Complementarity and Transition to Modern Economic Growth

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Abstract

In developing countries, income per capita typically remains stagnant for long periods before taking-off. We study this as the outcome of a *gradual* transition of the workforce from traditional to modern sectors. While exogenous productivity growth is present in the modern sector only, transition to the modern sector is gradual because work experience is *sector-specific* and *complements* labor. This generates an S-shaped income growth for the dual economy, the effect of which enters into TFP in an aggregate production function. We measure the theory using nationally representative micro data from Thailand (1976-1996). The technology parameters are estimated using cross-sectional earnings equations implied by the model. We find the model simulated at these estimates captures well the nonlinear dynamics of aggregate earnings growth and inequality in Thailand.

JEL: O11, O47, J31

Keywords: Complementarity, Modern Economic Growth, Transition Dynamics, TFP, Inequality

1 Introduction

In developing countries, income per capita typically remains stagnant for long periods before it takes off, and is followed by sustained growth. We study these phenomena as an outcome of a process of modernization, measured as a transition of the workforce from traditional to modern technology, in a dual economy. The identifying assumption of the modern sector is the existence of exogenous productivity growth, which is not present in the traditional sector.

Transition to the modern sector is gradual because of a particular adjustment cost. We assume labor and sector-specific work experience are complements in each sector, as in Chari and Hopenhayn (1991). Under this complementarity, entry into the modern sector among young agents who supply labor is limited by the stock of old agents who supply experience. Meanwhile, today's young entrants in turn determine tomorrow's stock of experience. Thus, the transition to the exclusive use of modern technology occurs gradually, despite the productivity growth gap between the two sectors. The implied path of income during transition is S-shaped (remaining stagnant initially, taking off with acceleration, and then decelerating). The speed and slope of the transition depend on the *initial distribution of work experience* across sectors and the magnitudes of the complementarities. In conventional growth accounting, this S-shaped income growth would enter into total factor productivity (TFP) growth, although the work experience is a component of human capital.

As the workforce shifts from the traditional to the modern sector, the labor-experience ratios change in both sectors. Due to the complementarity, this causes the within-sector experience-earnings profiles and the between-sector earnings gap, to vary over time. Thus, the modernization is a source of changes in earnings distribution. By linking the aggregate growth path to these dimensions of inequality, we highlight a novel nexus of growth and inequality.

Our model is motivated by observing a set of facts about earnings dynamics in the micro data from Thailand. There have been two groups of workers in Thailand: one group consists of occupations with no labor productivity growth, which we call the "traditional sector," and the other group with positive and stable labor productivity growth (at an average annual rate of 2.5%), which we call the "modern sector." Despite this productivity growth gap, the transition from the traditional sector to the modern sector has occurred only gradually. Furthermore, the two sectors have coexisted not only among old cohorts but also among the youngest cohorts entering the workforce. Figure 1 plots the non-parametrically estimated (using local polynomial fitting method) trend of the population
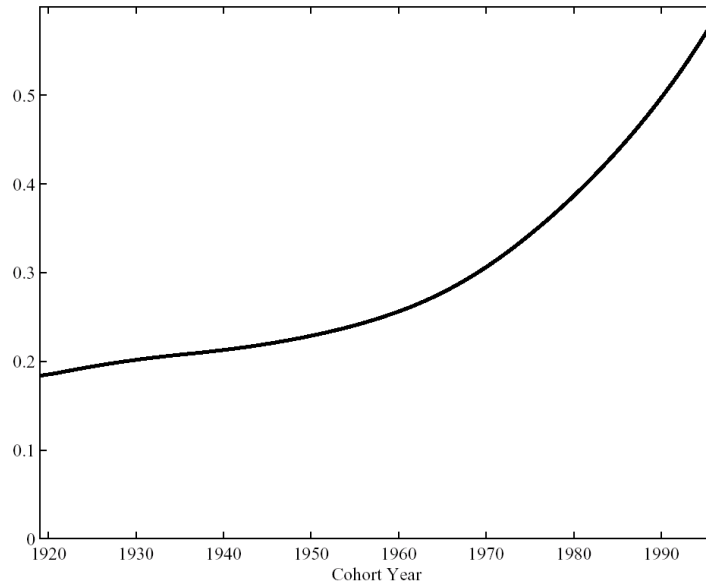
Figure 1: Cohort Share of Modern Sector in Thailand

share of workers in the modern sector across cohorts.[1] This shows that the Thai transition between sectors was not only gradual, but also accelerated, following a significant period of virtually no transition.

We also observe that experience-earnings profiles differ between traditional and modern sectors, and the within-sector experience-earnings profiles vary substantially over time. In particular, in each sector, the experience premium rises when experience becomes scarce relative to labor, consistent with the sector-specific complementarity between labor and experience. Figure 2.1 compares the series of experience premium and labor-experience ratio of the traditional sector for the 1976-1996 period.[2] The experience premium not only moved substantially over time, but also co-moved with the labor-experience ratio. Figure 2.2 displays a similar positive correlation between the experience premium and the labor-experience ratio for the modern sector. If the sector-specific experience simply adds on the effective units of labor within each sector (that is, experience and labor are substitutes), we would observe no systematic correlation between the movements of experience premia and the changes in the labor-experience ratios.

In sum, the documented data above confirm (i) the coexistence of two sectors with

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[1]We choose the bandwidth and weighting function following the Lowess procedure.

[2]The time-varying experience premia are estimated controlling for the typical income-generating socioeconomic characteristics such as years of schooling, geographic region, community type, and gender as well as the linear time trend and the year dummies within each sector. Direct information for actual work experience is not available in the Thai data and we follow the convention of the labor literature, measuring experience by potential experience, i.e., by (age - years of schooling - 6).
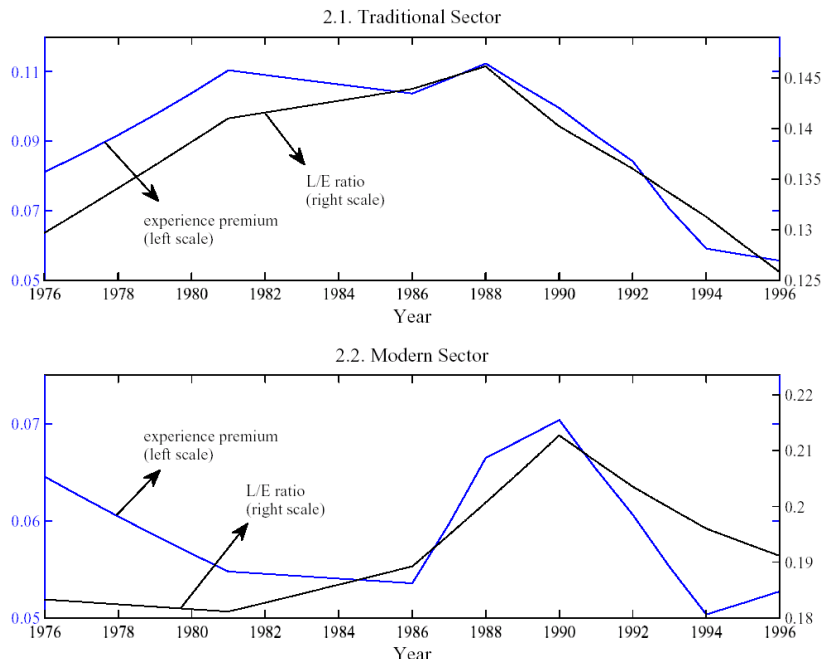
Figure 2: Co-movements between Experience Premium and Labor-Experience Ratio

a substantial gap in productivity growth, and (ii) the existence of sector-specific complementarity between labor and experience, the key specifications of our model.

We measure our theory using nationally representative micro data from the Socio-Economic Survey of Thailand (1976-1996). Specifically, the *parameters* of the model as well as the *partition* of economy into traditional and modern sectors (which is not directly measured in the data) are identified by estimating cross-sectional earnings equations as implied by the model. The source of the identification is the co-movement between the experience premia in the earnings equations and the sectoral labor-experience ratios over time within each sector. We then simulate the model to assess its quantitative importance. At these micro estimates of the parameters, the model simulates well the nonlinear dynamics of aggregate earnings growth and inequality, together with the gradual transition of the workforce from the traditional to modern sector. Specifically, the model captures the S-shaped path of aggregate earnings growth, the rise and fall of within-sector experience premia, and the converging then diverging between-sector earnings gap during transition observed in the Thai data.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the model. Section 4 estimates the model. Simulation results are compared to data in Section 5. Section 6 concludes.

4

2   Literature

Dual-economy models featuring transition from a stagnant traditional sector to a growing modern sector were pioneered by Lewis (1954) and Ranis and Fei (1961). A theoretical contribution of this paper is to show that a dual-economy model combined with the idea of labor-experience complementarity can generate long periods of stagnation followed by take-off to sustained growth. Unlike the assumptions of the early models, we consider all inputs to be priced at competitive margins in both sectors. Despite this and the constant returns to scale production technologies, we still generate the essential take-off dynamics. Furthermore, we show that the speed and slope of the transition depend on the *initial distribution of work experience* across the two sectors and hence that history matters for growth.

Our empirical contribution is that we identify the modern and traditional sectors together with their technology parameters by estimating micro earnings equations. Household or firm surveys do not directly collect data on the partition of an economy according to the use of traditional and modern technologies. Thus, the literature of dual-economy models approximates the distinction between the traditional and modern sectors by products type (agriculture versus manufacturing) or by community type (rural versus urban), although the original idea of the dual economy is characterized by the absence or presence of growth. We show that the traditional and modern sectors coexist within each subgroup of agriculture, manufacturing, services, rural and urban areas, and that population shifts from traditional to modern technology has occurred within each of them.

This paper applies the theory to a particular developing country Thailand, for the purpose of rich measurement using micro data.[3] Jeong and Kim (2006) show that a version of the model can also explain the differences in growth and evolution of per capita income inequality *across countries* since the time of the Industrial Revolution. In particular, they show that the S-shaped growth is a salient feature not only among developing countries, but also among today's rich countries over a long time horizon.

Chari and Hopenhayn (1991) consider the role of technology-specific complementarity between labor and experience (vintage human capital) in a steady state framework, implying *linear* growth. Jovanovic and Nyarko (1996) analyze the transition dynamics of technology adoption in a different context of Bayesian learning, while Kremer and Thomson (1998) study a multi-sector economy where labor and *skill* are complements. In these models, the aggregate transition path to steady states is *concave*. We show when labor and *experience* are complements, the transition path will be *convex* before

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[3]Conley and Udry (2005) also use a rich set of micro data to study the issue of technology diffusion in Ghana. They focus on identifying the micro mechanism of social learning for diffusion rather than its impact on macroeconomic dynamics, which is our focus.

becoming concave, generating the S-shaped transition, as observed in the data. This is a key to our analysis. Beaudry and Francois (2005) study a similar dual economy model of complementarity between unskilled worker and skilled managers and show how it can generate multiple steady states (including poverty traps). We consider a setting where the sustained productivity growth in the modern sector eventually overrides the persistent force of complementarities to imply a unique steady state.[4]

The importance of structural transformation in understanding the growth process is emphasized by Kuznets (1966), Lucas (2000, 2004) and Galor (2005) among others. Gollin, Parente and Rogerson (2002), and Hansen and Prescott (2002) highlight the role of either Stone-Geary type of non-homothetic preferences, the existence of a fixed input such as land, or some external barriers in making the transition gradual and varied. In the absence of these ingredients, we emphasize how gradual and varied transition is also possible solely because of *initial conditions.* As in Lucas (2004), we also emphasize the role of human capital for transition, but focus on human capital acquired through *work experience* rather than through schooling. Existing models are either silent or predict unrealistic (ever-widening income gap) income distribution dynamics during transition. Our model has clear predictions on both within-sector and between-sector inequality dynamics within a country, which we demonstrate are indeed borne out by the Thai data.

Aggregate income growth in our model is driven by endogenous changes in the *distribution* of experience across sectors, combined with exogenous productivity growth in the modern sector. In conventional growth accounting, both sources of growth would enter into total factor productivity (TFP) growth. Hence, our model provides a theory of TFP. This has two important implications. First, our model predicts that the observed S-shaped income growth is generated through TFP rather than through factor accumulation. Second, our model suggests that the relevant variable for explaining the differences in levels and growth rates of income is the *distribution* of experience across sectors, not the *aggregate* stock of experience. Klenow and Rodríguez-Clare (1997) and Caselli (2005) show that adding aggregate experience in measuring human capital plays virtually no role in reducing TFP. This is not surprising from our model's point of view. Incorporating instead the *distribution* of experience across sectors will reduce the size of TFP and magnify the importance of human capital.

Our analysis for Thailand suggests that the changes in demographic composition of the workforce is a key variable in explaining the earnings inequality dynamics via the movements of experience premia. The significance of relative cohort size in explaining the

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[4]Foster and Rosenzweig (2004) have studied the role of modern (non-farm) productivity growth on growth and inequality of the traditional (farm) sector in rural India during 1968-1999. We also emphasize the role of modern productivity growth in explaining the growth and inequality dynamics in both traditional and modern sectors.

change in U.S. wage structure has been emphasized by Welch (1979) and Katz and Murphy (1992). Kambourov and Manovskii (2005) emphasize the stock of occupation-specific experience in explaining the flattening earnings profiles. Our model emphasizes the relative *ratio* between labor and experience (not the cohort-specific size of labor or total stock of experience), in explaining changes in the earnings profile over time. We demonstrate how this implied relationship between experience premia and the labor-experience ratio can be explicitly brought to the data and tested. In a one-sector variant of the model, Jeong, Kim and Manovskii (2008) confirm that labor-experience complementarity plays a key role in explaining changes in earnings inequality for the U.S. and a sample of other developed countries as well.

3 Model

3.1 Two-period Model

Consider a two-period overlapping generations economy with constant population. Lifetime preferences of agents who are born at date t are

$$(1) \qquad U(c_{0t}, c_{1t+1}) = c_{0t} + \beta c_{1t+1},$$

where $\beta \in (0,1)$ is the time-discount factor, c_{0t} denotes the consumption when young, and $c_{1,t+1}$ the consumption when old. The lifetime budget constraint is given by

$$(2) \qquad c_{0t} + \frac{1}{R_{t+1}} c_{1t+1} = y_{0t} + \frac{1}{R_{t+1}} y_{1t+1},$$

where R_{t+1} is the interest factor. y_{0t} denotes the earnings when young and y_{1t+1} the earnings when old. Linear preferences imply $\frac{1}{R_{t+1}} = \beta$ for each date t. Agents have perfect foresight.

There are two sectors, traditional and modern, associated with different technologies that produce a homogenous good. Each young agent is endowed with one unit of raw labor that is inelastically supplied to either sector. When old, this agent acquires a skill from the work experience, specific to the sector he worked in when young. Old agents supply both this sector-specific experience and raw labor.[5] The effective units of labor and experience are subject to change when old by a factor λ, which we allow to be either a depreciation ($\lambda < 1$) or an appreciation factor ($\lambda > 1$).

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[5]Note that we split up the inputs into labor and experience for *each* agent unlike Chari and Hopenhayn (1991) who assume young and old agents supply different inputs. Our specification provides a more natural characterization of complementarity between young and old workers so that the two-period model can be easily generalized into a multi-period model, which we use for estimation and simulation later.

Let N_t and M_t denote the cohort shares of young agents who enter the traditional and modern sectors respectively at date t. Then, the aggregate measures of labor $L_{k,t}$ and experience $E_{k,t}$ of sector k ($k = T$ for traditional sector and $k = M$ for modern sector) at date t are given by

$$
\begin{aligned}
L_{T,t} &= N_t + \lambda N_{t-1}, \\
E_{T,t} &= \lambda N_{t-1}, \\
L_{M,t} &= M_t + \lambda M_{t-1}, \\
E_{M,t} &= \lambda M_{t-1}.
\end{aligned}
$$

Due to the resource constraints

$$
N_t + M_t = 1, \ \forall t,
$$

these can be simplified into a first-order difference equation system of a single state variable M_t such that

$$
\begin{aligned}
(3) \qquad L_{T,t} &= 1 - M_t + \lambda(1 - M_{t-1}), \\
E_{T,t} &= \lambda\left(1 - M_{t-1}\right), \\
L_{M,t} &= M_t + \lambda M_{t-1}, \\
E_{M,t} &= \lambda M_{t-1},
\end{aligned}
$$

given the initial state M_{-1}.

Let $Y_{T,t}$ and $Y_{M,t}$ denote efficiency units of output from the raw labor and experience in the traditional sector and the modern sector, respectively, such that

$$
\begin{aligned}
Y_{T,t} &= G\left(L_{T,t}, E_{T,t}\right), \\
Y_{M,t} &= \gamma^t X F\left(L_{M,t}, E_{M,t}\right),
\end{aligned}
$$

where $\gamma > 1$ is the exogenous growth factor available only in the modern sector and X denotes the productivity level of the modern sector relative to the traditional sector.[6] We assume $\beta\gamma < 1$. Then, aggregate labor earnings Y_t is given by

$$
(4) \qquad Y_t = G\left(L_{T,t}, E_{T,t}\right) + \gamma^t X F\left(L_{M,t}, E_{M,t}\right).^7
$$

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[6]The productivity growth factor γ may come from pure technical changes or from relative price changes (including changes in the quality of goods), which we do not distinguish.

[7]In Appendix A.2, we show how this aggregate earnings function can be derived from a general aggregate production function with physical capital, when the interest factor is constant (as implied by our assumption of linear perferences).

The functions G and F represent sector-specific technologies combining labor and experience subject to constant returns to scale. In each sector, labor and experience are complements in the sense that

$$\frac{\partial^2 G\left(L_{T,t}, E_{T,t}\right)}{\partial L_{T,t} \partial E_{T,t}} \geq 0 \text{ and } \frac{\partial^2 F\left(L_{M,t}, E_{M,t}\right)}{\partial L_{M,t} \partial E_{M,t}} \geq 0.$$

Thus, experience does not simply add to raw labor in contributing to efficiency units of output from labor.

Define $g\left(\frac{L_{T,t}}{E_{T,t}}\right) \equiv \frac{G\left(L_{T,t}, E_{T,t}\right)}{E_{T,t}}$. Then $g'\left(\frac{L_{T,t}}{E_{T,t}}\right)$ measures the marginal product of raw labor, and $\phi\left(\frac{L_{T,t}}{E_{T,t}}\right) \equiv g\left(\frac{L_{T,t}}{E_{T,t}}\right) - g'\left(\frac{L_{T,t}}{E_{T,t}}\right)\frac{L_{T,t}}{E_{T,t}}$ measures the marginal product of experience in the traditional sector. Similarly, define $f\left(\frac{L_{M,t}}{E_{M,t}}\right) \equiv \frac{F\left(L_{M,t}, E_{M,t}\right)}{E_{M,t}}$ and $\pi\left(\frac{L_{M,t}}{E_{M,t}}\right) \equiv$ $f\left(\frac{L_{M,t}}{E_{M,t}}\right) - f'\left(\frac{L_{M,t}}{E_{M,t}}\right)\frac{L_{M,t}}{E_{M,t}}$. Then, the earnings of young workers $y_{k,0t}$ in sector k at date t are

$$\widetilde{y}_{T,0t} = g'\left(\frac{L_{T,t}}{E_{T,t}}\right) \text{ for traditional sector,}$$

$$\widetilde{y}_{M,0t} = \gamma^t X f'\left(\frac{L_{M,t}}{E_{M,t}}\right) \text{ for modern sector,}$$

and the earnings of old workers $y_{k,1t}$ in sector k at date t are

$$\widetilde{y}_{T,1t} = \lambda\left[g'\left(\frac{L_{T,t}}{E_{T,t}}\right) + \phi\left(\frac{L_{T,t}}{E_{T,t}}\right)\right] \text{ for traditional sector,}$$

$$\widetilde{y}_{M,1t} = \lambda\gamma^t X\left[f'\left(\frac{L_{M,t}}{E_{M,t}}\right) + \pi\left(\frac{L_{M,t}}{E_{M,t}}\right)\right] \text{ for modern sector.}$$

The cross-sectional experience premia (measured as the ratio of experienced worker earnings to inexperienced worker earnings) for a given period t are given by

$$\frac{y_{T,1t}}{y_{T,0t}} = \lambda\left(1 + \frac{\phi\left(\frac{L_{T,t}}{E_{T,t}}\right)}{g'\left(\frac{L_{T,t}}{E_{T,t}}\right)}\right) \text{ for traditional sector,}$$

$$\frac{y_{M,1t}}{y_{M,0t}} = \lambda\left(1 + \frac{\pi\left(\frac{L_{M,t}}{E_{M,t}}\right)}{f'\left(\frac{L_{M,t}}{E_{M,t}}\right)}\right) \text{ for modern sector.}$$

Labor-experience complementarity implies that g' and f' are decreasing and ϕ and π are increasing in sector-specific labor-experience ratios. Thus, the experience premium is positively correlated with the movements of labor-experience ratios within each sector.

The lifetime earnings $W_{k,t}$ of an agent born at date t entering sector k are

$$W_{T,t} = g'\left(\frac{L_{T,t}}{E_{T,t}}\right) + \beta\lambda\left[g'\left(\frac{L_{T,t+1}}{E_{T,t+1}}\right) + \phi\left(\frac{L_{T,t+1}}{E_{T,t+1}}\right)\right] \quad \text{for traditional sector,}$$

$$W_{M,t} = \gamma^t X\left\{f'\left(\frac{L_{M,t}}{E_{M,t}}\right) + \beta\lambda\gamma\left[f'\left(\frac{L_{M,t+1}}{E_{M,t+1}}\right) + \pi\left(\frac{L_{M,t+1}}{E_{M,t+1}}\right)\right]\right\} \quad \text{for modern sector.}$$

If there is no sectoral reallocation of workers (i.e. when N_t and M_t are constant over time), within-sector labor-experience ratios are constant and equal across sectors (which defines a steady state), and we have

$$\frac{L_{T,t}}{E_{T,t}} = \frac{L_{M,t}}{E_{M,t}} = 1 + \frac{1}{\lambda}.$$

We assume that the lifetime earnings of an agent working in the traditional sector is weakly lower than that in the modern sector when there is no sectoral reallocation of workers

(5)
$$g'\left(1+\frac{1}{\lambda}\right) + \beta\lambda\left[g'\left(1+\frac{1}{\lambda}\right) + \phi\left(1+\frac{1}{\lambda}\right)\right]$$
$$\leq \gamma^t X\left\{f'\left(1+\frac{1}{\lambda}\right) + \beta\lambda\gamma\left[f'\left(1+\frac{1}{\lambda}\right) + \pi\left(1+\frac{1}{\lambda}\right)\right]\right\}, \quad \forall t,$$

which we will refer to as the "pivotal condition."

Transition is generated by the arrival of positive productivity growth in the modern sector. In the absence of such growth, there is a level $\tilde{X}$ for X such that the steady state sectoral distribution of agents is indeterminate, where

(6)
$$g'\left(1+\frac{1}{\lambda}\right) + \beta\lambda\left[g'\left(1+\frac{1}{\lambda}\right) + \phi\left(1+\frac{1}{\lambda}\right)\right]$$
$$\equiv \tilde{X}\left\{f'\left(1+\frac{1}{\lambda}\right) + \beta\lambda\left[f'\left(1+\frac{1}{\lambda}\right) + \pi\left(1+\frac{1}{\lambda}\right)\right]\right\}.$$

Note that $X = \tilde{X}$ is a sufficient condition for the pivotal condition in (5).

3.2 Equilibrium

A *competitive equilibrium* consists of a sequence of modern cohort shares $\{M_t\}_{t=0}^{\infty}$ and interest factor R such that

1. every agent earns his marginal products,

2. young agents decide which sector to work in and how much to consume to maximize their lifetime utility (1) subject to the budget constraint (2), and lifetime earnings

given by

$$
(7) \quad \max \left\{ \begin{array}{c} g'\left(\frac{L_{T,t}}{E_{T,t}}\right) + \frac{1}{R_{t+1}}\lambda\left[g'\left(\frac{L_{T,t+1}}{E_{T,t+1}}\right) + \phi\left(\frac{L_{T,t+1}}{E_{T,t+1}}\right)\right], \\ \gamma^t X\left[f'\left(\frac{L_{M,t}}{E_{M,t}}\right) + \frac{1}{R_{t+1}}\lambda\gamma\left[f'\left(\frac{L_{M,t+1}}{E_{M,t+1}}\right) + \pi\left(\frac{L_{M,t+1}}{E_{M,t+1}}\right)\right]\right] \end{array} \right\},
$$

3. the aggregate inputs are given by (3), and

4. the credit market clears in every period. (Linear preferences imply the credit market clearing condition is $\frac{1}{R_t} = \beta$, $\forall t$.)

If young agents enter both sectors in period t, i.e. $M_t \in (0,1)$, the following "participation constraint" should be satisfied during transition

$$
\begin{aligned}
(8) \quad & g'\left(1 + \frac{1-M_t}{\lambda(1-M_{t-1})}\right) + \beta\lambda\left[g'\left(1 + \frac{1-M_{t+1}}{\lambda(1-M_t)}\right) + \phi\left(1 + \frac{1-M_{t+1}}{\lambda(1-M_t)}\right)\right] \\
& = \gamma^t X\left\{f'\left(1 + \frac{M_t}{\lambda M_{t-1}}\right) + \beta\lambda\gamma\left[f'\left(1 + \frac{M_{t+1}}{\lambda M_t}\right) + \pi\left(1 + \frac{M_{t+1}}{\lambda M_t}\right)\right]\right\}.
\end{aligned}
$$

Lemma 1 *Let $\widehat{T}$ denote the first period at which an entire cohort works in the modern sector. Then, $M_{\widehat{T}} = 1$ implies $M_{\widehat{T}+s} = 1$ $\forall s \geq 1$.* Proof in Appendix A.1.

Combining Lemma 1 with the participation constraint (8), we get

$$
\begin{aligned}
(9) \quad & g'(1) + \beta\lambda\left[g'(1) + \phi(1)\right] \\
& \leq \gamma^t X\left\{f'\left(1 + \frac{1}{\lambda M_{t-1}}\right) + \beta\lambda\gamma\left[f'\left(1 + \frac{1}{\lambda}\right) + \pi\left(1 + \frac{1}{\lambda}\right)\right]\right\}, \quad \forall t \geq \widehat{T}.
\end{aligned}
$$

Equations (8) and (9) define a second-order difference equation system in M_t, which characterizes the equilibrium transition dynamics of the model.[8]

Proposition 1 *For a given initial state M_{-1}, (i) there exists a unique equilibrium transition path with $\widehat{T} < \infty$; (ii) $M_{t-1} \leq M_t$ $\forall t \geq 1$; (iii) M_t increases in M_{-1}, $\forall t \geq 0$; (iv) $\widehat{T}$ decreases in M_{-1}.* Proof in Appendix A.1.

The algorithm for constructing the equilibrium transition path (solving the second-order difference equation system in (8) and (9)) is explained in Appendix A.1. Note that the date for the completion of transition $\widehat{T}$ is *endogenous,* and hence the terminal condition of the difference equation system is not fixed, which makes the solution algorithm non-trivial.

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[8]Note that this competitive equilibrium allocation of workers across technologies coincides with the allocation of the following social planner's problem:

$$
\max_{\{M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t Y_t \text{ s.t. (3) and (4).}
$$

Proposition 1.(i) and 1.(ii) state that transition follows a unique path ending in finite time, and the modern population share does not decrease over time. Proposition 1.(iii) and 1.(iv) state that transition occurs faster if the initial share of modern sector is higher. In other words, when the initial modern share is very low, transition can be very slow and the economy can be stagnant for a very long while (appearing to be trapped), although the economy will eventually take off.

3.3 S-shaped Transition

During transition, lifetime earnings grow first slower than the rate γ and then faster than γ, i.e., in an S-shaped path. When transition is complete, i.e., everyone is in the modern sector, the economy will follow a constant steady-state growth path at the rate γ.

Two extreme cases provide intuition for this result. First, suppose there is no complementarity in the traditional sector only, i.e. $\frac{\partial^2 G(L_{T,t}, E_{T,t})}{\partial L_{T,t} \partial E_{T,t}} = 0$ and $\frac{\partial^2 F(L_{M,t}, E_{M,t})}{\partial L_{M,t} \partial E_{M,t}} > 0$. Then, the lifetime earnings of the traditional sector should be constant regardless of changes in labor-experience ratios during transition, and the participation constraint (8) implies the modern lifetime earnings is constant as well. Despite the modern productivity growth, lifetime earnings are constant during transition up to period $\widehat{T} - 1$ (one period before all young agents enter the modern sector), then converge to the steady-state lifetime earnings path by period $\widehat{T} + 1$, generating a kink shaped path.

Second, consider the opposite extreme case, where there is no complementarity in the modern sector only, i.e., $\frac{\partial^2 G(L_{T,t}, E_{T,t})}{\partial L_{T,t} \partial E_{T,t}} > 0$ and $\frac{\partial^2 F(L_{M,t}, E_{M,t})}{\partial L_{M,t} \partial E_{M,t}} = 0$. Then, the labor-experience ratios do not affect modern lifetime earnings, which simply grow at the rate γ. Again, participation constraint (8) implies traditional lifetime earnings must grow at the same rate. Here, lifetime earnings grow linearly at rate γ during and after transition. For general intermediate case where complementarities exist in both sectors, i.e. $\frac{\partial^2 G(L_{T,t}, E_{T,t})}{\partial L_{T,t} \partial E_{T,t}} > 0$ and $\frac{\partial^2 F(L_{M,t}, E_{M,t})}{\partial L_{M,t} \partial E_{M,t}} > 0$, we observe the S-shaped path of lifetime earnings growth.

Proposition 2 *During transition, (i) if lifetime earnings are rising over time, the population of the traditional sector is falling at a faster rate, $\frac{1-M_t}{1-M_{t-1}} > \frac{1-M_{t+1}}{1-M_t}$; (ii) if lifetime earnings are first rising slower than γ, then rising faster than γ, the population growth of the modern sector is single peaked, i.e., there exists unique period $S < \widehat{T}$ such that $\frac{M_t}{M_{t-1}} < \frac{M_{t+1}}{M_t}$ for all $t < S$ and $\frac{M_t}{M_{t-1}} \geq \frac{M_{t+1}}{M_t}$ for all $t \geq S$.* Proof is in Appendix A.1.

This result that the population growth of the modern sector is single peaked implies an S-shaped population shifts from traditional to modern sector. Combined with the S-shaped lifetime earnings growth, this S-shaped modernization process generates an S-shaped growth of aggregate earnings during transition.

3.4 Initial Condition and Diverse Transition

The curvature of the S-shaped transition paths depend on functional forms and the parameter space of F and G as well as on the initial cohort share of the modern sector. Later we will explicitly measure these technology parameters in a more general model using micro data for quantitative evaluation of the model. Here, we first illustrate how the difference in initial conditions can generate diverse growth patterns during transition by parameterizing the sectoral production functions G and F in the following CES forms

$$(10) \qquad G\left(L_{T,t}, E_{T,t}\right) \;=\; \left[\alpha_T L_{T,t}^{\rho_T} + (1-\alpha_T)E_{T,t}^{\rho_T}\right]^{\frac{1}{\rho_T}},$$

$$(11) \qquad F\left(L_{M,t}, E_{M,t}\right) \;=\; \left[\alpha_M L_{M,t}^{\rho_M} + (1-\alpha_M)E_{M,t}^{\rho_M}\right]^{\frac{1}{\rho_M}},$$

where $\rho_k \leq 1$ and $0 < \alpha_k < 1$, for $k = T$ and M.

The elasticity of substitution between labor and experience in each sector k is constant at $\frac{1}{1-\rho_k}$. The lower the ρ_k parameter, the greater the complementarity between labor and experience. Note that at the limit value of ρ_k at unity, labor and experience are perfect substitutes and the parameter $(1-\alpha_k)$ alone determines the experience premium, implying a constant experience premium over time.

We compare the transition paths by varying the initial modern cohort share M_{-1} over three economies, $M_{-1} = 0.1$ for Case 1, $M_{-1} = 0.001$ for Case 2 and $M_{-1} = 0.00001$ for Case 3, keeping the technology parameters constant at $\rho_T = \rho_M = -0.5$ and $\alpha_T = \alpha_M = 0.7$.[9] Figure 3.1 displays the path of the workforce transition into the modern sector. The lower the initial modern cohort share, the longer the delay of the workforce transition into the modern sector. Figure 3.2 compares the transition paths of aggregate output. Both figures illustrate that the transition dynamics are S-shaped for all three economies but the speed and slope of this transition depends on the initial cohort share of modern sector.

Comparing Case 1 and Case 2, the late catch-up economy (Case 2) grows faster than the early starter (Case 1), once it takes off. Comparing Case 2 and Case 3, this effect becomes more pronounced, i.e., the longer the period of stagnation, the faster the growth rate of catch-up.[10] The Case 3 economy with very small initial modern share stagnates for a very long while. During the first 100 years, the Case 3 economy does not grow at all, which may appear to be trapped although eventually it grows. In sum, among economies with otherwise identical characteristics, diverse patterns of growth, from stagnation to miracle, can be generated from differences in the initial share of modern sector.

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[9] Here, we assume people work for 60 years and adjust the parameter values to the 2-period OLG framework and the rest chosen parameters are $\beta = 0.8^{30}$, $\gamma = 1.022^{30}$ and $\lambda = 0.98^{30}$. To satisfy the pivotal condition, we set $X = \tilde{X}$ as in (6) at these parameters.

[10] Jeong and Kim (2006), call this the "catapult effect", and show that this effect indeed exists for the long run income growth paths across countries since 1820.
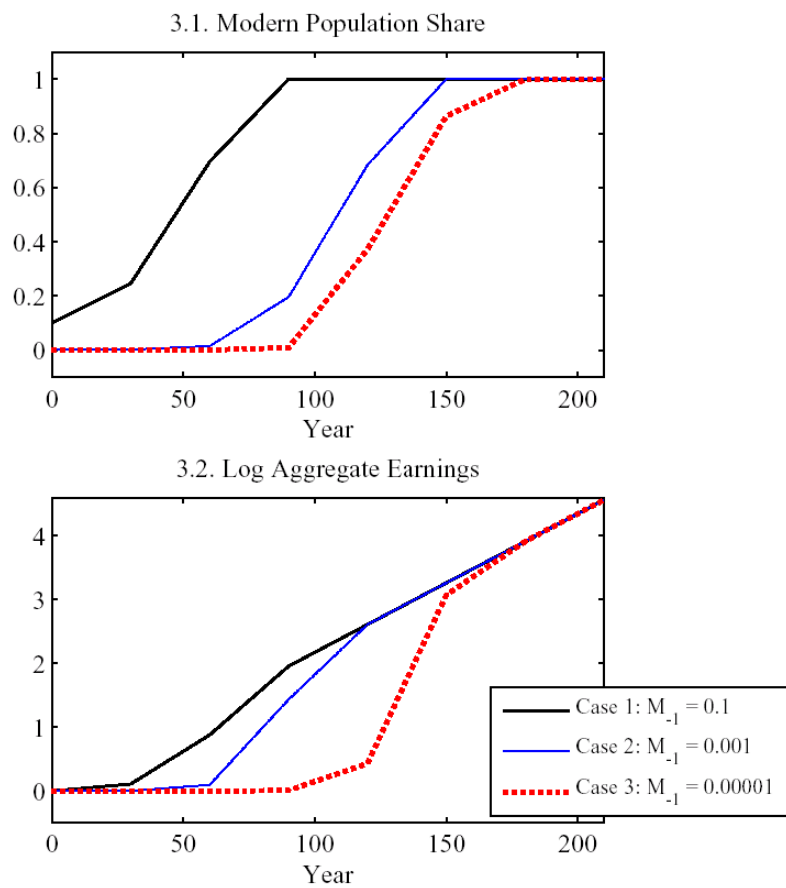
3.1. Modern Population Share

3.2. Log Aggregate Earnings

Case 1: $M_{-1} = 0.1$
Case 2: $M_{-1} = 0.001$
Case 3: $M_{-1} = 0.00001$

Figure 3: Initial Condition and Transition Dynamics

3.5 General J-period Model

We now generalize the model into a J-period overlapping-generations model for $2 \leq J < \infty$, which will be used in our estimation and simulation. Lifetime preferences of agents who are born at date t are

$$U_t = \sum_{j=0}^{J-1} \beta^j c_{j,t+j}. \tag{12}$$

As before, linear preferences imply $\frac{1}{R_t} = \beta$ for each date t, and the lifetime budget constraint is

$$\sum_{j=0}^{J-1} \beta^j c_{j,t+j} = \sum_{j=0}^{J-1} \beta^j y_{j,t+j}. \tag{13}$$

Each agent who has worked for j periods in sector k provides $\lambda_k(j)$ units of labor and $j\lambda_k(j)$ units of sector-specific experience, where $\lambda_k(j)$ reflects the change in effective units of labor and experience across experience j's.[11] The aggregate measures of sectoral labor and experience at date t are given by

$$L_{T,t} = \sum_{j=0}^{J-1} \lambda_T(j) D_{jt} N_{t-j}, \tag{14}$$

$$E_{T,t} = \sum_{j=0}^{J-1} j\lambda_T(j) D_{jt} N_{t-j}, \tag{15}$$

$$L_{M,t} = \sum_{j=0}^{J-1} \lambda_M(j) D_{jt} M_{jt-j}, \tag{16}$$

$$E_{M,t} = \sum_{j=0}^{J-1} j\lambda_M(j) D_{jt} M_{t-j}, \tag{17}$$

$$N_{t-j} + M_{t-j} = 1, \tag{18}$$

where D_{jt} denotes the total measure of agents with j periods of experience at date t. When workforce participation rates are constant across experience groups and over time, D_{jt} is constant over j and t. We allow D_{jt} to exogenously vary over j and t to capture the observed asymmetry in labor force participation rates across experience groups, which also fluctuates over time. The key state variable that endogenously evolves over time is $\{M_t\}_{t=0}^{\hat{T}-1}$ given the initial condition $\{M_{-j}\}_{j=1}^{J-1}$.

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[11] Allowing this general depreciation (or appreciation) factor helps to capture the observed schedule of experience-earnings profiles in a flexible way.

The cross-sectional earnings $\widetilde{y}_{k,t}(j)$ of workers with j periods of experience in sector k at date t are

$$(19) \quad \widetilde{y}_{T,t}(j) \;=\; \lambda_T(j) \left[g'\left(\frac{L_{T,t}}{E_{T,t}}\right) + \phi\left(\frac{L_{T,t}}{E_{T,t}}\right) j \right] \quad \text{for traditional sector,}$$

$$(20) \quad \widetilde{y}_{M,t}(j) \;=\; \lambda_M(j)\gamma^t X \left[f'\left(\frac{L_{M,t}}{E_{M,t}}\right) + \pi\left(\frac{L_{M,t}}{E_{M,t}}\right) j \right] \quad \text{for modern sector.}$$

The implied experience premia of workers with j periods of experience relative to zero-experienced workers are

$$(21) \quad \frac{\widetilde{y}_{T,t}(j)}{\widetilde{y}_{T,t}(0)} \;=\; \frac{\lambda_T(j)}{\lambda_T(0)} \left[1 + \frac{\phi\left(\frac{L_{T,t}}{E_{T,t}}\right)}{g'\left(\frac{L_{T,t}}{E_{T,t}}\right)} j \right] \quad \text{for traditional sector,}$$

$$(22) \quad \frac{\widetilde{y}_{M,t}(j)}{\widetilde{y}_{M,t}(0)} \;=\; \frac{\lambda_M(j)}{\lambda_M(0)} \left[1 + \frac{\pi\left(\frac{L_{M,t}}{E_{M,t}}\right)}{f'\left(\frac{L_{M,t}}{E_{M,t}}\right)} j \right] \quad \text{for modern sector,}$$

which increase with the respective sectoral labor-experience ratios due to the complementarity.

The lifetime earnings $W_{k,t}$ of a cohort born at date t entering sector k are given by

$$W_{T,t} \;=\; \sum_{j=0}^{J-1} \beta^j \lambda_T(j) \left[g'\left(\frac{L_{T,t+j}}{E_{T,t+j}}\right) + \phi\left(\frac{L_{T,t+j}}{E_{T,t+j}}\right) j \right] \quad \text{for traditional sector,}$$

$$W_{M,t} \;=\; \gamma^t X \sum_{j=0}^{J-1} \beta^j \lambda_M(j)\gamma^j \left[f'\left(\frac{L_{M,t+j}}{E_{M,t+j}}\right) + \pi\left(\frac{L_{M,t+j}}{E_{M,t+j}}\right) j \right] \quad \text{for modern sector.}$$

The pivotal condition becomes

$$(23) \quad \sum_{j=0}^{J-1} \beta^j \lambda(j) \left[g'\left(l_T^*\right) + \phi\left(l_T^*\right) j \right] \;\leq\; \gamma^t X \sum_{j=0}^{J-1} \beta^j \lambda(j)\gamma^j \left[f'\left(l_M^*\right) + \pi\left(l_M^*\right) j \right],$$

where l_k^* denotes the labor-experience ratio of sector k with no sectoral reallocation of workers, i.e. $l_T^* = \frac{\sum_{j=0}^{J-1} \lambda_T(j)}{\sum_{j=0}^{J-1} j\lambda_T(j)}$ and $l_M^* = \frac{\sum_{j=0}^{J-1} \lambda_M(j)}{\sum_{j=0}^{J-1} j\lambda_M(j)}$. In Appendix A.3, we outline the equilibrium construction procedure for this general J-period model.

4 Estimation

4.1 Data

We use a nationally representative household survey from Thailand, the Socio-Economic Survey (SES), for the 1976-1996 period. Eight rounds (1976, 1981, 1986, 1988, 1990, 1992,

16

1994, and 1996) of repeated cross-sections were collected during this period, using clustered random sampling, stratified by geographic regions (Bangkok and its Metropolitan vicinity region, Central region, Northern region, Northeast region, and South region).

The SES categorizes total income into wage, profits, property income, and transfer income.[12] The SES reports working status as employer, self-employed, employee, family worker, unemployed, or inactive. Combining the disaggregated income sources and work status data, we sort out earned income (i.e. wages for the employed workers and profits for the self-employed) from total income to construct our earnings measure. Property income and transfer income are all excluded in our measure of earnings, hence people who live only on these sources of income are excluded in our sample. We include only economically active people (excluding unemployed nor inactive people). Given this selection rule, the size of the selected sample is 178,428 individuals over all sample years.

4.2 Earnings Equations

Using the CES specification of the production functions as in (10) and (11), the cross-sectional earnings equations (19) and (20) of our general J-period OLG model are given by

$$\widetilde{y}_{k,t}(j) = \lambda_k(j)\gamma_k{}^t \left[\alpha_k \left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k} + (1-\alpha_k)\right]^{\frac{1}{\rho_k}-1} \left[\alpha_k \left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k-1} + j(1-\alpha_k)\right]$$

for an agent with j periods of experience in sector $k \in \{T, M\}$ at date t. In a typical aggregate production function, raw labor and experience are treated as perfect substitutes and experience simply adds to effective units of labor. This is a special limit case of the CES technology in our earnings function at $\rho_T = \rho_M = 1$. We take $J = 20$ and experience cohorts are formed in three year intervals. The sectoral labor and experience variables $L_{T,t}$, $E_{T,t}$, $L_{M,t}$, and $E_{M,t}$ are measured as in equations (14) to (17).

Note that we allow productivity growth factor parameters for both sectors γ_T and γ_M to take any values in our estimation. The model presumes $\gamma_M > \gamma_T = 1$. This is our verifying device in identifying the *partitioning* between the traditional and modern sectors. Thus, we can measure the sector partitioning (which is not directly measured in the data) together with the parameters of the model at the same time, using the same earnings equations.

In applying the earnings equations to the data, we allow for exogenous variation of individual productivity z_k, which depends on observable productive attributes χ_{it} and

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[12]The nominal income values are converted into real terms in 1990 baht value using the CPI indices differentiated by the regions.

unobservable attributes ϵ_{it} for individual i at date t. That is, the observed earnings $y_{k,it}(j)$ are

$$y_{k,it}(j) = z_k(\chi_{it}, \epsilon_{it})\widetilde{y}_{k,t}(j), \text{ for } k \in \{T, M\}.$$

We include years of schooling, gender, community type, geographic region, and constant terms in χ_{it} such that

$$z_k(\chi_{it}, \epsilon_{it}) = \exp\left[C_k\chi_{it} + \epsilon_{it}\right],$$

and ϵ_{it} are drawn from a mean-zero $i.i.d$ normal distribution over i and t.

The sector-specific depreciation schedule $\lambda_k(j)$ is approximated by the fifth-order polynomial (rather than the typical quadratic form) to flexibly capture the shape of the schedule observed in the data such that

$$\lambda_k(j) = 1 + \lambda_{k1}j + \lambda_{k2}j^2 + \lambda_{k3}j^3 + \lambda_{k4}j^4 + \lambda_{k5}j^5.{}^{13}$$

We normalize years setting $t = 0$ for 1976. Note the growth factor γ_k is replaced with $(1 + g_k)$ in order to facilitate the statistical significance test in identifying sectors (we test if g_T is estimated to be insignificant and g_M is significantly different from zero and positive at the chosen partition).

In sum, we estimate the following log-earnings equation

(24)
$$\ln y_{it} = \sum_{k \in \{T,M\}} d_{k,it}\left[t\ln(1 + g_k) + \ln\left(1 + \sum_{p=1}^{5}\lambda_{k,p}j^p\right) + \Psi_k\left(\frac{L_{k,t}}{E_{k,t}}, j\right) + C_k\chi_{it}\right] + \epsilon_{it},$$

where $d_{k,it}$ is an indicator variable for sector k, i.e. $d_{k,it} = 1$ if an individual i belongs to sector k at date t and 0 otherwise, and

(25)
$$\Psi_k\left(\frac{L_{k,t}}{E_{k,t}}, j\right) \equiv \left(\frac{1}{\rho_k} - 1\right)\ln\left[\alpha_k\left(\frac{L_{T,t}}{E_{T,t}}\right)^{\rho_k} + (1 - \alpha_k)\right] + \ln\left[\alpha_k\left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k - 1} + j(1 - \alpha_k)\right].$$

In typical Mincerian earnings regressions, only cross-sectional variations of individual income-generating attributes determine earnings. This is a special case of our earnings equations. With no complementarities, i.e. for ρ_k at the limit value of unity, the sectoral labor-experience ratio $\frac{L_{k,t}}{E_{k,t}}$ drops from the earnings equation (24). In the presence of the complementarities between labor and experience, however, the time-series variation of aggregate state variables (sectoral labor-experience ratios) also affect individual earnings. These ratios endogenously change during transition causing the experience premium to change. Thus, excluding the sectoral labor-experience ratios in earnings equations may bias the size and change of the experience premium, particularly for economies undergoing transition to modern growth.[14]

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[14]Note that the role of labor-experience complementarity can also be important for economies which

4.3 Identification

The technology parameters can be measured by estimating the cross-sectional earnings equation (24) by pooling the sample over time. This micro estimation strategy has two kinds of merit. First, no national income statistics exist to calibrate the labor-experience complementarity parameters in our model by distinguishing traditional and modern sectors. Furthermore, even if such data were available, it is well-known that identification of technology parameters from time series relationships between aggregate inputs and outputs suffers from endogeneity bias problems. Our micro estimation helps us avoid these problems. Furthermore, getting the standard errors from the structural estimation helps us to infer the parameter space of the model that conforms to the data. This is particularly helpful in finding the relevant range of parameters for sensitivity analysis.

Second, by not using the full data (such as aggregate time series, which are saved for the model evaluation stage) in parameter selection, the potential over-fitting problem can be avoided.[15] In this sense, we follow the original spirit of calibration, i.e. separation between parameter selection and model evaluation.

The parameters of the additively separable terms, i.e. $\{\gamma_k, \lambda_{k1}, \lambda_{k2}, \lambda_{k3}, \lambda_{k4}, \lambda_{k5}, C_k\}_{k=M,T}$ are easily identified. The remaining parameters α_k and ρ_k are identified from the non-linear terms in the function Ψ_k in (25). Note that the experience-earnings profile is time-invariant, and hence $(1 - \alpha_k)$ can be identified from the *cross-sectional variation* of experience through the second term, $\ln\left[\alpha_k \left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k - 1} + j(1 - \alpha_k)\right]$ in Ψ_k (note that at a given date t, the first term $\left(\frac{1}{\rho_k} - 1\right) \ln\left[\alpha_k \left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k} + (1 - \alpha_k)\right]$ and $\alpha_k \left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k - 1}$ are constant). Given α_k, the complementarity parameter ρ_k can be identified from the *time-series variation* of $\frac{L_{k,t}}{E_{k,t}}$ through the first term $\left(\frac{1}{\rho_k} - 1\right) \ln\left[\alpha_k \left(\frac{L_{k,t}}{E_{k,t}}\right)^{\rho_k} + (1 - \alpha_k)\right]$.

Note that the depreciation schedules λ_k's affect the sectoral labor and experience measures as shown in equations (14) to (17). Thus, the estimated depreciation schedules should be consistent with those used in constructing the sectoral labor and experience measures. To obtain such consistent estimates, we use an iterative guess-and-verify procedure. We first measure the sectoral labor and experience at an arbitrary initial depreciation schedule $\widetilde{\lambda}_{k,0}$ and then estimate depreciation schedule $\widetilde{\lambda}_{k,1}$, which is used in updating labor and experience measures. This in turn is used in estimating a new depreciation schedule $\widetilde{\lambda}_{k,2}$, and so on. We iterate this series of estimations until the distance between the guess $\widetilde{\lambda}_{k,q-1}$ and the follow-up estimate $\widetilde{\lambda}_{k,q}$ for iteration q becomes small enough for

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have already completed modern transition, when there are other forces driving demographic compositional changes in work force.

[15] See Granger (1999) for a discussion of the over-fitting issue in model evaluation.

each sector k.[16]

4.4   Parameter Estimates

Partitioning the workforce into traditional and modern sectors is a key measurement for the model. However, unlike other dual economy partitions (agriculture versus non-agriculture or rural versus urban), our partition does not have a direct counterpart in the data. Thus, we measure the sector partitioning following a *guess-and-verify* strategy using the same log earnings equation in (24). We guess the partition as follows. The SES provides us with detailed, three-digit occupational categories. We disaggregate the workforce using this three-digit occupational data. This disaggregation at the detailed occupation level, rather than one or two-digit industry level, helps us to group people by homogeneous activities. Then, we compute the rates of change in workforce shares between 1976-1996 for each occupational category, and order the occupational categories by the rates of net entry. The model predicts that occupations with positive net entry rates are likely to be in the modern sector. So, we guess a threshold level of rate of net entry around zero, occupational categories above which we assign to the modern sector.[17] It is important to note that we use the employment share growth data to get an initial guess for the sector partitioning, and not for the final identification of the modern sector.

Given the guessed partition, we estimate sectoral productivity growth rates g_T and g_M in the earnings equation (24) and *verify* if the estimates are consistent with the model, i.e. positive only for the modern sector and zero for the traditional sector, $g_M > g_T = 0$. If the estimates of the productivity growth rates agree with the model, we take the partition in the data as the one corresponding to the model. If not, we choose another guess, and verify again. This loop of guess-and-verify can be iterated until we find the right partition. A priori, there is no reason for having the productivity growth rates satisfying (i) $g_M > g_T$ *and* (ii) $g_T = 0$ between the entry and exit groups. However, consistent with the model, it turns out that using a guessed partition where the threshold level of net entry rate is set exactly at zero, we obtain estimates such that $g_M > g_T = 0$.

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[16] The distance between the guess $\widetilde{\lambda}_{k,q-1} = \left(\widetilde{\lambda}_{kp,q-1}\right)_{p=1}^{5}$ and the follow-up estimate $\widetilde{\lambda}_{k,q} = \left(\widetilde{\lambda}_{kp,q}\right)_{p=1}^{5}$ at iteration q is measured by the root-mean-squared errors criterion such that

$$\left\|\widetilde{\lambda}_{k,q} - \widetilde{\lambda}_{k,q-1}\right\| = \left[\frac{1}{p}\sum_{p=1}^{5}\left(\widetilde{\lambda}_{kp,q} - \widetilde{\lambda}_{kp,q-1}\right)^2\right]^{\frac{1}{2}}.$$

We stopped the iteration when $\left\|\widetilde{\lambda}_{k,q} - \widetilde{\lambda}_{k,q-1}\right\| \leq 0.0001$ for both sectors.

[17] The level and change of the population shares of occupational categories in the data are likely to be subject to sampling errors, and hence we vary the threshold level around zero rather than pinning it down at zero.

We use *nonlinear-least-squares* estimation to estimate the log earnings equation in (24).[18] Table 1 reports the estimates of the technology parameters (as well as the coefficients of control variables) at the chosen partition. Standard errors are in parentheses.

Table 1. Technology Parameter Estimates

Sector	Traditional	Modern
g_k	-0.005 (0.0005)	0.025 (0.0009)
α_k	7.68e-11 (4.36e-11)	0.033 (0.0197)
ρ_k	-10.95 (0.286)	-1.36 (0.376)
λ_{1k}	-0.2200 (0.0186)	-0.1594 (0.0386)
λ_{2k}	0.0586 (0.0046)	0.0248 (0.0095)
λ_{3k}	-0.0064 (0.0006)	-0.0018 (0.0011)
λ_{4k}	0.0003 (0.00003)	0.00004 (0.00006)
λ_{5k}	-5.19e-6 (6.83e-7)	-7.07e-10 (1.14e-6)
Schooling	0.160 (0.0012)	0.130 (0.0012)
Male	0.644 (0.0062)	0.409 (0.0096)
Urban	0.709 (0.0114)	0.320 (0.0123)
North	0.197 (0.0077)	0.028 (0.0172)
Central	0.575 (0.0085)	0.326 (0.0158)
South	0.557 (0.0115)	0.245 (0.0162)
Bangkok	0.943 (0.0133)	0.612 (0.0168)
Constant	3.883 (0.0478)	4.900 (0.0849)

Note: Number of observations = 178,428, $RMSE = 1.043711$.

The estimates confirm that there coexist two sectors in the economy, with a substantial gap in productivity growth. The estimate for g_T is close to zero at -0.005, and the estimate for g_M is 0.025. The estimates of ρ_T at -10.95 and ρ_M at -1.36 suggest that labor and experience are far from perfect substitutes. The implied elasticity of substitution between labor and experience $\frac{1}{1-\rho_k}$ is 0.084 for the traditional sector, and 0.424 for the modern sector. Complementarity is stronger in the traditional sector than in the modern sector.

The estimates for $\alpha_T = 7.68 * 10^{-11}$ and $\alpha_M = 0.033$ (the weights on raw labor in the CES production functions) are apparently small, although both are statistically significantly different from zero. The α_k's are scale parameters for raw labor *relative to* experience. Thus, small numbers for these parameters do not imply that shares of raw labor in earnings are this tiny. The share of raw labor for sector k earnings is

$$\frac{(\partial Y_{k,t}/\partial L_{k,t}) L_{k,t}}{Y_{k,t}} = \left[1 + \frac{1 - \alpha_k}{\alpha_k} \left(\frac{L_{k,t}}{E_{k,t}} \right)^{-\rho_k} \right]^{-1},$$

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[18] We use Gauss-Newton method for optimization.

which is determined by the combination of α_k and ρ_k, and also depends on the level of the labor-experience ratio. The implied average shares of raw labor at the above estimates are 0.18 and 0.24 for the traditional and modern sectors, respectively.[19] Thus, our low estimates of α_k's do not imply an odd parameter configuration for the CES production function. Still, we see that a major part of earnings is attributed to experience.

At the estimated depreciation schedule parameters in Table 1, the shapes of earnings profiles turn out to be very different between the two sectors. Modern earnings profiles display a clear hump-shape (as is typically observed in developed countries), peaking at experience interval 33-35. Traditional profiles are concave but without a hump.[20]

4.5   Within-subgroup Coexistence and Modernization

Note that the "modern" sector in our model does not necessarily correspond to urban areas or non-agriculture, as is typically proxied in the dual economy models or in the structural transformation literature. Our only identifying assumption for sector partitioning is the presence or absence of productivity growth. That is, we allow the coexistence of modern agriculture versus traditional agriculture, or modern manufacturing versus traditional manufacturing. We also allow the coexistence of the two sectors within each of rural and urban areas. At our estimated partition, we find that modern and traditional sectors coexist within each population subgroups of agriculture, manufacturing, services, rural areas and urban areas.

There are level gaps in modernization across the population subgroups. For example, the modern workforce share is higher in urban areas (53 percent on average) than rural areas (22 percent on average). However, the process of modernization, i.e., the population shift from traditional to modern sector has occurred in *every* group. Furthermore, the shape of the modernization process look similar between rural and urban areas, as shown in Figure 4.1. This suggests that there exists a driving force of modernization independent from urbanization. Figure 4.2 shows that the process of modernization has occurred within

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[19]Note that the labor share moves over time as the labor-experience ratio evolves and its time-series elasticity is determined by α_k and ρ_k. We found that traditional labor share fluctuates widely over time, decreasing from 0.28 in 1976 to 0.10 in 1988 and then increasing to 0.36 in 1996, averaging at 0.18. The modern labor share is more or less stable over time around 0.24. Thus, a constant labor share seems a good approximation for the modern sector but not for the traditional sector.

[20]The parameter estimates may depend on the specification of the control variables. In particular, Heckman, Lochner, and Todd (2003) document that the shape of the experience-earnings profiles are different across schooling groups, which in turn affects the estimate for returns to schooling. In principle, this may affect our estimates of the technology parameters. We experimented on the control-variable specification by allowing for the interaction between schooling and experience. We find that the coefficient of the interaction term is indeed negative, i.e. the slope of the experience-earnings profiles are steeper for lower than higher education groups, consistent with the findings of Heckman, Lochner, and Todd (2003). However, the estimates of the technology parameters remain robust to this specification change.

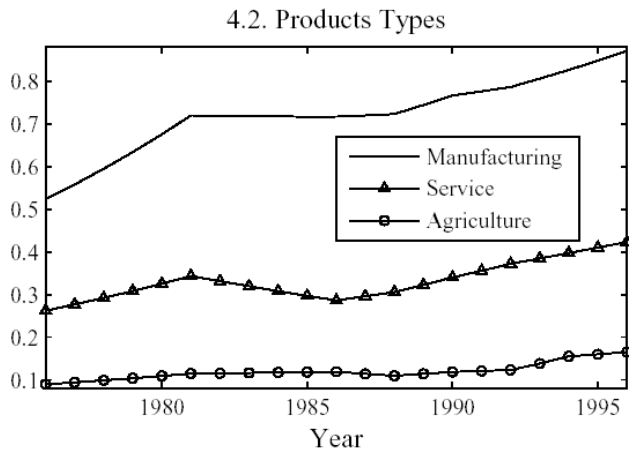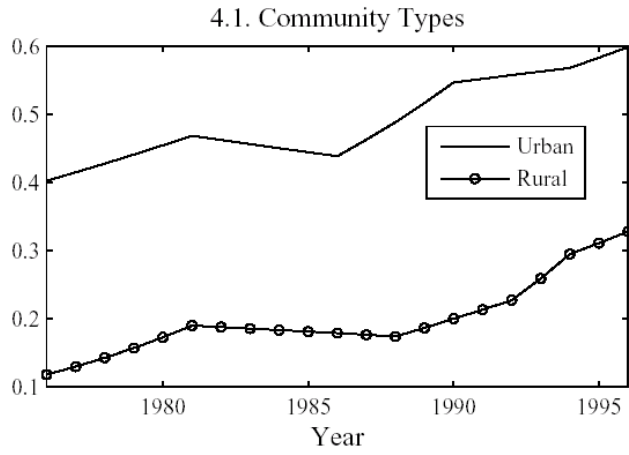4.1. Community Types

4.2. Products Types

Figure 4: Transition to Modern Sector within Subgroups

each of the agriculture, manufacturing, and services, although again the average levels of modernization differ across them (13 percent for agriculture, 35 percent in services, and 78 percent in manufacturing on average).

Table 2 lists examples of three-digit occupations for traditional and modern sectors by the final products type, illustrating the coexistence of the two sectors among workers who seem to provide (broadly defined) similar types of goods. For foods, for example, rice and field crop farmers are traditional while fruit or fishery farmers are modern. In fact, the use of modern farming technology and high productivity growth in shrimp farming, in contrast to the declining productivity in rice farming, are well-known phenomena in Thailand during our sample period 1976-1996. Our estimation correctly reflects this.

Among medical service workers, doctors and nurses belong to the modern sector while midwives and occupational therapists belong to the traditional sector. In manufacturing, both blacksmiths and sheet metal workers work on metal materials, but blacksmiths belong to the traditional sector while sheet metal workers to the modern sector. Both tailors and embroiders work in the textile industry, but the former belong to the traditional sector while the latter to the modern sector. This may suggest what determines being traditional or modern is *the way* that workers organize their activities rather than *the objects* that they produce.

Table 2. Examples of Partitioned Occupations

	Traditional Sector	Modern Sector
Agriculture	rice farming, field-crop farming	fishery, fruit farming
Manufacturing	metal caster, blacksmith	sheet metal maker, mechanic
	grain miller, tobacco maker	food and beverage processor
	tailor	pattern maker, embroider
	wood-paper-rubber product maker	electric/electronic engineer
Service	street and waterway vendor	insurance, real estate salesman
	midwife, occupational therapist	doctor, nurse
	legislative and government administrator	lawyer, judge
	journalist	physical/life scientist
	cook, cleaner, hairdresser, driver	accountant
	primary/secondary school teacher	pre-school/university teacher
	policeman, armed force	fireman

5 Simulation

We simulate a 20-period overlapping generations model at the estimated technology parameters of $\{\alpha_k, \rho_k, \gamma_k, \lambda_{k1}, \lambda_{k2}, \lambda_{k3}, \lambda_{k4}, \lambda_{k5}\}$ for $k = T$ and M, as reported in Table 1, setting the year 1976 as $t = 0$ for the model, as is done in estimation. Here, we set $\gamma_T = 1$

ignoring the negligible growth in the traditional sector. There remain two free parameters X (the relative productivity level gap between sectors in 1976) and β (time-discount factor).[21] They are calibrated at $X = 1.035$ and $\beta = 0.52$ (i.e. annual discount factor at 0.8) to match the path of modern cohort share for the period 1976-96.[22] Given these selected parameters, we verify if the pivotal condition is satisfied at the selected parameter values.

The initial state $(M_{-j})_{j=1}^{J-1}$ is set to the values of smoothed modern cohort shares in Figure 1, dating back to the cohort who entered the workforce in calender year 1919. Given the chosen parameters and the initial state, the series of modern cohort shares $\{M_t\}_{t=0}^{\widehat{T}}$ is simulated, where $\widehat{T}$ is the first period when an entire cohort enters into the modern sector. Sectoral labor and experience and individual earnings are constructed in accordance with the simulated modern cohort shares. Here, the constructed labor and experience measures, as in equations (14) to (17), depend on the relative size of the labor force of each experience group, i.e. $\{D_{jt}\}_{j=0}^{J-1}$. We exogenously embed $\{D_{jt}\}_{j=0}^{J-1}$ using the labor force participation rates, as observed in the SES data, reported in Table A.1 in the Data Appendix. Our benchmark simulation (labeled "Sim1") assumes the participation rates vary across experience groups but counterfactually ignores the time-series variation by averaging the participation rates of each experience group over time. We also simulate the model reflecting yearly deviations from the average participation rates (labeled "Sim2") as in the data. By comparing the two simulations, we can differentiate the deterministic trend effects arising from endogenous changes in the sectoral composition of experience groups, from the business cycle effects arising from exogenous shocks to participation rates across experience groups.

5.1   In-Sample Comparison

We first compare the simulated transition dynamics with data for the sample period. To make the comparison compatible, we filter out the effects of the control variables χ_{it} and the residual ϵ_{it} in the log-earnings equation (24), i.e., the factors outside our model. That

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[21] Given that there are categorical variables in χ_{it}, the estimated constant includes both X and the average income of the reference group in the modern sector. Thus, a simple comparison between the estimated sectoral constant terms does not identify X and it remains as a free parameter. The time-discount factor β does not enter the earnings equations.

[22] The chosen value for annual discount factor 0.8 seems lower than typical values which range between 0.9 and 0.99. This is due to the presumed linear preferences. Introducing concave utility function allows us to increase the discount factor into the typical range to match the same modern cohort share data. For example, simulating the model with a CRRA utility function at a relative risk aversion coefficient of 3 increases the annual discount factor to 0.95. Still we keep the linear preferences rather than introducing concave utility function in our analysis to isolate the effects of technology on earnings dynamics from the combined effects of consumption smoothing. Thus, calibrating β at the low value is a consistent restriction to this chosen specification.
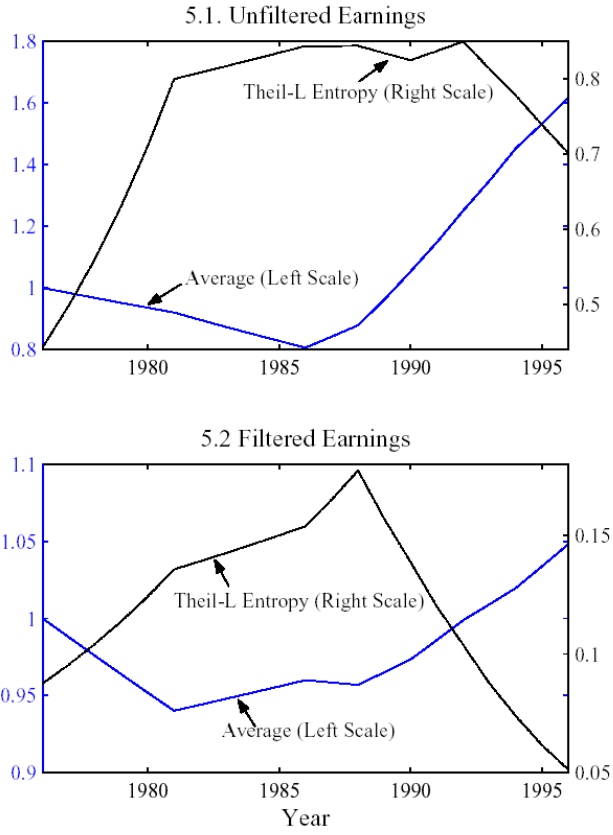
Figure 5: Growth and Inequality of Earnings

is, our filtered earnings y_{it}^F to be compared with simulation are

$$(26) \qquad y_{it}^F \equiv \exp\left(\ln y_{it} - \sum_{k \in \{T,M\}} d_{k,it} C_k \chi_{it} - \epsilon_{it}\right).$$

Figure 5.1 shows that average earnings grew with acceleration during the second decade, following a decade of stagnation. The earnings inequality, measured by the Theil-L entropy index, shows an inverted-U path.[23] When we filter out the effects from the control variables and the residual, the growth and inequality features become different, as shown in Figure 5.2. Despite the high modern-sector productivity growth, aggregate labor productivity (which we measure by the aggregate earnings after filtering out the effects of all control variables) remained virtually constant due to the dominance of the stagnant traditional sector. The inverted-U shape of the earnings inequality path becomes more pronounced.

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[23]This Thai inequality dynamics is robust to the choice of inequality indices including Gini coefficient, variance of log, coefficient of variation and Theil-T index. See Jeong (2008). Here, we use the Theil-L entropy index due to its decomposability that we will utilize later.
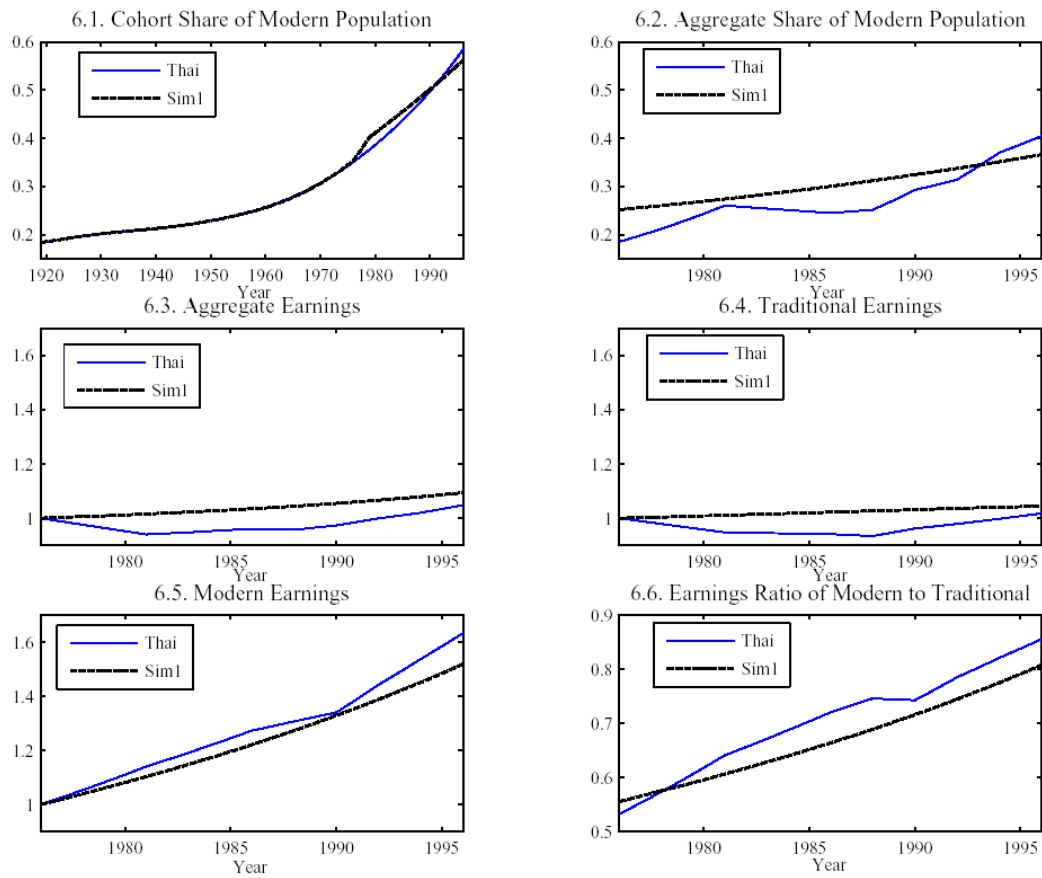
Figure 6: Aggregate Transition Dyanmics

We first compare the simulation results from Sim1 to isolate the performance of the model in explaining the trends (rather than fluctuation) of modernization and earnings, which are endogenously generated by the model. The trend of modernization, measured by the increase in the modern cohort share, is captured well by the model, as shown in Figure 6.1.[24] Figure 6.2 displays the aggregate share of the modern population (aggregated over cohorts at each given year), which the simulation predicts as slightly higher than in the data.

Aggregate earnings, indexed to initial year, are compared in Figure 6.3. After filtering the income-generating attributes as in (26), aggregate earnings in Thailand were more or less stagnant, slightly increasing during 1988-1996, following a mild recession during 1976-1988. On average, aggregate filtered earnings grew by only 0.24% per year. Note that this aggregate earnings growth can be interpreted as *aggregate labor productivity growth* from the perspective of an aggregate production function, entering as a component of TFP growth. The model does not predict the mild recession, but does capture the stagnation of aggregate earnings (growing only by 0.45% per year).

Note that this lack of TFP growth implied from the sluggish aggregate labor productivity growth does not mean that overall aggregate TFP did not grow in Thailand during the sample period. We already observed the rapid growth of *unfiltered* earnings in Figure 5.1. In fact, Jeong and Townsend (2007) show that Thai aggregate TFP did grow at a rate of 2.3 percent per year on average but the major source of this TFP growth was *financial deepening*. What our model points out is that there exist other forces which drag down the Thai TFP through the complementarities in the labor markets, consistent with the filtered earnings data in Figure 5.2.

Figure 6.4 shows that the stagnation of aggregate (filtered) earnings is due to the stagnation of the traditional sector in both model and data. In contrast, the aggregate earnings of the modern sector grew rapidly both in Thai data (at an annual rate of 2.5%) and simulation (at an annual rate of 2.1%), as shown in Figure 6.5.

The simulated ratio of modern average earnings to traditional average earnings increases from 0.56 in 1976 to 0.81 in 1996, shown in Figure 6.6. This ratio for the *filtered* earnings increases from 0.53 in 1976 to 0.86 in 1996 in Thailand as well.[25] Note that *average* earnings are lower in the *modern* sector for the entire two-decade sample period in

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[24] The modern cohort shares before 1976 are common between the model and the data because we took the initial state of the cohort shares of the model from the data. There is an initial jump in the simulated cohort share because of a slight mismatch between the data and simulation for the initial year 1976, in calibrating the free parameter X. This jump makes the overall shape of the simulated transition appear to be concave, which can be misleading. The simulated transition from 1977 onwards is convex throughout.

[25] For the *unfiltered* earnings, the ratio of modern average earnings to traditional average earnings is greater than one and increases from 1.6 to 2.0 between 1976 and 1996.
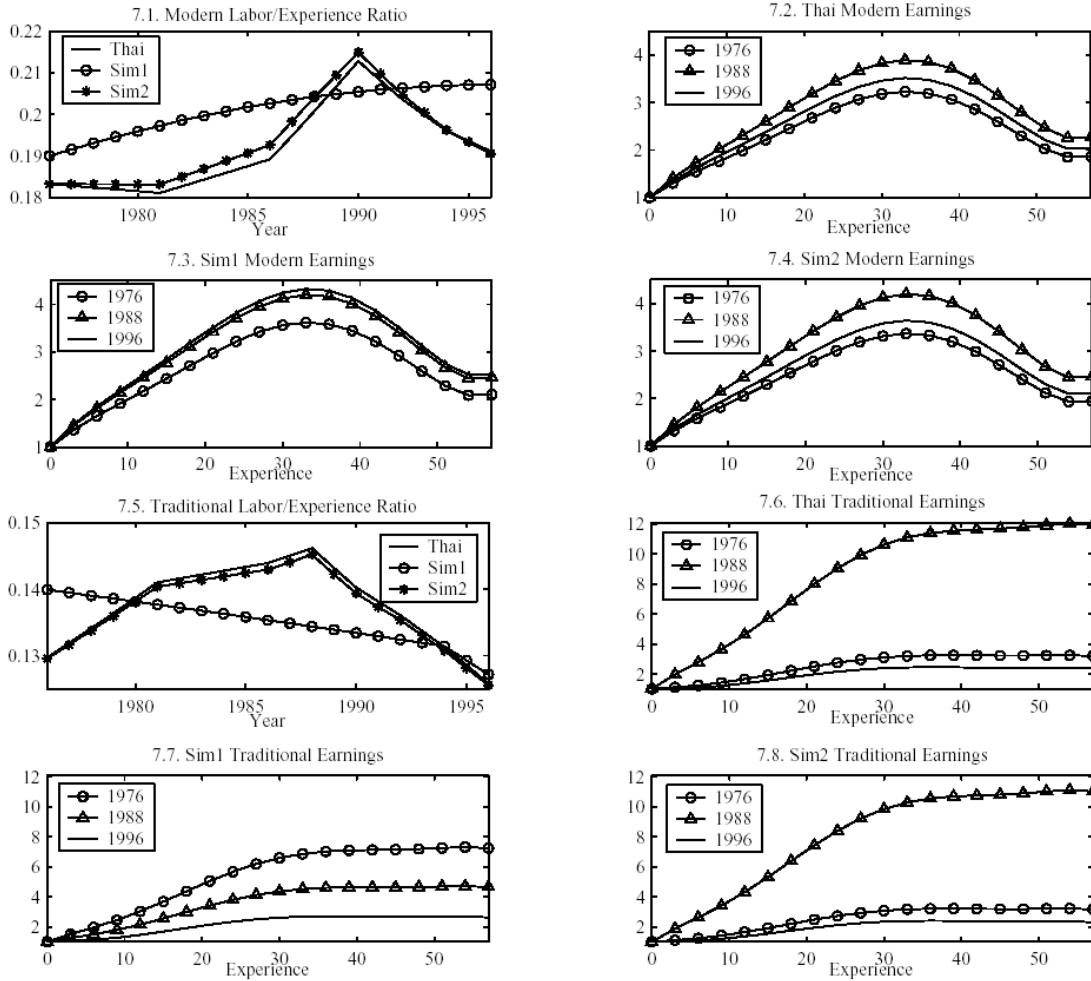
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Figure 7: Experience-Earnings Profile Dynamics

both model and data, although the productivity level gap parameter $X = 1.035$ exceeds one. This is possible because the proportion of highly paid experienced workers is lower in the modern sector than in the traditional sector. Due to the higher modern productivity growth and transition (hence the expansion of the experienced workers in the modern sector), this sectoral earnings gap narrows over time.

Figure 7.1 displays the evolution of modern labor-experience ratios. In the benchmark simulation (Sim1), the simulated modern labor-experience ratio moves around the levels in the data but increases monotonically, which does not capture the inverted-U shaped movement in the data. However, in Sim2, which adjusts for the yearly variation of participation rates by experience-group, the simulation captures the fluctuation very well.

Capturing the inverted-U dynamics of the modern labor-experience ratio is important

29

in explaining the rise and fall of the experience-earnings profiles in the data. Figure 7.2 displays the modern experience-earnings profiles (normalized to the zero-experience group) in Thailand for three years 1976, 1988, and 1996. The profiles are hump-shaped in each year. The experience premium increases until experience 33-35 (peaking at 3.2 in 1976, 3.9 in 1988 and 3.5 in 1996), and then decreases afterward (1.9 in 1976, 2.3 in 1988 and 2.0 in 1996 for the oldest group). The shape of the profiles changes over time: first shifting up and getting steeper between 1976 and 1988, and then shifting down and flattening between 1988 and 1996. In simulation Sim1, the monotone increase in the modern labor-experience ratio implies that profiles continue to shift up over time (Figure 7.3), which is different from the data. However, simulation Sim2 captures the non-monotonic movements of the modern earnings profiles in the data (Figure 7.4), as the simulated labor-experience ratio in Sim2 tracks the data.

Figure 7.5 displays the evolution of traditional labor-experience ratios. Again, Sim1 captures the level of the ratio in the data but not the fluctuation, which is captured by Sim2. The earnings profiles of the traditional sector are very different from the modern profiles. First, the traditional profiles are not hump-shaped in the data (Figure 7.6). That is, the experience premium does not decrease over experience in the traditional sector. Second, both the size and change of the experience premium are much larger in the traditional sector than modern sector. However, we still observe a positive correlation between the labor-experience ratio and the experience premium in the traditional sector. The earnings profile shifts up as the labor-experience ratio increases between 1976 and 1988, and flattens as the ratio decreases between 1988 and 1996. Again, simulation Sim2 mimics both the size and the dynamics of the traditional earnings profiles in the data (Figure 7.8).

These earnings dynamics imply an inverted-U shaped path of *within-sector inequality* for each sector over the sample period. The monotonically narrowing sectoral earnings gap (Figure 6.6) implies that *between-sector inequality* decreases over the same period. Using the Theil-L index, we can precisely decompose the contributions of within-sector versus between-sector inequalities, as shown in Figure 8.1.[26] Movements of the overall inequality

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[26] Given the earnings distribution $(y_{1t}, \cdots, y_{n_t t})$ at date t, Theil-L index measures the overall inequality I_t as follows:

$$I_t \equiv \frac{1}{n_t} \sum_{i=1}^{n_t} \ln \frac{\mu_t}{y_{it}},$$

where n_t denotes the sample size and μ_t the overall mean earnings. This is additively decomposed into within-sector inequality WI_t and between-sector inequality BI_t such that

$$I_t = WI_t + BI_t,$$
$$WI_t \equiv \sum_{k \in \{T,M\}} p_{kt} I_{kt} \text{ and } BI_t \equiv \sum_{k \in \{T,M\}} p_{kt} \ln \frac{\mu_t}{\mu_{kt}},$$

8.1. Thai Earnings Inequality

8.2. Sim1 Earnings Inequality

8.3. Sim2 Earnings Inequality
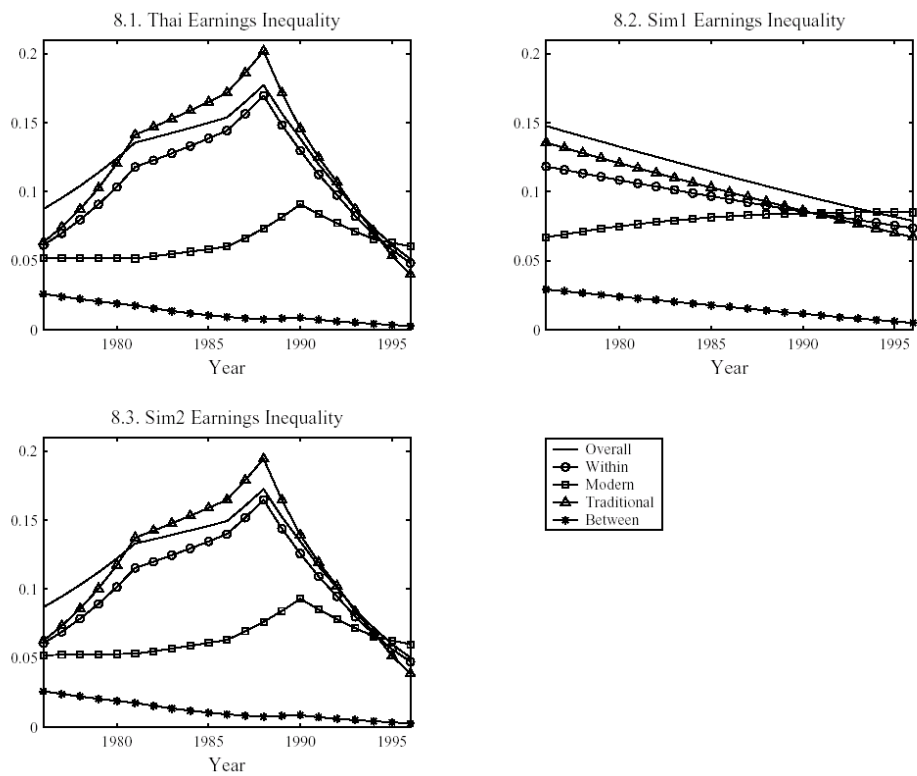
Overall
Within
Modern
Traditional
Between

Figure 8: Earnings Inequality Decomposition

are driven by *within-sector* inequality, which in turn is mainly driven by traditional-sector inequality. Thus, despite the monotone decrease of between-sector inequality, overall earnings inequality follows an inverted-U shape.

We apply the same decomposition to simulations, as shown in Figures 8.2 and 8.3 for Sim 1 and Sim 2, respectively, so that we can compare the decomposed inequality dynamics between the model and the data. In Sim1, the labor-experience ratio increases in the modern sector while it decreases in the traditional sector, inducing an increase in modern inequality and decrease in traditional inequality. Between-sector inequality decreases from the reduced sectoral earnings gap. The overall inequality turns out to be decreasing (Figure 8.2). After correcting for compositional changes of experience groups over time, Sim2 mimics both the overall and decomposed features of the Thai earnings inequality (Figure 8.3). This tells us that the source of the inverted-U shape of the filtered earnings inequality over the sample period is the exogenous compositional changes in workforce participation across experience groups, rather than the endogenous trends of transition.

We perform a sensitivity analysis by varying the technology parameters $\{\alpha_T, \rho_T, \alpha_M, \rho_M, \gamma_M\}$ within 95% confidence intervals using the standard errors of the estimates in Table 1, and check the robustness of the simulation results. We focus on the robustness of the modern cohort share, the building block of the simulation. For the calibrated parameters β and X, we experiment with $\pm 10\%$ deviations. We find that both the trend and level of the modern cohort shares remain robust to all these perturbations. Details of sensitivity analysis results are reported in Appendix A.4.

5.2   Long-run Forecast

We simulate the model beyond the sample period until the transition is complete in the benchmark simulation Sim1. The model predicts that the entire 2036 cohort will enter into the modern sector, and the entire workforce will be in the modern sector by 2096, as shown in Figures 9.1 and 9.2. These figures illustrate an S-shaped process of modernization in terms of workforce share.

Figure 9.3 shows that the ratio of modern average earnings to traditional average earnings is initially lower than one but keeps increasing and eventually exceeds one from the year 2006. Thus, the population shift from the traditional to modern sector delays the growth of aggregate earnings before 2006 and then accelerates aggregate earnings growth afterward.[27] As explained above, the initial "poverty" of the modern sector relative to

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where p_{kt} denotes the population fraction of sector k, I_{kt} the inequality within sector k (using the same Theil-L index formula), and μ_{kt} the mean earnings of sector k.

[27] Again, recall that this aggregate earnings in simulation measures the long-term trend of aggregate
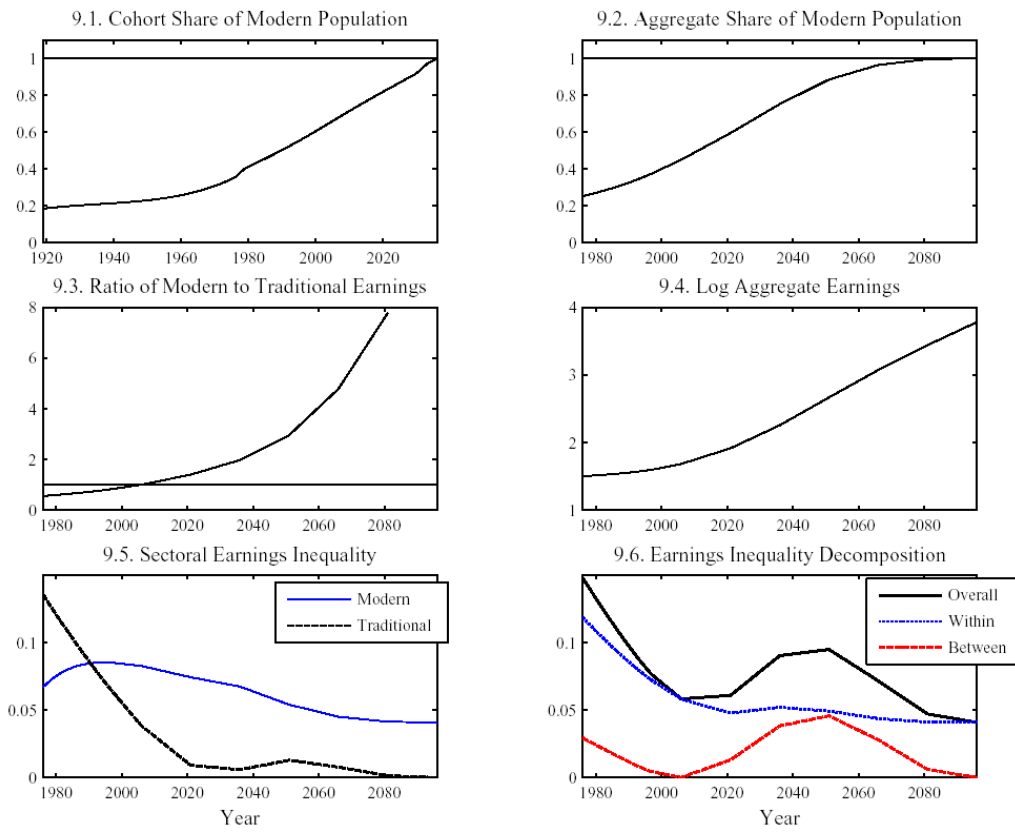
Figure 9: Long run forecast

traditional sector is due to the scarcity of the rich experienced workers in the modern sector at the early stages of transition. Thus, this force of modernization tends to *decrease* aggregate earnings at initial periods, but is eventually overturned as modern transition progresses.

Despite rapid modern transition, aggregate earnings are stagnant for the initial thirty years during 1976-2006 (Figure 9.4). Eventually, aggregate labor productivity takes off and its growth rate keeps increasing to a peak of 2.8% in 2051, and then decreases afterward, converging to the constant steady-state growth rate of 2.5%, the modern productivity growth rate. Thus, aggregate earnings dynamics display the typical S-shaped transition. Recall that this growth enters as a part of the TFP in the typical aggregate production function, implying that TFP tends to evolve in an S-shape during transition.

Figure 9.5 displays the long-run simulation of earnings inequality within each sector. In Figure 9.6, the overall inequality is decomposed into within-sector inequality and between-sector inequality. These figures show that the *long-run trend* of inequality can be non-monotonic as well, declining and then inverted-U shaped. Note that the inverted-U shape of the long-run inequality emerges after 2006, when the modern sector becomes richer than the traditional sector. The decomposition of the Theil-L index suggests this long wave of inverted-U dynamics of overall inequality is driven by *between-sector inequality*, as postulated by Kuznets (1955), while we find *within-sector inequality* declines monotonically. This contrasts with the finding in the previous in-sample comparison where the short-run movement of overall inequality is driven by within-sector inequality. After 2096, when the entire population enters into the modern sector, the labor-experience ratio stays constant and the modern sector inequality and aggregate inequality become constant.

6   Conclusion

Lucas (2004) states that "a useful theory of economic development will necessarily be a theory of transition." This paper provides a possible theory of transition in a dual economy combined with the idea of sector-specific complementarities between labor and experience. We emphasize the role of work experience, which has not been considered as a major factor in the growth and development literature. In particular, the initial distribution of work experience between the traditional and modern sectors determines the speed and slope of the growth process during transition. Work experience is an important component of human capital which should be incorporated into growth and development accounting. We show, however, it is the distribution of experience across

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labor productivity presuming all other TFP growth factors away.

sectors rather than the aggregate stock that would capture the role of experience and reduce the size of TFP.

We measured the theory using micro data from Thailand, where transition has occurred gradually out of a sector with no labor productivity growth (traditional sector) to a sector with positive (2.5% per year) labor productivity growth (modern sector). We estimated the partitioning of the sectors as well as the technology parameters from cross-sectional earnings equations. We verified the positive correlation between the experience premium and labor-experience ratio within each sector, as is implied by the assumption of labor-experience complementarity. At these estimated parameters, the model simulates well the observed nonlinear aggregate dynamics of earnings growth and inequality, together with the gradual transition of the labor force between sectors. In sum, the model is borne out by the Thai data.

We also document how the earnings dynamics differ between the modern and traditional sectors. Labor-experience complementarity is much stronger and the changes in the earnings profile are also much more pronounced in the traditional sector than the modern sector. Modern sector earnings profiles are hump-shaped as is typically observed in currently rich countries while traditional sector earnings profiles increase monotonically, and its experience premium is higher than in the modern sector. Therefore, the distinction between the traditional and modern sectors seems important in understanding the earnings dynamics of developing countries, where both sectors coexist and their composition changes over time.

Given the quantitative success of our model, incorporating sectoral labor-experience ratios into micro earnings equations and into aggregate production functions (in measuring human capital) seems important in understanding both growth and inequality for economies in transition. For our model to be considered a general theory of transition, further confirmation using micro data from other economies is needed.

References

[1] Beaudry, Paul and Patrick Francois (2005), "Managerial Skill Acquisition and the Theory of Economic Development," *NBER Working Paper* No. 11451.

[2] Caselli, Francesco (2005), "Accounting for Cross-Country Income Differences," *Handbook of Economic Growth,* ed. by P. Aghion and S. Durlauf, pp. 679-741, Elsevier, North-Holland.

[3] Chari, V.V. and Hugo Hopenhayn (1991), "Vintage Human Capital, Growth and the Diffusion of New Technology," *Journal of Political Economy*, vol. 99, no. 6, pp. 1142-1165.

[4] Conley, Timothy G. and Christopher R. Udry (2005), "Learning About a New Technology: Pineapple in Ghana," mimeo.

[5] Foster, Andrew and Mark Rosenzweig (2004), "Agricultural Development, Industrialization and Rural Inequality," mimeo.

[6] Galor, Oded (2005), "From Stagnation to Growth: Unified Growth Theory," *Handbook of Economic Growth*, North-Holland: Elsevier, pp.171-285.

[7] Gollin, Douglas, Stephen Parente and Richard Rogerson (2002), "The Role of Agriculture in Development," *American Economic Review* P&P, pp. 160-164.

[8] Granger, C. W. J. (1999), *Empirical Modeling in Economics: Specification and Evaluation*, Cambridge University Press.

[9] Hansen, Gary and Edward C. Prescott (2002), "Malthus to Solow," *American Economic Review*, vol. 92, pp. 1205-1217.

[10] Heckman, James, Lance Lochner, and Petra Todd (2003), "Fifty Years of Mincerian Earnings Regressions," NBER Working Paper Series No. 9732.

[11] Jeong, Hyeok (2008), "Assessment of Relationship between Growth and Inequality: Micro Evidence from Thailand," *Macroeconomic Dynamics*, Volume 12, pp. 155-197.

[12] Jeong, Hyeok and Yong J. Kim (2006), "S-shaped Transition and Catapult Effects," IEPR Working Paper Series No. 06.53, 2006.

[13] Jeong, Hyeok, Yong J. Kim and Iourii Manovskii (2008), "The Price of Experience," mimeo.

[14] Jeong, Hyeok and Robert M. Townsend (2007), "Sources of TFP Growth: Occupational Choice and Financial Deepening," *Economic Theory*,V. 32, No. 1, pp. 179-221.

[15] Jovanovic, Boyan, and Yaw Nyarko (1996), "Learning by doing and the Choice of Technology," *Econometrica*, V. 64: 1299-1310.

[16] Kambourov, Gueorgui, and Iourii Manovskii (2005), "Accounting for the Changing Life-Cycle Profile of Earnings," mimeo.

[17] Katz, Lawrence and Kevin M. Murphy (1992), "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics*, V.107, pp. 35-78.

[18] Kim, Dae-Il and Robert Topel (1995), "Labor Market and Economic Growth: Lessons from Korea's Industrialization, 1970-1990," in *Differences and Changes in Wage Structure*, pp. 227-264, Richard Freeman and Lawrence Katz eds., Chicago and London: The University of Chicago Press.

[19] Klenow, Peter J., and Andrés Rodríguez-Clare (1997), "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?," *NBER Macroeconomics Annual*, 73-102.

[20] Kremer, Michael and James Thomson (1998), "Why Isn't Convergence Instantaneous? Young Workers, Old Workers, and Gradual Adjustment," *Journal of Economic Growth*, March 3(1), pp. 5-28.

[21] Kuznets, Simon (1955), "Economic Growth and Income Inequality," *American Economic Review, Papers and Proceedings*, V. 45: 1-28.

[22] Kuznets, Simon (1966), *Modern Economic Growth*, New Haven: Yale University Press.

[23] Lewis, W. Arthur (1954), "Economic Development with Unlimited Supplies of Labor," *Manchester School of Economic and Social Studies*, vol. 22, pp. 139-191.

[24] Lucas, Robert E. (2000), "Some Macroeconomics for the 21st Century," *Journal of Economic Perspectives*, vol. 14(1), pp. 159-168.

[25] Lucas, Robert E. (2004), "Life Earnings and Rural-Urban Migration," *Journal of Political Economy*, vol. 112, pp. S29-S59.

[26] Ranis, G. and J. Fei (1961), "A Theory of Economic Development," *American Economic Review*, vol. 51, pp. 533-565.

[27] Welch, Finis (1979), "Effects of Cohort Size on Earnings: The Baby Boom Babies' Financial Bust," *Journal of Political Economy*, 87(5): S65-S97.

A Appendix

A.1 Proofs

Proof of Lemma 1. Proof by contradiction. Suppose the Lemma is not true, then we have $M_{t-1} = 1$ and $M_t < 1$. From (7) this implies, for period t,

$$
\begin{aligned}
& G\left(1,0\right) + \beta\lambda\left[g'\left(1 + \frac{1-M_{t+1}}{\lambda\left(1-M_t\right)}\right) + \phi\left(1 + \frac{1-M_{t+1}}{\lambda\left(1-M_t\right)}\right)\right] \\
= \ & \gamma^t X\left\{f'\left(1 + \frac{M_t}{\lambda}\right) + \beta\lambda\gamma\left[f'\left(1 + \frac{M_{t+1}}{\lambda M_t}\right) + \pi\left(1 + \frac{M_{t+1}}{\lambda M_t}\right)\right]\right\}.
\end{aligned}
$$

This condition combined with (5) implies $M_{t+1} < M_t$. Iterating forwards using the same argument we eventually must have a period s such that $M_s = 0$. The participation constraint for this period is,

$$
\begin{aligned}
& g'\left(1 + \frac{1}{\lambda\left(1-M_{s-1}\right)}\right) + \beta\lambda\left[g'\left(1 + \frac{1}{\lambda}\right) + \phi\left(1 + \frac{1}{\lambda}\right)\right] \\
\geq \ & \gamma^j X\left\{f'\left(1\right) + \beta\lambda\gamma\left[f'\left(1\right) + \pi\left(1\right)\right]\right\}
\end{aligned}
$$

This constraint contradicts pivotal condition (5) when noting that,

$$f'\left(1+\frac{1}{\lambda}\right) + \beta\lambda\gamma\left[f'\left(1+\frac{1}{\lambda}\right) + \pi\left(1+\frac{1}{\lambda}\right)\right]$$
$$< \quad f'(1) + \beta\lambda\gamma\left[f'(1) + \pi(1)\right]$$

Since $f'(x) + \beta\lambda\gamma\left[f'(x) + \pi(x)\right]$ is falling in x for values of $x < 1 + \frac{1}{\lambda}$. ∎

Proof of Proposition 1. The algorithm for constructing the equilibrium transition path is as follows:

Step 0: Given $M_{-1} < 0$, guess that $M_t = 1$ for $\forall t \geq 0$ (which by Lemma 1 is implied by $M_0 = 1$). Verify if $M_0 = 1$ by checking (9) for $\widehat{T} = 0$. If the inequality holds $\widehat{T} = 0$. If the inequality doesn't hold, $\widehat{T} > 0$ go to step 1.

Step 1: Given M_{-1}, determine $M_0 < 1$ guessing $M_t = 1$ for $\forall t \geq 1$. The participation constraint for M_0 is,

$$g'\left(1+\frac{1-M_0}{\lambda(1-M_{-1})}\right) + \beta\lambda\left[g'(1) + \phi(1)\right]$$
$$= \quad X\left\{f'\left(1+\frac{M_0}{\lambda M_{-1}}\right) + \beta\lambda\gamma\left[f'\left(1+\frac{1}{\lambda M_0}\right) + \pi\left(1+\frac{1}{\lambda M_0}\right)\right]\right\}$$

Since by construction $\frac{1}{M_0} > 1$, combining this constraint with (5) implies that $\frac{M_0}{M_{-1}} > 1 \Rightarrow \frac{1-M_0}{1-M_{-1}} < 1$. The left hand side of this constraint is rising in M_0, and the right hand side is falling in M_0. Thus, there exists a unique $M_0 \in (0,1)$ which solves this constraint. Verify if $M_1 = 1$ by checking (9) for $\widehat{T} = 1$. If the inequality holds, $\widehat{T} = 1$. If the inequality doesn't hold $\widehat{T} > 1$ go to step 2.

Step 2: Given M_{-1}, determine $M_0 < 0$, $M_1 < 0$ guessing $M_t = 1$ for $\forall t \geq 2$. The participation constraints for M_0, M_1 are,

$$g'\left(1+\frac{1-M_0}{\lambda(1-M_{-1})}\right) + \beta\lambda\left[g'\left(1+\frac{1-M_1}{\lambda(1-M_0)}\right) + \phi\left(1+\frac{1-M_1}{\lambda(1-M_0)}\right)\right]$$
$$= \quad X\left\{f'\left(1+\frac{M_0}{\lambda M_{-1}}\right) + \beta\lambda\gamma\left[f'\left(1+\frac{M_1}{\lambda M_0}\right) + \pi\left(1+\frac{M_1}{\lambda M_0}\right)\right]\right\}$$

$$g'\left(1+\frac{1-M_1}{\lambda(1-M_0)}\right) + \beta\lambda\left[g'(1) + \phi(1)\right]$$
$$= \quad X\gamma\left\{f'\left(1+\frac{M_1}{\lambda M_0}\right) + \beta\lambda\gamma\left[f'\left(1+\frac{1}{\lambda M_1}\right) + \pi\left(1+\frac{1}{\lambda M_1}\right)\right]\right\}$$

Since $\frac{1}{M_1} > 1$, combining the second constraint with (5) implies that $\frac{M_1}{M_0} > 1 \Rightarrow \frac{M_0}{M_{-1}} > 1$ using (5) combined with the first constraint. In the first constraint, given $M_1 \in (0,1)$, there exists a unique $M_0 \in (0,1)$ which solves the equality. In the second equation, given $M_0 \in (0,1)$, there exists a unique $M_1 \in (0,1)$ solving the equation. Verify if $M_2 = 1$ by checking (9) for $\widehat{T} = 2$. If the inequality holds $\widehat{T} = 2$, if the inequality doesn't hold $\widehat{T} > 2$ go to step 3, and so on.

This algorithm identifies an equilibrium with the lowest $\widehat{T}$. Next we show, by contradiction, that given such an equilibrium there cannot exist another equilibrium with higher $T' > \widehat{T}$. Suppose not so, given an equilibrium $\left\{M_0, ..., M_{\widehat{T}-1}, \widehat{T}\right\}$ there exists another equilibrium $\left\{M'_0, ..., M'_{T'-1}, T'\right\}$ where $T' > \widehat{T}$. From the participation constraints for the second equilibrium, using $\frac{1}{M'_{T'-1}} > 1 \Rightarrow \frac{M'_{T'-1}}{M'_{T'-2}} > 1$ in turn implies $\frac{M'_{t-1}}{M'_{t-2}} > 1$. The period $\hat{T}$ participation constraints in the two equilibria are,

$$g'(1) + \beta\lambda\left[g'(1) + \phi(1)\right]$$
$$\leq X\gamma\left\{f'\left(1 + \frac{1}{\lambda M_{\widehat{T}-1}}\right) + \beta\lambda\gamma\left[f'\left(1 + \frac{1}{\lambda}\right) + \pi\left(1 + \frac{1}{\lambda}\right)\right]\right\},$$

$$g'\left(1 + \frac{1 - M'_{\widehat{T}}}{\lambda\left(1 - M'_{\widehat{T}-1}\right)}\right) + \beta\lambda\left[g'\left(1 + \frac{1 - M'_{\widehat{T}+1}}{\lambda\left(1 - M'_{\widehat{T}}\right)}\right) + \phi(1 + \frac{1 - M'_{\widehat{T}+1}}{\lambda\left(1 - M'_{\widehat{T}}\right)})\right]$$

$$= X\gamma\left\{f'\left(1 + \frac{M'_{\widehat{T}}}{\lambda M'_{\widehat{T}-1}}\right) + \beta\lambda\gamma\left[f'\left(1 + \frac{M'_{\widehat{T}+1}}{\lambda M'_{\widehat{T}}}\right) + \pi\left(1 + \frac{M'_{\widehat{T}+1}}{\lambda M'_{\widehat{T}}}\right)\right]\right\}$$

Since $\frac{M'_{\widehat{T}+1}}{M'_{\widehat{T}}} > 1$, a comparison of these constraints implies $\frac{M'_{\widehat{T}}}{M'_{\widehat{T}-1}} > \frac{1}{M_{\widehat{T}-1}}$. Since $M'_{\widehat{T}} < 1$, the last inequality implies $M'_{\widehat{T}-1} < M_{\widehat{T}-1}$. Using $\frac{M'_{\widehat{T}}}{M'_{\widehat{T}-1}} > \frac{1}{M_{\widehat{T}-1}}$ and comparing the period $\hat{T} - 1$ participation constraints in the two equilibria implies $\frac{M'_{\widehat{T}-1}}{M'_{\widehat{T}-2}} > \frac{M_{\widehat{T}-1}}{M_{\widehat{T}-2}}$. Using the participation constraints repeatedly in this way implies, $\frac{M'_0}{M_{-1}} > \frac{M_0}{M_{-1}}$, so $M'_0 > M_0$. Combining the three implications that $M'_{\widehat{T}-1} < M_{\widehat{T}-1}$, that $M'_0 > M_0$ and that $\frac{M'_t}{M'_{t-1}} > \frac{M_t}{M_{t-1}} \, \forall t \geq 1$ leads to a contradiction.

To complete the proof for uniqueness an equilibrium $\{M_0, ..., M_{\widehat{T}-1}\}$ must be unique given $\widehat{T}$. Suppose not, so that there exists a $M'_t \neq M_t$ for some $t \in \{0, ..., \widehat{T} - 1\}$. Then participation constraints (8) imply that $M'_{\widehat{T}-1} \neq M_{\widehat{T}-1}$, so we just need to show that $M'_{\widehat{T}-1} \neq M_{\widehat{T}-1}$ leads to contradiction. Suppose $M'_{\widehat{T}-1} > M_{\widehat{T}-1}$, then to ensure the participation constraints (8) hold, $\frac{M'_t}{M'_{t-1}} < \frac{M_t}{M_{t-1}}$. Specifically, $\frac{M'_0}{M_{-1}} < \frac{M_0}{M_{-1}}$ implies $M'_0 < M_0$ given M_{-1}. Combining the three implications that $M'_{\widehat{T}-1} > M_{\widehat{T}-1}$, that $M'_0 < M_0$ and that $\frac{M'_t}{M'_{t-1}} < \frac{M_t}{M_{t-1}} \, \forall t \geq 1$ leads to a contradiction.

Now suppose the opposite, $M'_{\widehat{T}-1} < M_{\widehat{T}-1}$. Now to ensure the participation constraints hold, $\frac{M'_t}{M'_{t-1}} > \frac{M_t}{M_{t-1}} \Rightarrow M'_0 > M_0$ given M_{-1}. Combining the three implications that $M'_{\widehat{T}-1} < M_{\widehat{T}-1}$, that $M'_0 > M_0$ and that $\frac{M'_t}{M'_{t-1}} > \frac{M_t}{M_{t-1}} \, \forall t \geq 1$ leads to a contradiction.

Parts (iii) and (iv) are follow from participation constraints (8) and condition (9) for $\widehat{T}$. $\blacksquare$

Proof of Proposition 2. (i) Traditional sector lifetime income increasing over

time is given by

$$
g'\left(1+\frac{1-M_1}{\lambda\left(1-M_0\right)}\right)+\beta\lambda\left[g'\left(1+\frac{1-M_1}{\lambda\left(1-M_0\right)}\right)+\phi\left(1+\frac{1-M_2}{\lambda\left(1-M_1\right)}\right)\right]
$$

$$
<\quad...<g'\left(1+\frac{1-M_{\widehat{T}-2}}{\lambda\left(1-M_{\widehat{T}-3}\right)}\right)+\beta\lambda\left[g'\left(1+\frac{1-M_{\widehat{T}-1}}{\lambda\left(1-M_{\widehat{T}-2}\right)}\right)+\phi\left(1+\frac{1-M_{\widehat{T}-1}}{\lambda\left(1-M_{\widehat{T}-2}\right)}\right)\right]
$$

$$
<\quad g'\left(1+\frac{1-M_{\widehat{T}-1}}{\lambda\left(1-M_{\widehat{T}-2}\right)}\right)+\beta\lambda\left[g'\left(1\right)+\phi\left(1\right)\right].
$$

$\frac{1-M_{\widehat{T}-1}}{1-M_{\widehat{T}-2}}>0\Rightarrow\frac{1-M_{\widehat{T}-2}}{1-M_{\widehat{T}-3}}>\frac{1-M_{\widehat{T}-1}}{1-M_{\widehat{T}-2}}$ since $g'(\cdot)$ is decreasing and $\phi(\cdot)$ is increasing. Then by induction, we get the result.

(ii) Define $\tilde{t}$ such that, for $t<\tilde{t}$, modern sector lifetime income is growing slower than γ,

$$
f'\left(1+\frac{M_{t-1}}{\lambda M_{t-2}}\right)+\beta\lambda\gamma\left[f'\left(1+\frac{M_t}{\lambda M_{t-1}}\right)+\pi\left(1+\frac{M_t}{\lambda M_{t-1}}\right)\right]
$$

$$
>\quad f'\left(1+\frac{M_t}{\lambda M_{t-1}}\right)+\beta\lambda\gamma\left[f'\left(1+\frac{M_{t+1}}{\lambda M_t}\right)+\pi\left(1+\frac{M_{t+1}}{\lambda M_t}\right)\right],
$$

and for $t\geq\tilde{t}$, modern sector lifetime income is growing faster than γ,

$$
f'\left(1+\frac{M_{t-1}}{\lambda M_{t-2}}\right)+\beta\lambda\gamma\left[f'\left(1+\frac{M_t}{\lambda M_{t-1}}\right)+\pi\left(1+\frac{M_t}{\lambda M_{t-1}}\right)\right]
$$

$$
\leq\quad f'\left(1+\frac{M_t}{\lambda M_{t-1}}\right)+\beta\lambda\gamma\left[f'\left(1+\frac{M_{t+1}}{\lambda M_t}\right)+\pi\left(1+\frac{M_{t+1}}{\lambda M_t}\right)\right].
$$

For $t\geq\tilde{t}$, $\frac{M_{t+1}}{M_t}<\frac{M_t}{M_{t-1}}$ by an argument resembling that used in part (i). Thus, modern sector population growth peaks before lifetime income increases faster than γ, that is $S<\tilde{t}$.

The proof for $t<\tilde{t}$ is in two parts. First, by construction we have $\frac{M_S}{M_{S-1}}\geq\frac{M_{S+1}}{M_S}$. During transition, for $t<\tilde{t}$,

$$
f'\left(1+\frac{M_S}{\lambda M_{S-1}}\right)+\beta\lambda\gamma\left[f'\left(1+\frac{M_{S+1}}{\lambda M_S}\right)+\pi\left(1+\frac{M_{S+1}}{\lambda M_S}\right)\right]
$$

$$
>\quad f'\left(1+\frac{M_{S+1}}{\lambda M_S}\right)+\beta\lambda\gamma\left[f'\left(1+\frac{M_{S+2}}{\lambda M_{S+1}}\right)+\pi\left(1+\frac{M_{S+2}}{\lambda M_{S+1}}\right)\right],
$$

which then implies falling population growth $\frac{M_{S+1}}{M_S}>\frac{M_{S+2}}{M_{S+1}}$. By induction population growth is falling in $t\geq S$.

Second, by construction we have $\frac{M_{S-1}}{M_{S-2}}<\frac{M_S}{M_{S-1}}$. During transition, for $t<\tilde{t}$,

$$
f'\left(1+\frac{M_{S-2}}{\lambda M_{S-3}}\right)+\beta\lambda\gamma\pi\left(1+\frac{M_{S-1}}{\lambda M_{S-2}}\right)
$$

$$
>\quad f'\left(1+\frac{M_{S-1}}{\lambda M_{S-2}}\right)+\beta\lambda\gamma\pi\left(1+\frac{M_S}{\lambda M_{S-1}}\right),
$$

which then implies rising population growth $\frac{M_{S-2}}{M_{S-3}} < \frac{M_{S-1}}{M_{S-2}}$. By induction population growth is rising in $t < S$.

In period $\widehat{T}$, lifetime income is $X\gamma^{\widehat{T}} \left[f' \left(1 + \frac{1}{\lambda M_{T-1}} \right) + \beta\lambda\gamma \left[f' \left(1 + \frac{1}{\lambda} \right) + \pi \left(1 + \frac{1}{\lambda} \right) \right] \right]$. In period $\widehat{T} + 1$, lifetime income is $X\gamma^{\widehat{T}+1} \left[f' \left(1 + \frac{1}{\lambda} \right) + \beta\lambda\gamma \left[f' \left(1 + \frac{1}{\lambda} \right) + \pi \left(1 + \frac{1}{\lambda} \right) \right] \right]$. So between period $\widehat{T}$ and $\widehat{T} + 1$, lifetime income is growing faster than γ, and after period $\widehat{T} + 1$, it grows at rate γ. Thus, $S \leq \tilde{t} \leq \widehat{T}$.

There are three possibilities for the path of $\frac{M_{t+1}}{M_t}$: (i) it is rising until $t = \widehat{T} - 1$ and $S = \widehat{T} - 1$, (ii) it is falling and $S = 1$, and (iii) it is rising and then falling. Thus, the population growth of the modern sector is single peaked. ∎

A.2 Aggregate Output versus Aggregate Earnings

As before, $G\left(L_{T,t}, E_{T,t}\right)$ and $\gamma^t X F\left(L_{M,t}, E_{M,t}\right)$ denote efficiency units of labor, and let $K_{T,t}, K_{M,t}$ denote physical capital in the traditional and modern sectors respectively. In each sector, output is produced subject to constant returns to scale in all inputs. Aggregate output in period t is given by

$$
\begin{aligned}
\tilde{Y}_t &= \tilde{Y}_T \left[G\left(L_{T,t}, E_{T,t}\right), K_{T,t} \right] + \tilde{Y}_M \left[\gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right), K_{M,t} \right] \\
&\equiv \tilde{y}_T \left(\frac{K_{T,t}}{G\left(L_{T,t}, E_{T,t}\right)} \right) G\left(L_{T,t}, E_{T,t}\right) + \tilde{y}_M \left(\frac{K_{M,t}}{\gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right)} \right) \gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right) \\
&\equiv \tilde{y}_T\left(k_{T,t}\right) G\left(L_{T,t}, E_{T,t}\right) + \tilde{y}_M\left(k_{M,t}\right) \gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right).
\end{aligned}
$$

When the marginal product of capital is constant at $R = \frac{1}{\beta}$, the ratio of capital to efficiency units of labor is constant, and is implicitly given by

$$
R = \tilde{y}'_T\left(k_T^*\right) = \tilde{y}'_M\left(k_M^*\right)
$$

This in turn implies the labor share of output in each sector is a constant

$$
\begin{aligned}
s_T\left(k_T^*\right) &\equiv \frac{\tilde{y}_T\left(k_T^*\right) - \tilde{y}'_T\left(k_T^*\right) k_T^*}{\tilde{y}_T\left(k_T^*\right)} \\
s_M\left(k_M^*\right) &\equiv \frac{\tilde{y}_M\left(k_M^*\right) - \tilde{y}'_M\left(k_M^*\right) k_M^*}{\tilde{y}_M\left(k_M^*\right)}
\end{aligned}
$$

Then, we can express aggregate labor earnings as

$$
\widetilde{LY}_t = s_T\left(k_T^*\right) \tilde{y}_T\left(k_T^*\right) G\left(L_{T,t}, E_{T,t}\right) + s_M\left(k_M^*\right) \tilde{y}_M\left(k_M^*\right) \gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right).
$$

Re-normalizing the units of output by $s_T\left(k_T^*\right) \tilde{y}_T\left(k_T^*\right)$ and defining $X \equiv \frac{s_M\left(k_M^*\right) \tilde{y}_M\left(k_M^*\right)}{s_T\left(k_T^*\right) \tilde{y}_T\left(k_T^*\right)} \tilde{X}$, we get the formula for aggregate labor earnings Y_t, which is used in our analysis.

Consider the aggregate labor share of output, which can be written as

$$
\frac{\widetilde{LY}_t}{\tilde{Y}_t} = s_T\left(k_T^*\right) \frac{G\left(L_{T,t}, E_{T,t}\right) + \frac{s_M\left(k_M^*\right)}{s_T\left(k_T^*\right)} \frac{\tilde{y}_M\left(k_M^*\right)}{\tilde{y}_T\left(k_T^*\right)} \gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right)}{G\left(L_{T,t}, E_{T,t}\right) + \frac{\tilde{y}_M\left(k_M^*\right)}{\tilde{y}_T\left(k_T^*\right)} \gamma^t \tilde{X} F\left(L_{M,t}, E_{M,t}\right)}.
$$

When everyone is producing in the traditional sector this share is $s_T(k_T^*)$, and when everyone is producing in the modern sector this share is $s_M(k_M^*)$. Depending on whether $s_T(k_T^*) \lessgtr s_M(k_M^*)$, output grows faster or slower than earnings. In particular, if the capital share is higher in the modern sector, aggregate output grows faster than aggregate earnings during transition.

A.3 Equilibrium for J-period Model

A *competitive equilibrium* consists of a sequence of sectoral cohort shares $\{(N_t, M_t)\}_{t=0}^{\infty}$ and interest factor R such that

1. every agent earns his marginal product;

2. young agents decide on a sector to work in and how much to consume to maximize their lifetime utility (12) subject to the budget constraint (13), and lifetime earnings given by

$$
(27) \qquad \max \left\{ \begin{array}{l} \sum_{j=0}^{J-1} \beta^j \lambda_T(j) \left[g'\left(\frac{L_{T,t+j}}{E_{T,t+j}}\right) + \phi\left(\frac{L_{T,t+j}}{E_{T,t+j}}\right) j \right], \\ \gamma^t X \sum_{j=0}^{J-1} \beta^j \lambda_M(j) \gamma_M^j \left[f'\left(\frac{L_{M,t+j}}{E_{M,t+j}}\right) + \pi\left(\frac{L_{M,t+j}}{E_{M,t+j}}\right) j \right] \end{array} \right\}
$$

3. the resource constraints (14)-(18) are satisfied, and

4. the credit market clears in every period.

Using the resource constraints and the definitions of labor and experience from (16)-(18),

$$
(28) \quad \sum_{j=0}^{J-1} \beta^j \lambda_T(j) \left[g'\left(\frac{\sum_{i=0}^{J-1} \lambda_T(i)(1-M_{t+j-i})}{\sum_{i=0}^{J-1} i\lambda_T(i)(1-M_{t+j-i})}\right) + \phi\left(\frac{\sum_{i=0}^{J-1} \lambda_T(i)(1-M_{t+j-i})}{\sum_{i=0}^{J-1} i\lambda_T(i)(1-M_{t+j-i})}\right) j \right]
$$

$$
= \gamma^t X \sum_{j=0}^{J-1} \beta^j \lambda_M(j) \gamma^j \left[f'\left(\frac{\sum_{i=0}^{J-1} \lambda_M(i)M_{t+j-i}}{\sum_{i=0}^{J-1} i\lambda_M(i)M_{t+j-i}}\right) + \pi\left(\frac{\sum_{i=0}^{J-1} \lambda_M(i)M_{t+j-i}}{\sum_{i=0}^{J-1} i\lambda_M(i)M_{t+j-i}}\right) j \right]
$$

If young agents enter the modern sector only in period t, $M_t = 1$,

$$
(29) \quad \sum_{j=0}^{J-1} \beta^j \lambda_T(j) \left[g'\left(\frac{\sum_{i=0}^{J-1} \lambda_T(i)(1-M_{t+j-i})}{\sum_{i=0}^{J-1} i\lambda_T(i)(1-M_{t+j-i})}\right) + \phi\left(\frac{\sum_{i=0}^{J-1} \lambda_T(i)(1-M_{t+j-i})}{\sum_{i=0}^{J-1} i\lambda_T(i)(1-M_{t+j-i})}\right) j \right]
$$

$$
\leq \gamma^t X \sum_{j=0}^{J-1} \beta^j \lambda_M(j) \gamma^j \left[f'\left(\frac{\sum_{i=0}^{J-1} \lambda_M(i)M_{t+j-i}}{\sum_{i=0}^{J-1} i\lambda_M(i)M_{t+j-i}}\right) + \pi\left(\frac{\sum_{i=0}^{J-1} \lambda_M(i)M_{t+j-i}}{\sum_{i=0}^{J-1} i\lambda_M(i)M_{t+j-i}}\right) j \right]
$$

In the $J = 2$ model there was a single terminal period condition. In the general model, there are $(J-1)$ terminal period conditions. (28) and (29) characterize a simultaneous difference equation system of M_t of order $2(J-1)$.

Equilibrium construction for J-period overlapping generations model

Since $\gamma > 1$, and the lifetime product of agents working in the traditional sector is always finite, there exists a finite terminal period $T^* < \infty$ for which transition is complete. That is, there exists a $T^* < \infty$ for which the inequality (29) holds for $M_{T^*-(J-1)} = 1$ through to $M_{T^*-1} = 1$.

In our simulations, the algorithm for constructing the equilibrium transition path is as follows:

Guess that once $M_{\hat{T}} = 1$, $M_t = 1 \ \forall t \geq \hat{T}$, such that $T^* = \hat{T} + (J-1)$, and follow the steps below.

Step 0: If $M_{-1} < 1$, $T^* \geq J - 1$. $T^* = J - 1$ occurs if $M_t = 1 \ \forall t \geq 0$. Given $\{M_{-i}\}_{i=1}^{J-1}$, guess that $M_t = 1 \ \forall t \geq 0$.

Verify this by checking whether inequality (29) holds for $M_0 = 1$ through to $M_{J-2} = 1$. If these inequalities hold $T^* = J - 1$. If they do not all hold, $T^* > J - 1$ go to step 1.

Step 1: Given $\{M_{-i}\}_{i=1}^{J-1}$, guess that $M_t = 1 \ \forall t \geq 1$. Then determine $M_0 \in (0, 1)$ using participation constraint (28). The left hand side of this participation constraint is rising in M_0, and the right hand side is falling in M_0. Given $T^* > J - 1$, there exists a unique $M_0 \in (0, 1)$ which solves this equality.

Verify if $M_t = 1 \ \forall t \geq 1$ by checking whether inequality (29) holds for $M_1 = 1$ through to $M_{J-1} = 1$. If these inequalities hold $T^* = J$. If they do not all hold, $T^* > J$ go to step 2.

Step 2: Given $\{M_{-i}\}_{i=1}^{J-1}$, guess that $M_t = 1 \ \forall t \geq 2$. Then determine $M_1 \in (0, 1)$ using participation constraint (28), and M_0 using participation constraints (28) and (29).

Verify if $M_t = 1 \ \forall t \geq 2$ by checking whether inequality (29) holds for $M_2 = 1$ through to $M_J = 1$. If these inequalities hold $T^* = J + 1$. If they do not all hold, $T^* > J + 1$ go to step 3, and so on.

A.4 Sensitivity Analysis

We perform a sensitivity analysis by varying the technology parameters $\{\rho_T,\ \rho_M,\ \alpha_T,\ \alpha_M,\ \gamma_M\}$ by plus and minus one standard error (from the estimation), and check the robustness of the simulation results by focusing on the simulated modern cohort share from 1976 onwards. Both the trend and level of the cohort shares are typically robust to these changes, but the simulated cohort share for the initial year 1976 only, M_0, can deviate substantially. For instance, increasing α_M by one standard deviation increases M_0 by 10 per cent relative to the benchmark simulation, but subsequent cohort shares are only about 4 per cent higher. We also varied the parameters β and X, by plus and minus 10 per cent of their calibrated values and find a similar robustness of the modern cohort share.

In terms of direction of change, higher ρ_T and lower ρ_M tend to speed up transition, as do lower α_T and higher α_M. As expected, higher γ_M and higher X speeds up transition, and we find higher β tends to speeds up transition also.

While $\{\rho_T,\ \rho_M\}$ affect labor-experience complementarity, and $\{\alpha_T,\ \alpha_M\}$ affects the importance of raw labor in output, they also affect the level of productivity of each technology. One way to isolate the effects of complementarity and labor share from affects on productivity levels is to perform sensitivity analysis while also varying X. Specifically,

we re-conduct sensitivity checks for these variables allowing simulated X to vary in such a way that simulated M_0 always coincides with the data M_0. For several parameters this reversed the effect of parameter change on the speed of transition. Now lower ρ_T and higher ρ_M tend to speed up transition, as do lower α_T and lower α_M. These outcomes are consistent with the comparative statics exercises in the text where we also allow X to vary.

A.5   Data Appendix

Table A.1 Labor Force Participation Rates across Experience Groups (%)

Experience	Average	1976	1981	1986	1988	1990	1992	1994	1996
0	5.69	3.04	4.23	6.69	7.90	7.27	5.95	4.85	4.22
1	6.22	5.80	7.11	5.79	6.46	7.20	6.55	5.90	5.06
2	6.48	6.61	7.15	6.17	6.08	6.39	6.54	6.77	6.28
3	6.57	6.17	6.37	6.68	6.24	5.81	6.47	7.00	7.43
4	6.79	6.69	7.54	6.80	6.84	6.71	6.05	6.65	7.22
5	6.75	6.49	7.83	7.15	6.75	6.34	7.02	6.52	6.20
6	7.40	6.43	7.59	8.18	7.92	6.77	7.36	7.57	7.08
7	7.19	6.39	6.23	7.56	7.30	7.93	7.02	6.80	7.65
8	7.12	7.15	5.71	6.77	7.40	7.25	7.66	7.66	6.92
9	6.36	6.55	5.50	5.62	5.87	6.38	6.73	6.73	7.11
10	5.77	6.91	5.83	5.32	4.78	5.42	5.96	6.15	6.21
11	5.24	6.60	5.54	4.79	4.63	4.65	4.98	5.38	5.98
12	4.82	5.81	5.45	4.92	4.80	4.36	4.46	4.70	4.81
13	3.93	4.23	4.22	4.16	3.70	4.15	3.76	3.70	3.83
14	3.75	3.74	3.74	4.01	3.91	3.62	3.66	3.76	3.64
15	3.20	3.09	3.13	3.44	3.25	3.39	3.09	3.04	3.13
16	2.55	2.87	2.22	2.22	2.5	2.69	2.69	2.64	2.62
17	2.00	2.39	1.90	1.66	1.84	1.72	2.18	2.07	2.33
18	1.29	1.61	1.60	1.13	1.01	1.25	1.08	1.37	1.49
19	0.87	1.42	1.11	0.95	0.83	0.71	0.80	0.75	0.80