that:

$$\frac{B'(\sigma)}{B'(\sigma')} > \frac{B(\sigma)}{B(\sigma')}, \text{ for all } \sigma > \sigma'. \tag{15}$$

Property (15) has two possible interpretations. The literal one is a change in preferences such that for *any* pair of goods, consumers have a higher relative demand for the skill-intensive good in the economy characterized by B'. Another possible interpretation of Property (15) is skill-biased technological change (SBTC). If goods are reinterpreted as tasks or intermediates that are assembled into a final good according to Equation (1), then moving from B to B' can be viewed as a technological shock that raises the relative demand for skill-intensive intermediates. In either case, a shift from B to B' increases the relative demand for high-skill workers. In the rest of this paper, we say that:

Definition 4 *B' is skill-biased relative to B, denoted $B' \succ_s B$, if Property* (15) *holds.*

Let M and M' denote the matching functions associated with B and B', respectively. Using Equation (9), we can also show that:

Lemma 5 *Suppose $B' \succ_s B$. Then $M(s) < M'(s)$, for all $\underline{s} < s < \overline{s}$.*

This the demand version of the results on changes in factor supply derived in Section 4.1. Broadly speaking, if the relative demand for the skill-intensive goods rises, then market clearing conditions require workers to move towards sectors with higher skill intensities in order to maintain equilibrium. This implies sector upgrading at the worker level, and skill downgrading at the sector level.

Finally, let w and w' be the wage schedules associated with B and B', respectively. Combining Lemma 5, Equation (10), and the log-supermodularity of A, we now obtain

$$\frac{w'_s(s)}{w'(s)} > \frac{w_s(s)}{w(s)},$$

which after integration implies

$$\frac{w'(s)}{w'(s')} > \frac{w(s)}{w(s')}, \text{ for all } s > s'. \tag{16}$$

Moving from B to B' leads to a pervasive rise in inequality: for any pair of workers, the relative wage of the more skilled worker increases. The mechanism linking the matching function to the wage schedule is the same as in Section 4.1. By Lemma 5, an increase in the relative demand for goods with high skill intensities triggers a reallocation of workers towards such sectors. Given the log-supermodularity of A, this increases the marginal return of high-skill workers relatively more.

5 The World Economy

In the rest of this paper we consider a world economy comprising two countries, Home (H) and Foreign (F). Workers are internationally immobile while goods are traded without cost. In each country, preferences and technology are as described in Section 2. We further assume that preferences are identical across countries and that technological differences are Hicks-neutral, $A_i \equiv \gamma_i A$ for $i = H, F$ with $\gamma_i > 0$. Hence, differences in factor endowments, V_H and V_F, are the only rationale for trade. Within this environment, we study two polar cases, $V_H \succ_a V_F$ and $V_H \succ_d V_F$, which we interpret as North-South and North-North trade, respectively.

5.1 Free Trade Equilibrium

Before analyzing the consequences of international trade, we characterize a free trade equilibrium. Given our work in Section 3, this is a straightforward exercise. A competitive equilibrium in the world economy under free trade is a set of functions $(C_H, L_H, w_H, C_F, L_F, w_F, p)$ such that Conditions (5), (6), and (8) hold in both countries, and good markets clear

$$C_H(\sigma) + C_F(\sigma) = \int_{s \in S} \left[A_H(s,\sigma) L_H(s,\sigma) + A_F(s,\sigma) L_F(s,\sigma)\right] ds, \text{ for all } \sigma \in \Sigma.$$

Since technological differences are Hicks-neutral, our model is isomorphic to a model where technologies are identical around the world, but countries' factor supply are given by $\widetilde{V}_i \equiv \gamma_i V_i$ for $i = H, F$. Once Home and Foreign factors have been expressed in the same efficiency units, Theorem 1 directly implies the existence and uniqueness of a competitive equilibrium (C_W, L_W, p, w) in the integrated world economy. To demonstrate the existence of an equilibrium under free trade, we only need to find $(C_H, L_H, w_H, C_F, L_F, w_F, p)$ that replicate the integrated equilibrium. For $i = H, F$, we can set

$$
\begin{aligned}
C_i(\sigma) &\equiv C_W(\sigma) \cdot \left[\int_{s\in S} w(s)\,\gamma_i V_i(s)\,ds\right] \Big/ \left[\int_{s\in S} w(s)\,V_W(s)\,ds\right], \\
L_i(s,\sigma) &\equiv L_W(s,\sigma) \cdot [\gamma_i V_i(s)] / [V_W(s)], \\
w_i(s) &\equiv \gamma_i w(s),
\end{aligned}
$$

with $V_W \equiv \gamma_H V_H + \gamma_F V_F$ the world endowment of skills. By construction, if (C_W, L_W, p, w) is a competitive equilibrium in the integrated world economy, then $(C_H, L_H, w_H, C_F, L_F, w_F, p)$ is a free trade equilibrium. Since factors of production are perfect substitutes within each sector, factor price equalization necessarily holds in efficiency units; see Condition (6). From now on, we focus on the integrated equilibrium.

5.2 The Consequences of North-South Trade

In this subsection we assume that Home is skill abundant relative to Foreign: $V_H \succ_a V_F$. In a two-by-two Heckscher-Ohlin model, when the skill-abundant country opens up to trade: (i) the skill intensity of both sectors decreases; (ii) the skill-intensive sector expands; and (iii) the skill premium goes up. Conversely, when the unskill-abundant country opens up to trade: (i) the skill intensity of both sectors increases; (ii) the unskill-intensive sector expands; and (iii) the skill premium goes down. Prediction (iii) is the well-known Stolper-Samuelson effect. We now offer continuum-by-continuum generalizations of these classic results.[6] Our analysis builds on the following Lemma.

Lemma 6 *Suppose $V_H \succ_a V_F$. Then $V_W \equiv \gamma_H V_H + \gamma_F V_F$ satisfies $V_H \succ_a V_W$ and $V_W \succ_a V_F$.*

As in the two-factor model, if Home is skill-abundant relative to Foreign, then Home is skill-abundant relative to the World and the World is skill-abundant relative to Foreign.

Matching. We first consider the implications of North-South trade on the matching of workers to sectors. Let M_H and M_F be the matching functions at Home and Abroad, respectively, under autarky. By Lemmas 3 and 6, trade integration induces sector upgrading

[6] In the Appendix, we also prove the continuous analogue to the Heckscher-Ohlin Theorem. The intuition is the same as in Ohnsorge and Trefler (2007) and Costinot (2007). In the integrated equilibrium, the matching function is the same around the world. Thus, differences in factor endowments are mechanically reflected in the pattern of international specialization across countries.

at Home and sector downgrading Abroad:

$$M_H(s) < M_W(s) < M_F(s), \text{ for all } \underline{s} < s < \overline{s}. \tag{17}$$

At the sector level, this means skill downgrading at Home and skill upgrading Abroad. This is the counterpart to Effect (i) in the two-by-two Heckscher-Ohlin model. A direct corollary of Inequality (17) is that for any $\sigma_0 \in (\underline{\sigma}, \overline{\sigma})$:

$$\begin{array}{l} \int_{M_W^{-1}(\sigma_0)}^{\overline{s}} V_H(s)\,ds > \int_{M_H^{-1}(\sigma_0)}^{\overline{s}} V_H(s)\,ds; \\ \int_{\underline{s}}^{M_W^{-1}(\sigma_0)} V_F(s)\,ds > \int_{\underline{s}}^{M_F^{-1}(\sigma_0)} V_F(s)\,ds. \end{array} \tag{18}$$

According to Inequality (18), the employment share in sectors with high skill intensities, $\sigma \in (\sigma_0, \overline{\sigma})$, increases at Home, whereas the employment share in sectors with low skill intensities, $\sigma \in (\underline{\sigma}, \sigma_0)$, increases Abroad. This is the counterpart to Effect (ii) in the two-by-two Heckscher-Ohlin model.

Inequality. We now turn to the implications of North-South trade on inequality. Let w_H^A and w_F^A be the wage schedules at Home and Abroad, respectively, in autarky. As in Section 4.1, Inequality (17) and the log-supermodularity of A imply a pervasive rise in inequality:

$$\frac{w_H^A(s)}{w_H^A(s')} < \frac{w_H(s)}{w_H(s')} = \frac{w_F(s)}{w_F(s')} < \frac{w_F^A(s)}{w_F^A(s')}, \text{ for all } s > s'. \tag{19}$$

This is a strong Stolper-Samuelson effect: anywhere in the skill distribution, workers with higher skills get relatively richer at Home under free trade, whereas they get relatively poorer Abroad.

To get a better sense of this effect, denote by $W_H^A(q) \equiv \int_{s_q}^{\overline{s}} w_H^A(s)\,V_H(s)\,ds$ and $W_H(q) \equiv \int_{s_q}^{\overline{s}} w_H(s)\,V_H(s)\,ds$ the share of earnings of the top $q\%$ of the income distribution at Home under autarky and free trade, respectively. For any $1 \geq q > q' > 0$, Inequality (19) implies

$$\frac{W_H(q)}{W_H(q')} > \frac{W_H^A(q)}{W_H^A(q')}.$$

In other words, changes in inequality are fractal in nature: within any truncation of the earnings distribution, the rich are getting richer in the skill-abundant country. Similarly, in the unskill-abundant country, we have

$$\frac{W_F(q)}{W_F(q')} < \frac{W_F^A(q)}{W_F^A(q')}.$$

The fundamental forces linking trade integration and inequality are simple. In the skill-abundant country, trade integration induces skill downgrading. Thus, workers move into skill-intensive sectors, which increases the marginal-return to skill, and in turn, inequality. Proposition 1 summarizes our results on the consequences of North-South trade.

Proposition 1 *If Home is skill-abundant relative to Foreign then trade integration induces:* (i) *skill downgrading at Home and Skill upgrading Abroad;* (ii) *an increase in the employment share of sectors with high-skill intensities at Home and low-skill intensities Abroad; and* (iii) *a pervasive rise in inequality at Home and a pervasive fall in inequality Abroad.*

5.3 The Consequences of North-North Trade

In this subsection we assume that Home is more diverse than Foreign: $V_H \succ_d V_F$. Under this restriction, we demonstrate that the familiar mechanisms at work in North-South trade apply equally well to North-North trade. This allows us to generate new results on the consequences of international trade.[7] Compared to the North-South case, the main technical difference is that the methods used if $V_H \succ_a V_F$ need to be applied twice if $V_H \succ_d V_F$: first for $s < \widehat{s}$ and again for $s \geq \widehat{s}$. Our analysis of North-North trade builds on the following Lemma.

Lemma 7 *Suppose* $V_H \succ_d V_F$. *Then* $V_W \equiv \gamma_H V_H + \gamma_F V_F$ *satisfies* $V_H \succ_d V_W$ *and* $V_W \succ_d V_F$.

In other words, if Home is more diverse than Foreign, then Home is more diverse than the World and the World is more diverse than Foreign.

Matching. Consider the Home country. By Lemmas 4 and 7, trade integration induces sector downgrading for low-skill workers and sector upgrading for high-skill workers. Formally, there exists $s_H \in (\underline{s}, \overline{s})$ such that

$$\begin{aligned} M_H(s) &> M_W(s), \text{ for all } \underline{s} < s < s_H; \\ M_H(s) &< M_W(s), \text{ for all } s_H < s < \overline{s}. \end{aligned} \tag{20}$$

[7] Like in the North-South case, results on the pattern of trade have been relegated to the Appendix.

Accordingly, trade integration leads to skill upgrading in sectors with low-skill intensity, $\sigma < \sigma_H$, and skill downgrading in sectors with high-skill intensity, $\sigma > \sigma_H$, with $\sigma_H \equiv M_H(s_H)$.

The converse is true in the Foreign country. Namely, there exists $s_F \in (\underline{s}, \overline{s})$ such that

$$\begin{aligned} M_F(s) < M_W(s), &\text{ for all } \underline{s} < s < s_F; \\ M_F(s) > M_W(s), &\text{ for all } s_F < s < \overline{s}. \end{aligned} \tag{21}$$

Inequality. The differential impact of trade integration on the sectoral choices of high- and low-skill workers has stark implications on inequality in the two countries. At Home, Inequality (20) and the log-supermodularity of A imply

$$\begin{aligned} \tfrac{w_H^A(s)}{w_H^A(s')} > \tfrac{w_H(s)}{w_H(s')}, &\text{ for all } \underline{s} \leq s' < s \leq s_H; \\ \tfrac{w_H^A(s)}{w_H^A(s')} < \tfrac{w_H(s)}{w_H(s')}, &\text{ for all } s_H \leq s' < s' \leq \overline{s}. \end{aligned} \tag{22}$$

Moving from autarky to free trade leads to a *polarization* of inequality in the more diverse country. Among the least skilled workers, those with lower skills get relatively richer, whereas the converse is true among the most skilled workers. Similarly, in the less diverse country we have

$$\begin{aligned} \tfrac{w_F^A(s)}{w_F^A(s')} < \tfrac{w_F(s)}{w_F(s')}, &\text{ for all } \underline{s} \leq s' < s \leq s_F; \\ \tfrac{w_F^A(s)}{w_F^A(s')} > \tfrac{w_F(s)}{w_F(s')}, &\text{ for all } s_F \leq s' < s' \leq \overline{s}. \end{aligned} \tag{23}$$

Inequality (23) implies *convergence* Abroad, as the "middle-class" benefits relatively more from free trade. Proposition 2 summarizes our results on the consequences of North-North trade.

Proposition 2 *If Home is more diverse than Foreign, then trade integration induces* (i) *skill upgrading in sectors with low-skill intensities at Home and high-skill intensities Abroad;* (ii) *skill downgrading in sectors with high-skill intensities at Home and low-skill intensities Abroad; and* (iii) *polarization of inequality at Home and convergence Abroad.*

Note that Proposition 2 has no clear implications on the overall level of inequality. Under North-North trade, the relative wage *between* the high- and low-skill workers—as well as the price of the goods they produce—may either increase or decrease. The consequences of North-North trade are to be found at a higher level of disaggregation. When trading partners vary in terms of skill diversity, changes in inequality occur *within* low- and high-skill workers, respectively. Similarly, Proposition 2 does not predict a decrease (or increase) in the employment shares of the skill-intensive sectors. According to our theory, North-North trade leads to a U-shape (or inverted U-shape) relationship between sectors' employment growth and their skill-intensity.

6 Comparative Statics in the World Economy

In this section we consider the impact of technological diffusion and skill-biased technological change in a global economy. In order to avoid a taxonomic exercise, we restrict ourselves to the North-South case, $V_H \succ_a V_F$, and assume that $\gamma_H > \gamma_F$. In other words, the skill-abundant country also is the technologically advanced country.

6.1 Global Skill-Biased Technological Change

We first analyze the impact of global skill-biased technological change (SBTC), modelled as a shift from B to B' such that $B' \succ_s B$. In line with subsection 4.3, we denote M_W and M'_W the matching functions in the integrated equilibrium under B and B', respectively, and w and w' the associated wage schedules. Therefore, the wage schedules at Home and Abroad are given by $w_i^{(\prime)} = \gamma_i w^{(\prime)}$ for $i = H, F$.

From our previous work in a closed economy, we already know that global SBTC induces sector upgrading/skill downgrading in both countries:

$$M_W(s) < M'_W(s), \text{ for all } \underline{s} < s < \overline{s}.$$

We also know that this change in matching implies

$$\frac{w'_i(s)}{w'_i(s')} > \frac{w_i(s)}{w_i(s')}, \text{ for all } s > s' \text{ and } i = H, F.$$

which leads to a pervasive rise in inequality within each country.[8] Compared to a closed economy, however, we can further ask how global SBTC affects inequality *between* countries. Let $W_i^{(\prime)} \equiv \int_{\underline{s}}^{\overline{s}} w_i^{(\prime)}(s) V_i(s) ds$ denote total income in country $i = H, F$. Our predictions about the impact of global SBTC on cross-country inequality can be stated as follows.

[8] Autor, Katz, and Kearney (2008) argue that technological change has decreased the relative demand for intermediate skill levels and has caused polarization of inequality. Our model can also capture this idea by introducing "extreme-biased" technological change. We say that B' is extreme-biased relative to B, denoted $B' \succ_e B$, if $B \succ_e B'$ for $s < \widehat{s} \in (\underline{s}, \overline{s})$, $B' \succ_e B$ for $s \geq \widehat{s}$, $B'(\underline{s}) = B'(\overline{s})$, and $B(\underline{s}) = B(\overline{s})$. If $B' \succ_e B$, then moving from B to B' induces (i) workers to reallocate out of intermediate σ sectors/occupations and towards extreme σ sectors/occupations in both countries; and (ii) a polarization of inequality in both countries.

Lemma 8 *Suppose $V_H \succ_a V_F$ and $B' \succ_s B$. Then total income satisfies $W'_H/W'_F > W_H/W_F$.*

According to Lemma 8, an increase in the relative labor demand for skill-intensive goods worldwide increases inequality between Home and Foreign. Intuitively, high-skill agents gain relatively more from such a change, and Home has relatively more of them. In our model, within- and between-country inequality tend to go hand in hand: *ceteris paribus*, changes in matching that increase inequality in both countries also increase inequality across countries. Proposition 3 summarizes our results on the consequences of global SBTC.

Proposition 3 *Global SBTC induces:* (i) *skill downgrading in each country;* (ii) *a pervasive rise in inequality in each country; and* (iii) *an increase in inequality across countries.*

6.2 Offshoring

For our final comparative statics exercise, we analyze the impact of an increase in Foreign workers' productivities from $\gamma_F A(s,\sigma)$ to $\gamma_H A(s,\sigma)$. A natural way—though not the only way—to think about such a technological change is *offshoring*, i.e. the ability of Domestic firms to hire Foreign workers using Home's superior technology. This is the interpretation we adopt in the rest of this subsection.

Our analysis of offshoring builds on two simple observations. First, as far as the integrated equilibrium is concerned, increasing the productivity of all Foreign workers by γ_H/γ_F is similar to increasing their supply by γ_H/γ_F. Second, since Foreign is relatively unskill-abundant, an increase in Foreign factor supply, from $\gamma_F V_F$ to $\gamma_H V_F$, makes the World relatively less skill abundant, as we show in the following Lemma.

Lemma 9 *Suppose $V_H \succ_a V_F$. Then $V_W \equiv \gamma_H V_H + \gamma_F V_F$ and $V'_W \equiv \gamma_H V_H + \gamma_H V_F$ satisfy $V_W \succ_a V'_W$.*

To sum up, if domestic firms offshore their production, it is *as if* the World distribution were relatively less skill abundant. Therefore, the results of Section 4.1 directly imply that

$$M_H(s) < M_W(s) < M'_W(s) < M_F(s)\text{, for all } \underline{s} < s < \overline{s}.$$

where M_W and M'_W are the World matching functions without and with offshoring, respectively. By Lemmas 3 and 9, offshoring induces sector upgrading, as the World's matching function moves closer towards Foreign's matching function under autarky. This implies a pervasive rise in inequality in both countries:

$$\frac{w_i'(s)}{w_i'(s')} > \frac{w_i(s)}{w_i(s')}, \text{ for all } s > s' \text{ and } i = H, F.$$

For any pair of workers in either country, the relative wage of the more skilled worker increases as a result of offshoring. In the integrated equilibrium, offshoring is similar to an increase in the relative size of the Foreign country. As Foreign grows relative to Home, World prices converge to those that hold in Foreign under Autarky. Since the wage schedule is steeper Abroad than at Home under autarky, offshoring increases inequality in both countries. Proposition 4 summarizes our results on the consequences of offshoring.

Proposition 4 *Offshoring in the world economy induces:* (i) *skill downgrading in both countries; and* (ii) *a pervasive rise in inequality in both countries.*

7 Concluding Remarks

Though much attention has been paid to the potential impact of technology and globalization on the relative wage of skilled and unskilled workers, large changes in inequality—whatever their causes may be—occur within these two broad categories. In this paper we have developed a rich, yet tractable general equilibrium model that delivers sharp predictions about the full distribution of earnings for a wide range of comparative statics exercises.

In our model, changes in relative factor supply or demand affect matching between workers and sectors. Changes in matching, in turn, affect workers' relative productivities and wages. Because of complementarities in production, this simple mechanism yields a rich set of conclusions. For example, we have shown that North-North trade integration leads to polarization in the more diverse country and convergence in the less diverse country. By contrast, North-South trade integration leads to a pervasive rise in inequality in the skill-abundant country and a pervasive fall in the unskill-abundant country. Finally, global skill-biased technological change and offshoring induce a pervasive rise in inequality in both countries.

A Proofs

Proof of Lemma 1. We first demonstrate Property (i) by contradiction. Suppose that there is a pair of workers $s_0 < s_1$ such that $w(s_0) \geq w(s_1)$. Now take a sector σ_0 such that $L(s_0, \sigma_0) > 0$. By Condition (8), there must be at least one. Then Condition (6) implies

$$p(\sigma_0) A(s_0, \sigma_0) - w(s_0) = 0 \tag{24}$$

Since $A(s, \sigma_0)$ is strictly increasing in s and $w(s_0) \geq w(s_1)$, Equation (24) implies

$$p(\sigma_0) A(s_0, \sigma_1) - w(s_1) > 0$$

which contradicts Condition (6). We now demonstrate Property (ii). In the rest of this proof, we denote $S(\sigma) \equiv \{s \in S \mid L(s, \sigma) > 0\}$ and $\Sigma(s) \equiv \{\sigma \in \Sigma \mid L(s, \sigma) > 0\}$. Clearly $s \in S(\sigma)$ if and only if $\sigma \in \Sigma(s)$. We proceed in 5 steps.

Step 1: *$S(\sigma) \neq \emptyset$ for all $\sigma \in \Sigma$ and $\Sigma(s) \neq \emptyset$ for all $s \in S$.*

$S(\sigma) \neq \emptyset$ derives from Conditions (5) and (7). $\Sigma(s) \neq \emptyset$ derives from Condition (8).

Step 2: *$S(\cdot)$ and $\Sigma(\cdot)$ are weakly increasing in the strong set order.*

We first show that $S(\cdot)$ is weakly increasing in the strong set order by contradiction. Suppose there are a pair of sectors $\sigma_0 < \sigma_1$ and a pair of workers $s_0 < s_1$ such that $s_0 \in S(\sigma_1)$ and $s_1 \in S(\sigma_0)$. Condition (6) implies

$$\begin{aligned} p(\sigma_1) A(s_0, \sigma_1) - w(s_0) &= 0 && (25) \\ p(\sigma_0) A(s_1, \sigma_0) - w(s_1) &= 0 && (26) \\ p(\sigma_0) A(s_0, \sigma_0) - w(s_0) &\leq 0 && (27) \\ p(\sigma_1) A(s_1, \sigma_1) - w(s_1) &\leq 0 && (28) \end{aligned}$$

By Equation (25) and Inequality (27), we have

$$p(\sigma_0) A(s_0, \sigma_0) \leq p(\sigma_1) A(s_0, \sigma_1) \tag{29}$$

By Equations (26) and Inequality (28), we have

$$p(\sigma_1) A(s_1, \sigma_1) \leq p(\sigma_0) A(s_1, \sigma_0) \tag{30}$$

Combining Inequalities (29) and (30), we obtain

$$A(s_0, \sigma_0) A(s_1, \sigma_1) \leq A(s_0, \sigma_1) A(s_1, \sigma_0)$$

which contradicts $A(s, \sigma)$ strictly log-supermodular. Hence, $S(\cdot)$ is weakly increasing in the strong set order. Since $s \in S(\sigma)$ if and only if $\sigma \in \Sigma(s)$, $\Sigma(\cdot)$ must be weakly increasing in the strong set order as well.

Step 3: *$S(\sigma)$ is a singleton for all but a countable set of σ.*

Let Σ_0 be the subset of sectors σ such that $\mu[S(\sigma)] > 0$, where μ is the Lebesgue measure over $\mathbb{R}$. We first show that Σ_0 is a countable set. Choose an arbitrary $\sigma \in \Sigma_0$ and let $\underline{s}(\sigma) \equiv \inf S(\sigma)$ and $\overline{s}(\sigma) \equiv \sup S(\sigma)$. The fact that $\mu[S(\sigma)] > 0$ has strictly positive measure yields $\underline{s}(\sigma) < \overline{s}(\sigma)$. Because $S(\sigma)$ is weakly increasing in σ, we must have $\sum\limits_{\sigma \in \Sigma_0} [\overline{s}(\sigma') - \underline{s}(\sigma')] \leq \overline{s} - \underline{s}$. So for any $\sigma \in \Sigma_0$, there must be $j \in \mathbb{N}$ such that $\overline{s}(\sigma) - \underline{s}(\sigma) \geq (\overline{s} - \underline{s})/j$; and for any $j \in \mathbb{N}$, there must be at most j points $\{\sigma\}$ in Σ_0 for which $\overline{s}(\sigma) - \underline{s}(\sigma) \geq [\overline{s} - \underline{s}]/j$. Since the union of countable sets is countable, the two previous observations imply that Σ_0 is a countable set. Now take $\sigma \notin \Sigma_0$. To show that $S(\sigma)$ is a singleton, we proceed by contradiction. If $S(\sigma)$ is not a singleton, then there are $s < s''$ such that $s, s'' \in S(\sigma)$. Using Step 1 and the fact that $\mu[S(\sigma)] = 0$, there also is $s < s' < s''$ such that $s' \in S(\sigma')$ with $\sigma' \neq \sigma$, which contradicts Step 2.

Step 4: *$\Sigma(s)$ is a singleton for all but a countable set of s.*

Since $\Sigma(s) \neq \emptyset$ and $\Sigma(\cdot)$ is weakly increasing in the strong set order, this follows from the same argument as in Step 3.

Step 5: *$S(\sigma)$ is a singleton for all σ.*

To obtain a contradiction, suppose that there exists $\sigma \in \Sigma$ for which $S(\sigma)$ is not a singleton. By the same argument as in Step 3, we must have $\mu[S(\sigma)] > 0$. By Step 4, $\Sigma(s) = \{\sigma\}$ for μ-almost all $s \in S(\sigma)$. Hence, Condition (8) implies

$$L(s, \sigma) = V(s)\, \delta \left[1 - \mathbb{1}_{S(\sigma)}\right], \text{ for } \mu\text{-almost all } s \in S(\sigma) \tag{31}$$

where δ is a Dirac delta function. By Step 3 and Condition (8), we must also have $\sigma' \in \Sigma$ for which $S(\sigma') = \{s'\}$ with $s' \in S$ such that

$$L(s', \sigma') \leq V(s')\, \delta \left[1 - \mathbb{1}_{S(\sigma')}\right] \tag{32}$$

To conclude, we use Conditions (5) and (7), which imply

$$\begin{aligned} B(\sigma) &= \int_{s\in S(\sigma)} A(s,\sigma)\, L(s,\sigma)\, ds \Big/ \left[\int_{s\in S} w(s)\, V(s)\, ds\right] \\ B(\sigma') &= \int_{s\in S(\sigma')} A(s,\sigma')\, L(s,\sigma')\, ds \Big/ \left[\int_{s\in S} w(s)\, V(s)\, ds\right] \end{aligned}$$

and in turn,

$$\frac{B(\sigma)}{B(\sigma')} = \frac{\int_{s\in S(\sigma)} A(s,\sigma)\, L(s,\sigma)\, ds}{\int_{s\in S(\sigma')} A(s,\sigma')\, L(s,\sigma')\, ds} \tag{33}$$

Since $\mu[S(\sigma)] > 0$ and $\mu[\sigma(s')] = 0$, Equations (31) and (32) imply $\frac{B(\sigma)}{B(\sigma')} = \infty$, which contradicts $B(\sigma') > 0$.

Steps 2 and 5 imply the existence of a strictly increasing function $M : S \to \Sigma$ such that $L(s,\sigma) > 0$ if and only if $M(s) = \sigma$. Step 1 requires $M(\underline{s}) = \underline{\sigma}$ and $M(\overline{s}) = \overline{\sigma}$. **QED**. ■

Proof of Lemma 2. We first consider Equation (10). By Condition (6) and Lemma 1 (ii), we know that

$$p[M(s)]\, A[s, M(s)] - w(s) = \max_{s'} \{p[M(s)]\, A[s', M(s)] - w(s')\}$$

for all $s \in S$. Since $w(s)$ is strictly increasing by Lemma 1 (i), it is differentiable almost everywhere. Thus, w must satisfy the following first-order condition

$$w_s(s) = p[M(s)]\, A_s[s, M(s)] \tag{34}$$

By Condition (6), we also know that

$$p[M(s)]\, A[s, M(s)] - w(s) = 0$$

Thus, we can rearrange Equation (34) as

$$w_s(s) = \frac{w(s)\, A_s[s, M(s)]}{A[s, M(s)]}$$

This completes the first part of our proof. We now turn to Equation (9). Lemma 1 (ii) and

Condition (8) imply that, for all $s \in S$,

$$L(s,\sigma) = V(s)\,\delta\left[\sigma - M(s)\right] \tag{35}$$

where δ is a Dirac delta function. Now consider Condition (7). At $\sigma = M(s)$, we have

$$C\left[M(s)\right] = \int_{s \in S} A\left[s', M(s)\right] L\left[s', M(s)\right] ds'$$

Using Equation (35), we can rearrange the previous expression as

$$C\left[M(s)\right] = \int_{s' \in S} A\left[s', M(s)\right] V(s')\,\delta\left[M(s) - M(s')\right] ds'$$

Now set $\sigma' = M(s')$. Since M is a bijection from S to Σ, we have

$$C\left[M(s)\right] = \int_{\sigma' \in \Sigma} A\left[M^{-1}(\sigma'), M(s)\right] V\left[M^{-1}(\sigma')\right] \delta\left[M(s) - \sigma'\right] \frac{1}{M_s\left(M^{-1}(\sigma')\right)} d\sigma'$$

By definition of the Dirac delta function, this simplifies into

$$M_s(s) = \frac{A\left[s, M(s)\right] V(s)}{C\left[M(s)\right]} \tag{36}$$

Equation (36) and Condition (5) imply

$$M_s(s) = \frac{A\left[s, M(s)\right] V(s)}{\left[\int_{s' \in S} w(s') V(s')\, ds'\right] B\left[M(s)\right]}$$

This completes the second part of our proof. $M(\underline{s}) = \underline{\sigma}$ and $M(\overline{s}) = \overline{\sigma}$ directly derive from the fact M is an increasing bijection from S onto Σ. **QED**. ■

Proof of Theorem 1. We first show that there exists a unique solution to the system of Equations (10)-(9) such that $M(\underline{s}) = \underline{\sigma}$ and $M(\overline{s}) = \overline{\sigma}$. We proceed in 2 steps.

Step 1: *There exist a unique $I^* > 0$ and a unique function $M^* : S \to \Sigma$ such that:*

$$M_s^*(s) = \frac{A\left[s, M^*(s)\right] V(s)}{I^* B\left[M^*(s)\right]} \tag{37}$$

$$M^*(\underline{s}) = \underline{\sigma} \tag{38}$$

$$M^*(\overline{\sigma}) = \overline{\sigma} \tag{39}$$

Let us define $H(M, s, I)$ such that

$$H(M, s, I) \equiv \frac{A[s, M] V(s)}{IB[M]}$$

for all $M \in \Sigma$, $s \in S$, and $I > 0$. Since A, B, and V are continuous, H is a continuous function of (M, s, I). By the Cauchy Theorem, we know that, for any $I > 0$, there exists a unique function $M(s|I)$ from S into Σ such that:

$$\begin{aligned} M_s(s|I) &= H[M(s|I), s, I] && (40)\\ M(\underline{s}|I) &= \underline{\sigma} && (41) \end{aligned}$$

By Equations (40) and (41), we know that

$$\begin{gathered} \lim_{I \to +\infty} M(s|I) = \underline{\sigma}, \text{ for all } s > \underline{s} \\ \lim_{I \to 0} M(s|I) = +\infty, \text{ for all } s > \underline{s} \end{gathered}$$

We also know that, for any $s \in S$, $M(s|I)$ is continuous in I . We now show that, for any $s > \underline{s}$, $M(s|I)$ is strictly increasing in I. We proceed by contradiction. Suppose that there exist $s > \underline{s}$ and $I_1 < I_2$ such that $M(s|I_1) \geq M(s|I_2)$. Since $M(\underline{s}|I_1) = M(\underline{s}|I_2) = \underline{\sigma}$ and $M_s(\underline{s}|I_1) > M_s(\underline{s}|I_2)$ by Equation (40), $M(s|I_1) \geq M(s|I_2)$ implies the existence of $s_0 \leq s$ such that $M(s_0|I_1) = M(s_0|I_2)$ and $M_s(s_0|I_1) < M_s(s_0|I_2)$. Combining the two previous conditions with Equation (40), we get $\frac{1}{I_1} < \frac{1}{I_2}$, which contradicts $I_1 < I_2$. Hence, $M(s|I)$ is strictly increasing in I. To conclude, fix $s = \overline{s}$. Since $M(\overline{s}|I)$ is continuous, strictly increasing, and satisfies $\lim_{I \to +\infty} M(\overline{s}|I) = \underline{\sigma}$ and $\lim_{I \to 0} M(\overline{s}|I) = +\infty$, the Intermediate Value Theorem implies the existence of a unique $I^* > 0$ such that

$$M(\overline{s}|I^*) = \overline{\sigma}$$

By construction, $M^* \equiv M(\cdot|I^*)$ is the unique function from S onto Σ that satisfies Equations (37)-(39). This concludes the proof of Step 1.

Step 2: *There exist a unique function $w^* : \Omega \to \mathbb{R}^+$ such that:*

$$\begin{aligned} w_s^*(s) &= \frac{w^*(s) A_s[s, M^*(s)]}{A[s, M^*(s)]} && (42)\\ \int_{s \in S} w^*(s) V(S) \, ds &= I^* && (43) \end{aligned}$$

By Cauchy Theorem, we know that, for any $\underline{w} > 0$, there exists a unique function $w\left(s|\underline{w}\right)$ from S into $\mathbb{R}^+$ such that:

$$w_s\left(s|\underline{w}\right) = \frac{w\left(s|\underline{w}\right) A_s\left[s, M^*\left(s\right)\right]}{A\left[s, M^*\left(s\right)\right]} \tag{44}$$

$$w\left(s|\underline{w}\right) = \underline{w} \tag{45}$$

Combining Equations (44) and (45), we get

$$w\left(s|\underline{w}\right) = \underline{w} \exp\left\{\int_{\underline{s}}^{s} \frac{A_s\left[s', M^*\left(s'\right)\right]}{A\left[s', M^*\left(s'\right)\right]} ds'\right\} \tag{46}$$

Equation (46) implies

$$\int_{s\in S} w\left(s|\underline{w}\right) V\left(s\right) ds = \underline{w} \int_{s\in S} \exp\left\{\int_{\underline{s}}^{s} \frac{A_s\left[s', M^*\left(s'\right)\right]}{A\left[s', M^*\left(s'\right)\right]} ds'\right\} V\left(s\right) ds$$

Now let us define

$$\underline{w}^* = I^* \Big/ \left[\int_{s\in S} \exp\left\{\int_{\underline{s}}^{s} \frac{A_s\left[s', M^*\left(s'\right)\right]}{A\left[s', M^*\left(s'\right)\right]} ds'\right\} V\left(s\right) ds\right]$$

By construction, $w^* \equiv w\left(\cdot|\underline{w}^*\right)$ is the unique function from S into $\mathbb{R}^+$ that satisfies Equations (42)-(43). This concludes the proof of Step 2.

Steps 1 and 2 imply the existence of a unique solution (w^*, M^*) to the system of Equations (10)-(9) such that $M\left(\underline{s}\right) = \underline{\sigma}$ and $M\left(\overline{s}\right) = \overline{\sigma}$. Since the previous equations must be satisfied in a competitive equilibrium, this implies uniqueness. To demonstrate existence, we consider:

$$\begin{aligned}
C\left(\sigma\right) &= B\left(\sigma\right) \cdot \left[\int_{s\in S} w^*\left(s\right) V\left(s\right) ds\right], \text{ for all } \sigma \in \Sigma \\
L\left(s, \sigma\right) &= V\left(s\right) \delta\left[\sigma - M^*\left(s\right)\right], \text{ for all } s \in S \text{ and } \sigma \in \Sigma \\
w\left(s\right) &= w^*\left(s\right), \text{ for all } s \in S \\
p\left(\sigma\right) &= w^*\left(s\right) A\left[s, M^*\left(s\right)\right], \text{ for all } \sigma \in \Sigma
\end{aligned}$$

By construction, (C, L, w, p) satisfy Conditions (5)-(8). So, a competitive equilibrium exists. **QED**. ■

Proof of Lemma 3. We proceed by contradiction. Suppose that there exists an $s \in (\underline{s}, \overline{s})$ at which $M\left(s\right) \geq M'\left(s\right)$. By Lemma 2, $M\left(\underline{s}\right) = M'\left(\underline{s}\right) = \underline{\sigma}$, $M\left(\overline{s}\right) = M'\left(\overline{s}\right) = \overline{\sigma}$, and

$M(s)$ and $M'(s)$ are continuous functions. So, there must exist $\underline{s} \leq s_1 < s_2 \leq \overline{s}$ and $\underline{\sigma} \leq \sigma_1 < \sigma_2 \leq \overline{\sigma}$ such that $M'(s_1) = M(s_1) = \sigma_1$ and $M'(s_2) = M(s_2) = \sigma_2$ while

$$\begin{aligned} M_s'(s_1) &\leq M_s(s_1) \\ M_s'(s_2) &\geq M_s(s_2) \end{aligned}$$

The two inequalities above imply $M_s'(s_1)/M_s'(s_2) \leq M_s(s_1)/M_s(s_2)$. This is equivalent to

$$\frac{V'(s_2)}{V'(s_1)} \geq \frac{V(s_2)}{V(s_1)}$$

which contradicts $V \succ_a V'$. **QED.** ■

Proof of Lemma 4. Throughout this proof, we use the fact that $\underline{\sigma} = M(\underline{s}) = M'(\underline{s})$, $\overline{\sigma} = M(\overline{s}) = M'(\overline{s})$, and $M(s)$ and $M'(s)$ are both continuous functions, by Lemma 2. We proceed in two steps.

Step 1: *There exists no $s_1 \in (\underline{s}, \overline{s})$ at which $M(s_1) = M'(s_1)$ and $M_s(s_1) \geq M_s'(s_1)$.*

To obtain a contradiction, suppose that such an s_1 exists. There are two possibilities: $s_1 < \widehat{s}$ and $s_1 \geq \widehat{s}$. First, consider $s_1 < \widehat{s}$. There must exist $\underline{s} \leq s_0 < s_1 < \widehat{s}$ at which $M(s_0) = M'(s_0)$ and $M(s_1) = M'(s_1)$ such that

$$\begin{aligned} M_s(s_0) &\leq M_s'(s_0) \\ M_s(s_1) &\geq M_s'(s_1) \end{aligned}$$

These two inequalities imply $M_s(s_0)/M_s(s_1) \leq M_s'(s_0)/M_s'(s_1)$, which is equivalent to

$$\frac{V(s_1)}{V(s_0)} \geq \frac{V'(s_1)}{V'(s_0)},$$

which contradicts $V' \succ_a V$ over the support $s < \widehat{s}$. Second, consider $s_1 \geq \widehat{s}$. There must exist $\widehat{s} \leq s_1 < s_2 \leq \overline{s}$ at which $M(s_1) = M'(s_1)$ and $M(s_2) = M'(s_2)$ such that

$$\begin{aligned} M_s(s_1) &\geq M_s'(s_1) \\ M_s(s_2) &\leq M_s'(s_2) \end{aligned}$$

These two inequalities imply $M_s(s_1)/M_s(s_2) \geq M'_s(s_1)/M'_s(s_2)$, which is equivalent to

$$\frac{V(s_2)}{V(s_1)} \leq \frac{V'(s_2)}{V'(s_1)}$$

which contradicts $V \succ_a V'$ over the support $s \geq \widehat{s}$. This concludes the proof of Step 1.

Step 2: *Either $M(s) > M'(s)$ or $M(s) < M'(s)$ for all $s \in (\underline{s}, \overline{s})$ is impossible.*

To obtain a contradiction, suppose that $M(s) > M'(s)$ for all $s \in (\underline{s}, \overline{s})$. This requires that $M_s(\underline{s}) > M'_s(\underline{s})$ and that $M_s(\overline{s}) < M'_s(\overline{s})$, which implies $M_s(\underline{s})/M_s(\overline{s}) > M'_s(\underline{s})/M'_s(\overline{s})$. This is equivalent to

$$\frac{V(\underline{s})}{V(\overline{s})} > \frac{V'(\underline{s})}{V'(\overline{s})}.$$

which violates the assumption that $V(\underline{s}) = V(\overline{s})$ and $V'(\underline{s}) = V'(\overline{s})$. The proof that $M(s) < M'(s)$ for all $s \in (\underline{s}, \overline{s})$ is impossible is similar. This concludes the proof of Step 2.

Steps 1 and 2 imply that M crosses M' once and only once from above. **QED**. ■

Proof of Lemma 5. We proceed by contradiction. To obtain a contradiction, suppose that there exists an $s \in (\underline{s}, \overline{s})$ at which $M(s) \geq M'(s)$. By Lemma 2, $M(\underline{s}) = M'(\underline{s}) = \underline{\sigma}$, $M(\overline{s}) = M^*(\overline{s}) = \overline{\sigma}$, and $M(s)$ and $M'(s)$ are continuous functions. So, there must exist $\underline{s} \leq s_1 < s_2 \leq \overline{s}$ and $\underline{\sigma} \leq \sigma_1 < \sigma_2 \leq \overline{\sigma}$ such that $M'(s_1) = M(s_1)$ and $M'(s_2) = M(s_2)$ while

$$\begin{aligned} M'_s(s_1) &\leq M_s(s_1) \\ M'_s(s_2) &\geq M_s(s_2) \end{aligned}$$

The two inequalities above imply $M'_s(s_1)/M'_s(s_2) \leq M_s(s_1)/M_s(s_2)$. This is equivalent to

$$\frac{B'(\sigma_2)}{B'(\sigma_1)} \leq \frac{B(\sigma_2)}{B(\sigma_1)}$$

which violates Property (15). Hence, $M'(s) > M(s)$ for all $s \in (\underline{s}, \overline{s})$. **QED**. ■

Proof of Lemma 6. To show that $V_H \succ_a V_F \iff V_H \succ_a V_W$, note that

$$\frac{V_H(s)}{V_H(s')} > \frac{V_W(s)}{V_W(s')} \iff \frac{V_H(s)}{V_H(s')} > \frac{\gamma_H V_H(s) + \gamma_F V_F(s)}{\gamma_H V_H(s') + \gamma_F V_F(s')} \iff \frac{V_H(s)}{V_H(s')} > \frac{V_F(s)}{V_F(s')}.$$

The proof that $V_H \succ_a V_F \iff V_W \succ_a V_F$ is similar. **QED**. ■

Proof of Lemma 7. By definition, $V_H \succ_d V_F$ is equivalent to $V_F \succ_a V_H$, for all $s < \widehat{s}$, and $V_H \succ_a V_F$, for all $s \geq \widehat{s}$. Thus, Lemma 7 follows from Lemma 6 applied separately to $s < \widehat{s}$ and $s \geq \widehat{s}$. **QED**. ■

Proof of Lemma 8. Define $W(i,j) \equiv \gamma_i \int_{\underline{s}}^{\overline{s}} w(s,j)\, V(s,i)\, ds$, where $i = 1$ for Foreign and $i = 2$ for Home; $j = 1$ under B and $j = 2$ under B'; $w(s,j)$ is the World wage function for $j = 1, 2$; and $V(s,i) = V_i(s)$. The fact that $V_H \succ_a V_F$ implies that $V(s,i)$ is log-supermodular. According to Inequality (16), $w(s,j)$ is also log-supermodular. Since log-supermodularity is preserved by multiplication and integration, $W(i,j)$ is log-supermodular; see Karlin and Rinott (1980). This can be rearranged as $\frac{W(2,2)}{W(1,2)} > \frac{W(2,1)}{W(1,1)}$, which is equivalent to $\frac{W'_H}{W'_F} > \frac{W_H}{W_F}$. **QED**. ■

Proof of Lemma 9. To show that $V_H \succ_a V_F \iff V_W \succ_a V'_W$, note that if $\gamma_H > \gamma_F$,

$$\frac{V_W(s)}{V_W(s')} > \frac{V'_W(s)}{V'_W(s')} \iff \frac{\gamma_H V_H(s) + \gamma_F V_F(s)}{\gamma_H V_H(s') + \gamma_F V_F(s')} > \frac{V_H(s) + V_F(s)}{V_H(s') + V_F(s')} \iff \frac{V_H(s)}{V_H(s')} > \frac{V_F(s)}{V_F(s')}.$$

QED. ■

B The Pattern of Trade

B.1 North-South

In the two-by-two model, the Heckscher-Ohlin theorem predicts that the skill-abundant country will export the skill-intensive good, while the unskill-abundant country will export the unskill-intensive good. We now derive the continuum-by-continuum counterpart to this result.

Let $V_H \succ_a V_F$ and denote by $E_H(\sigma) \equiv p(\sigma)[Y_H(\sigma) - C_H(\sigma)]$ the value of Home's net exports in sector σ. Using Equation (**??**) and our solution for $L_W(s,\sigma)$, we can substituting in for $Y_H(\sigma)$ to get

$$E_H(\sigma) = p(\sigma) \left\{ A\left[M_W^{-1}(\sigma), \sigma\right] V_H\left[M_W^{-1}(\sigma)\right] \frac{dM_W^{-1}}{d\sigma} - C_W(\sigma) \right\},$$

where M_W is the matching function in the integrated economy. By Equation (9), this simplifies to

$$E_H(\sigma) = p(\sigma) C_H(\sigma) \left\{ \frac{V_H\left[M_W^{-1}(\sigma)\right]}{V_W\left[M_W^{-1}(\sigma)\right]} - \frac{C_H(\sigma)}{C_W(\sigma)} \right\}. \tag{47}$$

According to Equation (47), Home's net exports are positive in sector σ if and only if $V_H\left[M_W^{-1}(\sigma)\right] / V_W\left[M_W^{-1}(\sigma)\right] > C_H(\sigma) / C_W(\sigma)$. The right-hand side of this inequality is constant because preferences are homothetic. The left-hand side is strictly increasing because $V_H \succ_a V_W$ by Lemma (3) and M_W is strictly increasing by Theorem 1. Thus, Home's net exports, $E_H(\sigma)$, are strictly increasing in σ. A similar argument implies that Foreign's net exports, $E_F(\sigma)$, are strictly decreasing in σ. Balanced trade then directly implies the following:

Proposition B1 *If Home is skill-abundant relative to Foreign, $V_H \succ_a V_F$, then there exists a unique sector $\widetilde{\sigma} \in (\underline{\sigma}, \overline{\sigma})$ such that Home exports goods $\sigma < \widetilde{\sigma}$ and Foreign exports goods $\sigma > \widetilde{\sigma}$.*

This is the continuous analogue to the Heckscher-Ohlin Theorem. Ohnsorge and Trefler (2007) and Costinot (2007) each contains a similar result. The intuition is clear. In the integrated equilibrium, the matching function is the same in both countries. Thus, differences in factor endowments are mechanically reflected in the pattern of international specialization. If Home has relatively more high-skill workers, then it will produce relatively more in the sectors to which those workers are matched, i.e. the skill-intensive ones. With identical and homothetic preferences, the pattern of trade follows.

In addition to extending the Heckscher-Ohlin theorem, our model allows us to predict the cross-sectoral variation of world trade volumes. Let $T_W(\sigma) \equiv 2 \cdot |E_H(\sigma)| / p(\sigma) C_W(\sigma)$ be the share of world exports in world consumption for good σ. By Equation (47), we have:

$$T_W(\sigma) = 2 \cdot \left| \frac{V_N\left[M_W^{-1}(\sigma)\right]}{V_N\left[M_W^{-1}(\sigma)\right]} - \frac{C_H[\sigma]}{C_W[\sigma]} \right|.$$

Because Home is skill-abundant relative to the World, $T_W(\sigma)$ is strictly decreasing for $\sigma \leq \widetilde{\sigma}$ and is strictly increasing for $\sigma \geq \widetilde{\sigma}$. In our North-South environment, the share of world exports in world consumption is U-shape. This result has no natural counterpart in a two-good model.

B.2 North-North

Although the traditional two-by-two Heckscher-Ohlin model predicts that there is no trade between countries with similar aggregate factor endowments, Grossman and Maggi (2000) and subsequent work have demonstrated that the distribution of factors, and not just the aggregate endowment, can shape the pattern of trade. In order to provide context for our novel results on the consequences of North-North trade, we now make this point within our framework.

Recall that the value of Home's net exports in sector σ can be expressed as

$$E_H(\sigma) = p(\sigma) C_W(\sigma) \left\{ \frac{V_H\left[M_W^{-1}(\sigma)\right]}{V_W\left[M_W^{-1}(\sigma)\right]} - \frac{C_H(\sigma)}{C_W(\sigma)} \right\} \tag{48}$$

According to Equation (48), Home's net exports are positive in sector σ if and only if $V_H\left[M_W^{-1}(\sigma)\right]/V_W\left[M_H^{-1}(\sigma)\right] > C_H(\sigma)/C_W(\sigma)$. As in the case of North-South trade, the right-hand side of this inequality is constant because preferences are homothetic. However, unlike the case of North-South trade, the left-hand side is no longer a monotonic function. It is strictly decreasing for all $\sigma < \widehat{\sigma}$ and strictly increasing for all $\sigma > \widehat{\sigma}$ with $\widehat{\sigma} \equiv M(\widehat{s})$. Thus, Home's net exports, $E_H(\sigma)$, are a U-shape function of σ. Balanced trade and the symmetry of V_i for $i = N, S, W$ then directly imply the following Proposition.

Proposition B2 *If Home is more diverse than Foreign, $V_H \succ_d V_F$, then there exists a unique pair of sectors, $\sigma_1 \in (\underline{\sigma}, \widehat{\sigma})$ and $\sigma_2 \in (\widehat{\sigma}, \overline{\sigma})$, such that Home exports goods $\sigma < \sigma_1$ and $\sigma > \sigma_2$, and Foreign exports goods $\sigma \in (\sigma_1, \sigma_2)$.*

The intuition is the same as in the case of North-South trade. Under free trade, workers of a given skill match into the same industries in both countries. With identical and homothetic preferences, the pattern of trade is then completely determined by the relative distribution of skills. Home has a relative abundance of both the least- and most-skilled workers. Thus, Home exports the goods that have the lowest and highest skill intensities. Foreign has a relative abundance of workers with intermediate skill levels. As a result, Foreign exports the sectors into which such workers match.

As in the case of North-South trade, we can also predict how the share of world exports in world consumption, $T(\sigma)$, varies across the continuum of goods. Recall that

$$T_W(\sigma) = 2 \cdot \left| \frac{V_H\left[M_W^{-1}(\sigma)\right]}{V_W\left[M^{-1}(\sigma)\right]} - \frac{C_H[\sigma]}{C_W[\sigma]} \right|$$

Because Home is more diverse than the World, $T_W(\sigma)$ is strictly decreasing for $\sigma \in (\underline{\sigma}, \sigma_1) \cup (\widehat{\sigma}, \sigma_2)$ and is strictly increasing for $\sigma \in (\sigma_1, \widehat{\sigma}) \cup (\sigma_2, \overline{\sigma})$. In a North-North environment, the share of trade in world consumption is W-shape: it tends to be high in the sectors with extreme skill intensities, as in North-South trade, but also in the sectors with intermediate skill intensities, into which workers with average skill levels are matched.

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