

# Disaster Relief Funds: Policy Implications for Catastrophe Insurance

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## Abstract

We study the policy implications that arise from introducing a disaster relief fund to an insurance monopoly. Such a form of government intervention can promote higher demand and lower prices in the market for catastrophe insurance. Our model predicts buyers to increase their demand by 15% and the seller to lower prices by 30%. The optimal design of the fund contains two components. On average, we find that 20% of the fund serves as complementary insurance while 80% effectively serves as catastrophe reinsurance. The model also predicts a positive correlation between the size of the disaster relief fund and the fraction that serves as complementary insurance. The downside of the design is that the seller may be overhedged, and it becomes more costly for a potential new firm to enter the market.

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## Abstract

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**JEL Classification:** D42, G22, H30.

# 1 Introduction

Severe catastrophes frequently fuel the discussion about the necessity of public intervention in the private market for insurance. One form of public intervention that has emerged in several forms is disaster relief programs. Popular examples include the California Earthquake Authority (CEA) and the Federal Emergency Management Agency (FEMA), or more recently the Terrorism Risk Insurance Act (TRIA) - a government reinsurance facility aiming to provide support for the insurance industry subject to terrorism risk, see Cummins and Mahul (2008) for an international survey.

The goal of this paper is to provide a normative approach to public intervention in catastrophe insurance. We build a case for intervention by assuming the existence of market power. Specifically, we model an economic agent who can introduce a disaster relief fund to an insurance monopoly. The agent's goal is to improve risk sharing by mitigating the effects of monopoly power, while ensuring that the insurance seller continues to produce for the private market. There are of course several market imperfections that could be associated with inefficient risk sharing in cat risk insurance, as summarized in Gollier (2008), but our objective is to study one friction in isolation. Market power can generally be present in cat risk insurance, see Froot (2001), but it seems particularly relevant after the occurrence of a severe catastrophe. Typically, fewer firms serve the primary market in those times, see Cummins (2006), often resulting in the introduction of new disaster relief funds for which specific policies need to be determined - a task that we undertake in this paper.

The key players in our model are insurance buyers and a seller, as well as a risk-neutral government entity. Our proposed fund has the form of an ex-ante program, as argued for by

Kunreuther and Pauly (2006). All agents know about its existence and account for disaster relief payments in their decision making process. The fund collects premia proportionally from buyer and seller such that the fund is funded at expected losses, suggested by Jaffee (2006). Buyer and seller in our model are assumed to be risk averse. We calibrate the risk tolerance values such that the model delivers average prices in the private market of approximately 1.3 times expected losses without public intervention. This price appears slightly below the current pricing of catastrophe insurance, which is justified since market power is typically not the only source for high prices, as in Froot and O'Connell (2008). Comparative statics of the policy implications with respect to the buyers and sellers risk tolerance are shown in the main body of the text.

Our model provides support for the existence of a disaster relief fund, since the buyer strictly prefers having access to the additional risk sharing mechanism – the absence of disaster relief is never optimal given our parameterizations. On average, we find that 20% of the fund serves as complementary insurance while 80% effectively serves as catastrophe reinsurance. The model also predicts a positive correlation between the size of the disaster relief fund and the fraction that serves as complementary insurance. Specifically, we find that a 1% increase in the fraction of the fund serving the buyer is associated with a .36% larger fund size. Our predictions about the key role of reinsurance are, for example, consistent with Kunreuther (1996), who argues for additional reinsurance coverage for cat losses to protect insurers against potential insolvency. Our results, however, are obtained solely based on the objective of risk sharing.

We find that the introduction of a disaster relief fund can promote higher demand and

lower prices in the market for catastrophe insurance. We predict buyers to increase their demand by 15% and the seller to lower prices by 30%, which is approximately 50% of the distance to an otherwise competitive market without public intervention. The model shows that the introduction of a disaster relief fund will decrease the extent of monopoly rents, a prediction consistent with insurers' negative abnormal performance found in Brown et al. (2004) due to the "make available rule" of TRIA. Interestingly, the design of TRIA as a reinsurance facility also matches the 80% policy resulting from our model, which is the reason why the insurer continues to produce although being worse off in the private market. The downside of the design is that the seller may ultimately be overhedged, and it becomes more costly for a potential new firm to enter the market.

The modeling structure of arrival probabilities plus conditional loss distributions are important to capture the nature of catastrophe risk, as compared to more traditional forms of insurance. Diversification does not play a role in our model. The disaster fund that we investigate is effectively a non-tradable asset whose payoffs are perfectly correlated with catastrophic risk. The fund has, however, a different risk and return trade-off than the insurance product offered in the private market. This stems from the possibility that a governmental agency is in a position to time-diversify even catastrophic risks and can provide government funding should the net balance of the fund be negative, as argued in Lewis and Murdock (1996). Comparative statics show that funding the fund at prices higher than expected losses can have a severe impact. In general, the optimal size of the fund will shrink and its distribution policy will favor the seller.

## 2 The Model

We consider a one-period economy in which three agents interact. The first agent is facing risks stemming from the impact of a catastrophic event, and she is aiming to protect herself by buying insurance in the primary market. The second agent, a monopolistic seller, sets the price of the insurance given the buyer's inverse demand function. While the buyer and the seller lock in a simple insurance contract, they also take into account the possible existence of an additional risk sharing mechanism, i.e. a disaster relief fund. The third agent - a social planner or governmental agency - determines the design of this disaster fund.

The catastrophic event may occur with probability  $\lambda \in [0; 1]$  at the end of the period. Conditional on the occurrence of a catastrophe, the potential impact corresponds to a loss distribution  $l \in [0; 1]$ , characterized by expected losses given by  $\bar{l} \in [0; 1]$ , as well as uncertainty about losses given by  $\sigma_l^2 = Var[l] = \bar{l}^2 - \bar{l}^2$ . All agents are equally informed about the source and the extent of catastrophic risk. Our model has four choice variables: First, the buyer maximizes utility by choosing demand  $q$ . Second, the buyer maximizes utility by setting the price  $p$ . Third, the governmental agency determines the size of the disaster relief fund  $\nu \in [0; 1]$ , and the fraction of the fund paid for and received by the buyer,  $\kappa \in [0; 1]$ . Her objective is to implement a fund that improves risk sharing and is weakly preferred by the buyer *and* by the seller.

### 2.1 Buyer's Demand Policy

Without loss of generality, we assume the buyer's initial wealth is normalized to one unit,  $W_{b,0} = 1$ . The buyer is facing a quadratic utility function defined over end of period wealth

given by  $W_b$ , such that

$$U_b = -\frac{1}{2}E[(A_b - W_b)^2], \quad 0 \leq W_b < A_b. \quad (1)$$

The parameter  $A_b > 1$  characterizes the buyer's risk tolerance, where higher values of  $A_b$  correspond to lower degrees of risk aversion. End of period wealth is uncertain and it depends on the price paid for insurance, the catastrophe contingent payoff, as well as the extent of the disaster relief fund. We consider an insurance contract in which the buyer pays the seller a fraction of the total insured wealth at the beginning of the period. If the cat event occurs, the buyer is fully compensated for losses of the *insured* wealth. Due to normalization, the *insured* wealth is a *fraction*  $q \in [0; 1]$  of total wealth  $W_{b,0}$ , such that the parameter  $q$  also represents the buyer's demand policy.

An indirect form of insurance is offered by the properties of the fund. At the occurrence of a catastrophe, the buyer receives amount  $\nu_b = \kappa\nu$ , where as the remaining amount  $\nu_s = (1 - \kappa)\nu$  is paid to the seller. Such policies require additional funding which we assume to be collected proportionally. The buyer pays amount  $C_b = (\lambda\bar{l} + \Delta C)\kappa$  per unit of  $\nu$  to the third agent, the seller pays amount  $C_s = (\lambda\bar{l} + \Delta C)(1 - \kappa)$ . For most of our analysis we assume that the disaster relief fund is financed at expected losses, i.e.  $\Delta C = 0$ . However, we will leave  $\Delta C$  as an exogenous parameter to be able to derive some comparative statistics with respect to disaster funding costs.

In summary, the distribution of buyer's end of period wealth is given by

$$W_b = \begin{cases} 1 - pq - C_b\nu, & prob = 1 - \lambda \\ 1 - pq - C_b\nu - l(1 - q - \nu_b), & prob = \lambda. \end{cases} \quad (2)$$

We define the insurance loading factor as  $\Delta p = p - \lambda \bar{l}$ , and it will be useful to introduce a rescaled loading factor as  $x = \Delta p \sqrt{\gamma}$ , where

$$\gamma = \left( \lambda \bar{l}^2 - (\lambda \bar{l})^2 \right)^{-1} = \left( \lambda (1 - \lambda) \bar{l}^2 + \lambda \sigma_l^2 \right)^{-1}. \quad (3)$$

Intuitively, the parameter  $\gamma$  corresponds to the inverse of the unconditional variance of losses.

### Proposition 1

The buyer's demand policy  $q^* = \arg \max_q [U_b]$  is given by

$$q^*(\nu, \kappa, x) = (1 - \nu_b) \frac{1 - \alpha x}{1 + x^2}, \quad (4)$$

where

$$\alpha(\nu, \kappa) = \frac{\sqrt{\gamma} \left( (A_b - 1) + \lambda \bar{l} (1 - \nu_b) + \nu C_b \right)}{1 - \nu_b}. \quad (5)$$

It follows from equation (4) that when insurance is actuarially fair, the buyer will protect all her wealth by using private insurance plus the disaster relief fund,  $q^* + \nu_b = 1$ .<sup>1</sup> However, the loading factor is typically positive and the buyer will protect a fraction strictly less than one. All comparative statics of the buyer's demand function are standard. *Ceteris paribus* and given a positive loading, the buyer's demand for insurance is larger the higher the level of risk aversion, and the demand is larger for higher levels of conditional expected losses or conditional variance, respectively.

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<sup>1</sup>Since we assumed that  $\nu_b$  does not depend on the buyer's demand  $q$ , it could happen that total coverage is greater than 100%, i.e.  $q + \nu_b > 1$ . This is a waste of resources and does not occur in equilibrium.



## 2.2 Seller's Price Policy

The insurance seller is also risk averse and faces a quadratic utility function with risk tolerance parameter  $A_s$ . She derives utility from selling insurance in the primary market and from possible participation in the disaster relief fund given by

$$U_s = -\frac{1}{2}E[(A_s - W_s)^2], \quad 0 \leq W_s < A_s, \quad (6)$$

where  $W_s$  is the seller's end of period wealth such that

$$W_s = \begin{cases} pq - C_s\nu, & \text{prob} = 1 - \lambda \\ pq - l(q - \nu_s) - C_s\nu, & \text{prob} = \lambda. \end{cases} \quad (7)$$

The previous equation can also be written as

$$W_s = \Delta pq^*(\Delta p) + \lambda \bar{l} \nu_s - C_s \nu - \Delta \tilde{l} (q^*(\Delta p) - \nu_s), \quad (8)$$

while introducing unexpected losses defined by  $\Delta \tilde{l} = \tilde{l} - \lambda \bar{l}$ . The seller sets the optimal insurance loading  $\Delta p^*$  given the buyer's inverse demand function (4). It will be useful to consider two new quantities, the maximal loading accepted by the buyer,  $\Delta p_2$ , and the minimal loading acceptable for the seller,  $\Delta p_1$ . The corresponding rescaled loading factors are given by

$$x_2(\nu, \kappa) = (\alpha(\nu, \kappa))^{-1}, \quad (9)$$

and

$$x_1(\nu, \kappa) = (\beta(\nu, \kappa))^{-1}, \quad (10)$$

respectively, where the function  $\beta(\nu, \kappa) > \alpha(\nu, \kappa)$  is given by

$$\beta(\nu, \kappa) = \frac{\sqrt{\gamma} \left( (A_b + 2A_s - 1) + \lambda \bar{l} (1 - \nu_b - 2\nu_s) + \nu (C_b + 2C_s) \right)}{1 - \nu_b - 2\nu_s}.$$

As expected,  $x_1$  vanishes in the limit  $A_s \rightarrow \infty$  as the seller becomes risk-neutral.

## Proposition 2

The seller's price policy  $\Delta p^* = \arg \max_{\Delta p} [U_s]$  is characterized by the optimal rescaled loading factor

$$x^*(\nu, \kappa) = \left( \sqrt{1 + z^2} - z \right), \quad (11)$$

where

$$z(\nu, \kappa) = \frac{1 - x_1 x_2}{x_1 + x_2}. \quad (12)$$

Combining the results of Propositions 1 and 2, the buyer's and seller's and indirect utilities are summarized in the following Theorem.

## Theorem 1

The buyer's demand yields

$$q^*(\nu, \kappa) = \frac{(1 - \nu_b)}{2} \left( 1 - \frac{x_1}{1 - z x_1} \sqrt{1 + z^2} \right), \quad (13)$$

leading to indirect utilities

$$U_s^* = -\frac{1}{2} \left( \frac{\nu_s^2}{\gamma} + (A_s - \lambda \bar{l} \nu_s + \nu C_s)^2 \right) + \frac{(1 - \nu_b)(1 - \nu_b - 2\nu_s)(x_2 - x^*)(x^* - x_1)}{2\gamma x_1 x_2 (1 + (x^*)^2)}, \quad (14)$$

and

$$U_b^* = -\frac{(1 - \nu_b)^2}{2\gamma x_2^2} \left( 1 + x_2^2 - \frac{(x_2 - x^*)^2}{1 + (x^*)^2} \right). \quad (15)$$

We expect a single insurance seller to extract monopoly rents. However, the degree to which this happens is affected by the size and the policy of the disaster relief fund. The demand for insurance  $q^*$  can either increase or decrease due to a larger value of  $\nu$ , depending on the policy parameter  $\kappa$ . Hence, there is one region in which a disaster relief fund will serve as a substitute for private market insurance. But there is also a region in which the existence of a disaster fund will *increase* the demand for insurance in the primary market, a channel that we explore further in the next section.

### 2.3 Size and Policy of the Disaster Fund

The third agent is able to observe the buyer's and seller's policies and their corresponding indirect utilities. Therefore, she is able to compare how the market for catastrophe insurance is affected by the introduction of a disaster relief fund. In the first step, we identify the set of disaster relief policies that buyer and seller weakly prefer over the economy without disaster relief, i.e. the set  $\Theta$  containing those values of  $\nu$  and  $\kappa$  satisfying

$$U_b^*(\nu, \kappa) \geq U_b^*(0, \kappa), \text{ and} \quad (16)$$

$$U_s^*(\nu, \kappa) \geq U_s^*(0, \kappa).$$

Note that the indirect utilities  $U_b^*(0, \kappa)$  and  $U_s^*(0, \kappa)$  correspond to the case when there is no disaster relief fund, and therefore  $U_a^*(0, \kappa) = U_a^*(0, 0)$  for  $a \in b, s$ . In a second step, we

assume the third agent implements the design that maximizes the buyer's utility. Formally,

$$(\nu^*, \kappa^*) = \arg \max_{(\nu, \kappa) \in \Theta} [U_b^*]. \quad (17)$$

Our social planner or governmental agency acts on behalf of the buyer of insurance, while making sure the seller continues to produce in the primary market. This is motivated by the feature that the buyer faces the catastrophic risk to begin with and originates the economic problem at hand.

In many cases, we should expect this optimization problem to lead to a solution in which  $U_s^*(\nu^*, \kappa^*) = U_s^*(0, \kappa)$ . This is driven by the observation that the seller's monopoly profits will diminish while the disaster relief fund acts as a secondary provider of insurance. The seller can, however, benefit from participation in the disaster relief fund – from her perspective a form of catastrophe *reinsurance*. Hence the loss in utility from diminishing monopoly profits is compensated by a gain in utility through the third agent acting also as an effective reinsurer - a feature that several disaster funds display in reality. From the perspective of the buyer, introducing a disaster relief fund to the set of risk sharing mechanisms can lead to strict Pareto improvement, i.e.  $U_b^*(\nu^*, \kappa^*) > U_b^*(0, \kappa)$ . Two mechanisms are at work: First, the buyer might insure an additional fraction of wealth directly through the disaster relief fund at more reasonable prices. Second, the seller can reduce prices in the primary market due to the existence of reinsurance.

## 3 Results and Interpretation

### 3.1 Properties of Buyer's and Seller's Policies

We first show some general properties of the seller's demand policy and buyer's loading policy. We parameterize the model with equal risk tolerance for buyer and seller,  $A_s = A_b = 2$ , not necessarily implying that buyer and seller have equal effective risk aversion. Buyer and seller both face the monotone region of a quadratic utility function, and thereby only care about unconditional expected losses and standard deviation, given by  $\bar{l}\lambda = 8\%$  and  $\gamma^{-.5} = 20\%$ . As we will see later, the modeling structure of arrival intensities plus conditional loss properties is important to capture the nature of catastrophe risk, as compared to more traditional forms of insurance.

The effect of introducing a disaster relief fund on the buyer's demand in the private market is ambiguous, and Figure 1 displays some interesting effects. The demand for private market insurance generally decreases with the size of government intervention for high values of  $\kappa$ . This is the region in which the disaster relief fund acts as a substitute for private market insurance, being preferred by the buyer due to a lower price. As can be seen in the right graph of Figure 1, any value of  $\kappa$  will lead to a smaller loading factor. Hence, it is not surprising that the demand approaches zero with a unit disaster payment and  $\kappa = 1$ . Interestingly, when a large fraction of the disaster payment is routed to the seller, in our case  $\kappa < .4$ , the demand is larger as compared to not having access to the disaster fund. This feedback effect is driven by the risk sharing that occurs on the seller's side. Suppose all disaster relief is paid to the seller, then such a mechanism acts as a form of reinsurance. Some amount of catastrophic risk is transferred to the third agent, being implicitly less risk averse compared

to the seller, by charging no loading while financing the fund. To rule out the possibility that this effect is the result of our choice of parameters, we show the more general case.

### **Proposition 3**

*The buyer's optimal demand  $q^*(\nu, \kappa)$  is increasing in the volume  $\nu$  at sufficiently low values of the policy parameter  $\kappa$ .*

The following Proposition summarizes the feedback effects on the seller's policy.

### **Proposition 4**

*The seller's optimal rescaled loading factor  $x^*(\nu, \kappa)$ , and the seller's income  $q^*(\nu, \kappa)p^*(\nu, \kappa)$  are decreasing in the volume of the disaster fund  $\nu$ .*

In addition to the feedback effects on private market insurance, we might ask what the *net* positions of buyer and seller yield after the introduction of disaster relief, see Figure 2. The buyer's total insured wealth is strictly increasing in  $\nu$ , and the total price for both forms of insurance per unit of insured wealth is strictly decreasing, due to no loading charged by the third agent. Hence, we should also expect the seller's monopoly rents to be reduced. This effect is further strengthened by the price the seller has to pay for her fraction of disaster relief, as shown in the lower left graph of Figure 2.

The disaster relief fund can also have a severe effect on the net risk position of the insurance seller, in which the seller ends up being overhedged through the risk transfer to the third agent. Given our parametrization, a negative net risk position occurs for low values of  $\kappa$  with values of  $\nu$  larger than .5. Please note that such cases do not correspond to an arbitrage opportunity from the perspective of the seller, even though insurance and

disaster fund payments are perfectly correlated. Claims on the disaster relief fund are not marketable, and the fund's implementation is governed by the social planner.

Before moving to the *optimal* policy of the disaster relief fund, we show the general effects on the buyer's and seller's indirect utilities given the same set of parameters, see Figure 3. The net effect on either agent is ambiguous. It is not surprising that the buyer prefers access to disaster relief for almost all fund sizes and policy values. We can identify a small region,  $\nu < .4$  and  $\kappa < .04$ , for which the buyer is not better off. This occurs in cases where the feedback effect on prices in the private insurance market is so small, that even though the buyer's insured wealth has slightly increased, the net effect on the buyer's utility is negative. We can also identify a region in which the seller weakly prefers the economy with access to disaster relief. If the fraction of the fund received by the buyer is too large, however, then the seller is not better off due to a loss in monopoly rents which can not be compensated by any benefits that might arise from the structure of the fund. This is an important feature of the model in order to sustain *optimal* policies, therefore shown formally.

### **Proposition 5**

*The seller's indirect utility  $U_s^*(\nu, \kappa)$  is increasing in the volume  $\nu$  at sufficiently low values of the policy parameter  $\kappa$ .*

## **3.2 Optimal Disaster Relief Fund Policies**

We compute optimal disaster relief policies according to the objective outlined in Section 2.3. To gauge cross-sectional variation, we generate 10,000 random draws from uniform distributions with the following limits: We allow for risk tolerance values between 1 and 3,

and assume unconditional expected losses to be within a range between 2% and 14%, and the unconditional variance to be between 10% and 30%. The average values of the model parameters correspond to the case analyzed in the previous subsection.

The average optimal size of the disaster fund is .58, a significant value given that the buyer's initial wealth is normalized to one unit. As shown in Figure 4, the maximum fund size is .75, the smallest is .35. It is also noteworthy that the absence of disaster relief is never optimal, since the buyer strictly prefers having access to the additional risk sharing mechanism. Given the size of the fund, on average 20% should be paid to the buyer and the remaining 80% to the seller. This confirms our earlier finding that a key feature of disaster relief is being able to provide reinsurance. We also observe significant variation in the policy parameter  $\kappa$ , ranging between 0 and .45. The mass, however, is centered between .10 and .25. Looking at both choice variables jointly, the model predicts a strong positive correlation between size and policy, see equation (C3). Specifically, we find that a 1% increase in the fraction of the fund owned by the buyer is associated with a .36% larger fund size.

To better understand the cross-sectional variation of these values we show the results of a regression analysis with the dependent variables *volume* and *policy*, respectively, see equations (C1) and (C2) in Figure 4. Ceteris paribus, disaster relief is larger for lower risk aversion of the buyer and higher risk aversion of the seller, with the buyer's risk tolerance having a significantly larger sensitivity. The sign on buyers's risk tolerance is somewhat surprising, as disaster relief at zero loading could be more attractive the higher the buyer's risk aversion. However, the demand for private insurance at the optimal policy is decreasing in the buyers' risk tolerance, see equation (C4), so that buyers are left with a larger fraction of uninsured wealth for which the existence of disaster relief can be beneficial. This follows from



equation (4) and is also confirmed by the results in equation (C2), suggesting that a larger fraction of the fund should be paid for and received by the buyer for lower degrees of risk tolerance. The largest part of the variation in *volume* is explained by the second statistical moment, such that higher variance of losses suggests more government intervention, but with increasing participation by the seller, see equation (C2).

We utilize the same random draws to compute the corresponding values for demand and loading, see Figure 5. The results show that equilibrium demand in the private market increases with the size of the disaster fund, and it decreases significantly in the fraction of the fund owned by the buyer, see regression (C6). This suggests that in equilibrium disaster relief does not serve as a substitute but as a *complement* for the private market. A similar effect occurs with respect to the price, see regression (C7), again indicating that offering reinsurance to the seller has a significant feedback effect on the buyer's well-being. We note that on average the introduction of disaster relief increases the demand for private market insurance by 15%, and it decreases loading factors by 30%, see Figure 6.

In the previous subsection we pointed towards the possibility that disaster relief could result in the seller having negative net risk exposure to catastrophic risk. Hence, we investigated whether this is the case under the *optimal* design. The results are summarized in Figure 7, and we find that in 70% of all cases the seller does indeed overhedge. Although the average risk exposure is only -3%, this does point towards a risk sharing distortion that might arise from our design. It stems from the "optimality" requirement where the seller should not be made worse off with disaster relief. Since monopoly rents will decrease, the seller requires a benefit in order to continue producing for the private market, and especially for large amounts of uncertainty in catastrophic losses as seen in equation (C11).

### 3.3 Effects on the Impact of Monopoly Pricing

As seen above, the introduction of the disaster relief fund affects the pricing of insurance in the primary market. But we would like to explore more specifically to which extent is monopoly pricing reduced, or how close is the new economy to an otherwise competitive market?

One diagnostic of the competitive limit is the minimal accepted (rescaled) loading by the seller in case of no disaster relief,  $x_1(0, \kappa)$ . Since we allow the seller to be risk averse, this value does not equal zero and it corresponds to the smallest loading factor at which the seller starts producing. Hence, the distance  $x^*(0, \kappa) - x_1(0, \kappa)$  is a measure of monopoly pricing, and the ratio  $(x^*(0, \kappa) - x^*(\nu^*, \kappa^*)) / (x^*(0, \kappa) - x_1(0, \kappa))$  a measure of the disaster fund impact.

Results are summarized in Figure 8, left graph. The initial spread due to monopoly pricing is reduced by 50% at the optimal design of disaster relief. In 90% of the cases the reduction is between 0% and 100%. We note that in 10% of the cases the new loading in the primary market is even lower than the competitive limit without disaster relief. These are the cases in which a large fraction of catastrophic risk is ultimately transferred to the social planner, essentially a risk-neutral agent who is willing to accept risks below the risk averse competitive limit set by the seller.

As a second diagnostic, we attempt to measure how a potentially *new entrant* is affected by the introduction of disaster relief. A Bertrand competition among insurance sellers has to be treated carefully since one can show there is no symmetric Nash equilibrium in our model. Intuitively, the entrant has to always undercut the monopolist by lowering the loading, in

which case the entrant attracts all demand and effectively becomes a monopolist herself. We can, however, treat the value  $x_1(\nu^*, \kappa^*)$  as a reference point at which a potential entrant is indifferent between entering and not entering. We first obtain the following general property.

**Proposition 6**

*The spread between the optimal and the lowest acceptable loadings  $x^*(\nu, \kappa) - x_1(\nu, \kappa)$  is increasing in the volume  $\nu$  at sufficiently low values of the policy fraction  $\kappa$ .*

Results are summarized in Figure 8, right graph, showing the ratio  $(x^*(\nu^*, \kappa^*) - x_1(\nu^*, \kappa^*)) / (x^*(0, \kappa) - x_1(0, \kappa))$ . A potential entrant is hurt by the existence of the optimal fund in all cases. Parameter combinations at which the entry cost decreases do exist in theory, but this does not occur in equilibrium. The entry cost increases by 15% on average. In summary, the introduction of disaster relief does mitigate several of the effects due to monopoly power, but it does not promote forces of market competition in the private market.

**3.4 Catastrophe Insurance versus Traditional Insurance**

Buyer and seller face quadratic utility functions and therefore base their decision on mean and variance. This could relate to any source of uncertainty, but we intentionally chose to incorporate three parameters in order to capture the nature of catastrophic risk. Namely, the case of catastrophe insurance typically corresponds to high conditional losses but relatively low arrival probabilities, while more traditional insurance is characterized by relatively low conditional losses and high arrival probabilities.

To illustrate the difference, we assume the level of unconditional expected losses is fixed at  $\lambda \bar{l} = \mu = 0.08 = 8\%$ , and analyze the resulting equilibria for different values of the arrival

probability  $\lambda \in (0.08; 0.30]$ . It follows that the level of unconditional variance becomes a function of  $\lambda$ , and based on equation (3) we obtain

$$\gamma^{-1} = \lambda(1 - \lambda) \left(\frac{\mu}{\lambda}\right)^2 + \lambda\sigma_l^2 = \left(\frac{1}{\lambda} - 1\right)\psi^2 + \frac{1}{\lambda}\sigma_\psi^2 = \frac{1}{\lambda}(\psi^2 + \sigma_\psi^2) - \psi^2. \quad (18)$$

Therefore, the effective variance  $\gamma^{-1}$  monotonically increases as the arrival rate  $\lambda$  decreases. Even under the assumption of zero conditional standard deviation, the unconditional standard deviation will range between 12% and 27%.

A set of simulation results is summarized in Figure 9, where  $\lambda$  is the dependent variable under observation. The coefficient in the volume regression is large and negative, suggesting that more frequent events are indeed associated with a smaller size of the disaster relief fund. Furthermore, the coefficient in the policy regression is positive, suggesting that for more frequent events the optimal policy is to favor the buyer. This result is of course driven by the amount of standard deviation in the economy, but it shows that “disaster relief” indeed emerges from catastrophe risk as compared to more traditional types of insurance.

### 3.5 Funding Disaster Relief with a Small Loading

We operate under the assumption that the fund is funded at expected losses. While this could be seen as reasonable from the perspective of a risk-neutral social planner who can time diversify even catastrophic events, one could also make an argument that at least operational costs should be covered on average. Hence, we analyze a comparative static in which buyer and seller contribute to the disaster fund based on a small positive loading, i.e.  $\Delta C = .001$ . How the size of optimal disaster relief and the policy are affected is shown in Figure 10.

We find that charging a small loading reduces the size of the fund in 88% of the cases, in some cases significantly. The price feedback due to the seller having access to more expensive reinsurance is not attractive to the buyer. Interestingly, for 12% of the cases the size of optimal disaster relief increases. Those are cases in which the buyer's demand in the private market decreases and those buyers find complementary insurance through the fund, even with a small loading, more beneficial. A comparison of loading versus no loading, however, should be treated with care since on average a small fraction of financial wealth leaves the economic system.

As an aside, a positive result for a disaster relief fund that charges *market prices* can not be sustained. Although such a design might be preferred by a buyer in several scenarios, it is never weakly preferred by the seller. The seller has no other source of income in our model, and this would require the seller to contribute to the fund at the *same* price levels she extracts rents at, leading to a strictly inferior situation.

## 4 Conclusion

We studied a role for government intervention in a monopolistic setting. Our results stem from the objective of risk sharing, in which a disaster relief fund represents an additional asset that is perfectly correlated with catastrophic risks, but has a different trade-off between risk and return than the insurance product offered in the private market. We conclude that the introduction of government funded disaster relief can mitigate some of the negative effects of monopoly power. Such a fund, optimally designed, promotes higher demand and lower prices in the private market for cat insurance. Our model delivers specific predictions regarding the

size and the policy of the fund, resulting in a prominent role of reinsurance. It also shows that large, perhaps too large, cat risk exposure may be transferred to the government entity.

In future research, it might be useful to investigate additional imperfections that are present in the cat insurance market but not considered in our model, namely moral hazard and adverse selection. Further modeling work is needed to better understand the interactions between government intervention and issues such as insurance deductables, insurer insolvency, or product discrimination.

## 5 Appendix

### 5.1 Proofs

#### Proof of Proposition 1

The buyer's expected utility can be written as

$$-2U_b = ((A_b - 1) + \lambda\bar{l}(1 - \nu_b) + \nu C_b + \Delta pq)^2 + \frac{(1 - q - \nu_b)^2}{\gamma}. \quad (\text{B1})$$

Applying the FOC and minimizing the r.h.s. with respect to  $q$ , we obtain for  $q^* =$

$$\arg \min_q [-U_b] = \arg \max_q [U_b]$$

$$q^* = (1 - \nu_b) \frac{1 - \alpha\sqrt{\gamma}\Delta p}{1 + \gamma(\Delta p)^2}, \quad (\text{B2})$$

with  $\gamma$  as defined in equation (3) and  $\alpha$  as in equation (5).

*Q.E.D.*

#### Proof of Proposition 2

The seller's expected utility can be written as

$$-2U_s = (A_s - \Delta pq - \lambda\bar{l}\nu_s + \nu C_s)^2 + \frac{(q - \nu_s)^2}{\gamma}, \quad (\text{B3})$$

which can be written in terms of the rescaled loading  $x = \sqrt{\gamma}\Delta p$  in the form of

$$2U_s = \text{const} + B \frac{(x_2 - x)(x - x_1)}{1 + x^2}, \quad (\text{B4})$$

where the constant does not depend on  $\Delta p$ , and

$$B = B_1 B_2 > 0, \tag{B5}$$

with

$$B_1 = ((A_b - 1) + \lambda \bar{l} (1 - \nu_b) + \nu C_b) > 0,$$

$$B_2 = ((A_b + 2A_s - 1) + \lambda \bar{l} (1 - \nu_b - 2\nu_s) + \nu (C_b + 2C_s)) > 0.$$

The maximal and minimal loadings are defined as in equations (10) and (9) with

$$\beta(\nu, \kappa) = \frac{\sqrt{\gamma} ((A_b + 2A_s - 1) + \lambda \bar{l} (1 - \nu_b - 2\nu_s) + \nu (C_b + 2C_s))}{1 - \nu_b - 2\nu_s}. \tag{B6}$$

Applying the FOC and maximizing the r.h.s. with respect to  $x$ , we obtain

$$x^*(\nu, \kappa) = \left( \sqrt{1 + z^2} - z \right), \tag{B7}$$

with

$$z(\nu, \kappa) = \frac{1 - x_1 x_2}{x_1 + x_2}. \tag{B8}$$

*Q.E.D.*

## Proof of Theorem 1



Making use of equations (11) and (4), we obtain

$$q^*(\nu, \kappa) = (1 - \nu_b) \frac{1 - \alpha x^*}{1 + (x^*)^2} = (1 - \nu_b) \frac{1 - \frac{x^*}{x_2}}{1 + (x^*)^2}. \quad (\text{B9})$$

Combining (B9) and (12), we obtain

$$\begin{aligned} q^*(\nu, \kappa) &= \frac{(1 - \nu_b)}{2} \frac{1 - \frac{z+x_1}{1-zx_1} (\sqrt{1+z^2} - z)}{1 - z (\sqrt{1+z^2} - z)} \\ &= \frac{(1 - \nu_b)}{2(1 - zx_1)} \frac{1 - z (\sqrt{1+z^2} - z) - x_1 \sqrt{1+z^2}}{1 - z (\sqrt{1+z^2} - z)} \\ &= \frac{(1 - \nu_b)}{2} \left( 1 + \frac{zx_1}{1 - zx_1} - \frac{x_1}{1 - zx_1} \frac{\sqrt{1+z^2}}{1 - z (\sqrt{1+z^2} - z)} \right). \end{aligned} \quad (\text{B10})$$

The last equation can be transformed to

$$\begin{aligned} q^*(\nu, \kappa) &= \frac{(1 - \nu_b)}{2} \left( 1 - \frac{x_1}{1 - zx_1} \left( \frac{\sqrt{1+z^2}}{1 - z (\sqrt{1+z^2} - z)} - z \right) \right) \\ &= \frac{(1 - \nu_b)}{2} \left( 1 - \frac{x_1}{1 - zx_1} \frac{\sqrt{1+z^2} - z + z^2 (\sqrt{1+z^2} - z)}{1 - z (\sqrt{1+z^2} - z)} \right) \\ &= \frac{(1 - \nu_b)}{2} \left( 1 - \frac{x_1}{1 - zx_1} \frac{(\sqrt{1+z^2} - z) (1 + z^2)}{1 - z (\sqrt{1+z^2} - z)} \right). \end{aligned}$$

Multiplying numerator and denominator of the last equality by  $(\sqrt{1+z^2} + z)$  and using the

identity  $(\sqrt{1+z^2}-z)(\sqrt{1+z^2}+z)=1$ , we obtain

$$\begin{aligned} q^*(\nu, \kappa) &= \frac{(1-\nu_b)}{2} \left( 1 - \frac{x_1}{1-zx_1} \frac{(\sqrt{1+z^2}-z)(1+z^2)}{1-z(\sqrt{1+z^2}-z)} \right) \\ &= \frac{(1-\nu_b)}{2} \left( 1 - \frac{x_1}{1-zx_1} \frac{(\sqrt{1+z^2}-z)(\sqrt{1+z^2}+z)(1+z^2)}{(1-z(\sqrt{1+z^2}-z))(\sqrt{1+z^2}+z)} \right) \\ &= \frac{(1-\nu_b)}{2} \left( 1 - \frac{x_1}{1-zx_1} \frac{(1+z^2)}{\sqrt{1+z^2}} \right) = \frac{(1-\nu_b)}{2} \left( 1 - \frac{x_1}{1-zx_1} \sqrt{1+z^2} \right), \end{aligned}$$

which proves the first claim of the Theorem 1. The second claim follows from  $q^*(\nu, \kappa)$ , plus equations (6) and (11). The third claim is obtained combining  $q^*(\nu, \kappa)$ , plus equations (B1) and (11).

*Q.E.D.*

### Proof of Propositions 3 and 4

We first show the effect on the seller's price policy. Making use of (B7) with (B8), we obtain

$$\frac{dx^*(\nu, \kappa)}{d\nu} = \frac{dx^*(\nu, \kappa)}{dz} \frac{dz}{d\nu}, \quad (\text{B11})$$

with

$$\frac{dz}{d\nu} = \frac{\partial z}{\partial x_1} \frac{\partial x_1(\nu, \kappa)}{\partial \nu} + \frac{\partial z}{\partial x_2} \frac{\partial x_2(\nu, \kappa)}{\partial \nu}. \quad (\text{B12})$$

We notice that

$$\frac{dx^*(\nu, \kappa)}{dz} = - \left( 1 - \frac{z}{\sqrt{1+z^2}} \right) \leq 0, \quad (\text{B13})$$

and

$$\frac{dz}{d\nu} = \frac{\left( (1+x_2^2) x_2 \frac{\kappa}{1+\lambda l} + (1+x_1^2) x_1 \frac{2-\kappa}{5+\lambda l} \right)}{\sqrt{\gamma} (x_1+x_2)^2} \geq 0. \quad (\text{B14})$$

Substituting (B13) and (B14) into (B11), we conclude that

$$\frac{dx^*(\nu, \kappa)}{d\nu} \leq 0. \quad (\text{B15})$$

Next, we show the effect on the buyer's demand policy

$$\frac{\partial q^*(\nu, 0)}{\partial \nu} = \frac{\partial q^*(\nu, 0)}{\partial x^*} \frac{dx^*(\nu, \kappa)}{d\nu}, \quad (\text{B16})$$

with

$$\frac{\partial q^*(\nu, 0)}{\partial x^*} = -\frac{1 + 2x_2x^* - (x^*)^2}{x_2(1 + (x^*)^2)^2}. \quad (\text{B17})$$

The positive root  $r_2$  of the concave quadratic function  $f(y) = 1 + 2x_2y - y^2$  is given by

$$r_2 = x_2 + \sqrt{1 + x_2^2} \geq x_2 \geq x^*, \quad (\text{B18})$$

and hence  $f(x^*) \geq 0$ . Therefore,  $\frac{\partial q^*(\nu, 0)}{\partial x^*} \leq 0$ . Combining this with (B15) and substituting into (B16), we conclude that

$$\frac{dq^*(\nu, 0)}{d\nu} \geq 0. \quad (\text{B19})$$

Finally, the effect on the seller's income, analogous to the above

$$\frac{\partial (x^*q^*(\nu, 0))}{\partial \nu} = \frac{\partial (x^*q^*(\nu, 0))}{\partial x^*} \frac{dx^*(\nu, \kappa)}{d\nu}, \quad (\text{B20})$$

with

$$\begin{aligned} \frac{\partial (x^* q^*(\nu, 0))}{\partial x^*} &= -\frac{(1 + 2x_2 x^* - (x^*)^2) x^*}{x_2 (1 + (x^*)^2)^2} \\ &\quad + \frac{x_2 - x^*}{x_2 (1 + (x^*)^2)}. \end{aligned} \tag{B21}$$

Collecting terms, we obtain

$$\frac{\partial (x^* q^*(\nu, 0))}{\partial x^*} = -\frac{\left(-1 + \frac{2}{x_2} x^* + (x^*)^2\right)}{(1 + (x^*)^2)^2}. \tag{B22}$$

The positive root of the convex quadratic function  $g(y) = -1 + \frac{2}{x_2} y + y^2$  is given by

$$r_2 = \sqrt{1 + \frac{1}{x_2^2} - \frac{1}{x_2}} \geq x^*, \tag{B23}$$

and hence  $g(x^*) \leq 0$ . Therefore,  $\frac{\partial (x^* q^*(\nu, 0))}{\partial x^*} \geq 0$ . Combining this with (B15) and substituting into (B20), we conclude that

$$\frac{d(x^* q^*(\nu, 0))}{d\nu} \leq 0. \tag{B24}$$

*Q.E.D.*

### Proof of Proposition 5

Rewriting the seller's expected utility in the form

$$-2\gamma U_s = (b - qx + \nu(1 - \kappa)\chi)^2 + (q - \nu(1 - \kappa))^2, \tag{B25}$$

with the dimensionless loading of the fund  $\chi$  defined by

$$\chi = \sqrt{\gamma} \Delta C. \quad (\text{B26})$$

Differentiating (B25) and making use of the envelope property, we obtain

$$\gamma \frac{dU_s^*}{d\nu} = \theta(\nu, \kappa) \left( \frac{\partial q}{\partial \nu} \left( 2x + \frac{q}{q_x} \right) - (1 - \kappa) \left( \chi + x + \frac{q}{q_x} \right) \right), \quad (\text{B27})$$

where

$$\theta(\nu, \kappa) = (b - qx + \nu(1 - \kappa)\chi) \geq 0. \quad (\text{B28})$$

Note that  $q_x = \frac{\partial q}{\partial x} \leq 0$  and  $\frac{\partial q}{\partial \nu} \leq 0$  are given by

$$\frac{\partial q}{\partial x} = - \frac{(2x^*(1 - \nu\kappa) + (\alpha + \nu\kappa\chi)(1 - (x^*)^2))}{(1 + (x^*)^2)^2}, \quad (\text{B29})$$

and

$$\frac{\partial q}{\partial \nu} = -\kappa \frac{1 + \kappa\chi}{1 + (x^*)^2}, \quad (\text{B30})$$

respectively. Collecting the terms, we obtain a condition that  $\frac{dU_s^*}{d\nu} \geq 0$ , *iff*

$$\Theta(\nu, \kappa) \geq \Gamma(\nu, \kappa), \quad (\text{B31})$$

with

$$\Theta(\nu, \kappa) = \frac{1 - \nu\kappa - x^*(\alpha + \nu\kappa\chi)}{2x^*(1 - \nu\kappa) + (1 - (x^*)^2)}, \quad (\text{B32})$$

and

$$\Gamma(\nu, \kappa) = \frac{(1 - \kappa)(\chi + x) + \frac{2\kappa x^*}{1+(x^*)^2}(1 + \chi x^*)}{((1 - \kappa)(1 - (x^*)^2) + \kappa(1 + \chi x^*))}. \quad (\text{B33})$$

In the limit  $k = 0$ , the above condition is simplified to

$$\frac{(1 - \alpha x^*)(1 - (x^*)^2)}{2x^* + \alpha(1 - (x^*)^2)} \geq x, \quad (\text{B34})$$

which is satisfied if  $h(x^*) \leq 0$  with

$$h(y) = -\frac{1}{3} + \frac{2}{3}\alpha y + y^2. \quad (\text{B35})$$

Analyzing the roots of the quadratic function  $h(y)$  analogous to how this was done before, we conclude that the condition  $h(x^*) \leq 0$  can always be satisfied.

*Q.E.D.*

### **Proof of Proposition 6**

Making use of (B7) and (B8), we obtain

$$x^* - x_1 = \frac{\sqrt{1 + x_1^2}}{\sqrt{1 + x_1^2} + \sqrt{1 + x_2^2}}(x_2 - x_1). \quad (\text{B36})$$

Differentiation yields

$$\frac{\partial(x^* - x_1)}{\partial\nu} = \frac{\sqrt{1 + x_1^2}}{\sqrt{1 + x_1^2} + \sqrt{1 + x_2^2}} \frac{1}{\sqrt{\gamma}} \Omega(\nu, \kappa), \quad (\text{B37})$$

with

$$\Omega(\nu, \kappa) = \frac{2 - \kappa}{5 + \lambda \bar{l}} (1 - \varphi) - \frac{\kappa}{1 + \lambda \bar{l}} (1 - \psi), \quad (\text{B38})$$

and

$$\varphi(\nu, \kappa) = \frac{\sqrt{1 + x_2^2}}{\sqrt{1 + x_1^2} + \sqrt{1 + x_2^2}} \frac{x_1}{1 + x_1^2} < 1, \quad (\text{B39})$$

$$\psi(\nu, \kappa) = \frac{\sqrt{1 + x_2^2}}{\sqrt{1 + x_1^2} + \sqrt{1 + x_2^2}} \frac{x_2}{1 + x_2^2} < 1. \quad (\text{B40})$$

Note that  $\Omega(\nu, \kappa)$  is a smooth function in both arguments. We have  $\Omega(\nu, 0) =$

$\frac{2}{5 + \lambda \bar{l}} (1 - \varphi(\nu, 0)) \geq 0$ , and therefore

$$\frac{\partial (x^*(\nu, 0) - x_1(\nu, 0))}{\partial \nu} \geq 0. \quad (\text{B41})$$

From the smoothness argument formulated above, it follows that

$$\frac{\partial (x^*(\nu, \kappa) - x_1(\nu, \kappa))}{\partial \nu} \geq 0, \quad (\text{B42})$$

holds in the finite segment  $\kappa \in [0; \kappa_{\max}]$  with  $\kappa_{\max} > 0$ .

*Q.E.D.*

## 5.2 Figures and Numerical Results

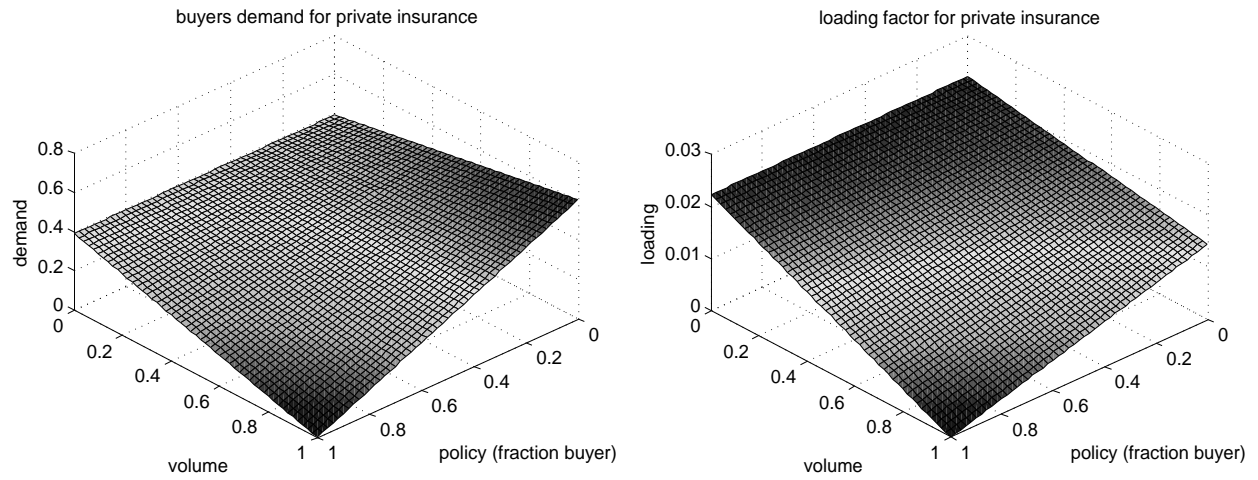
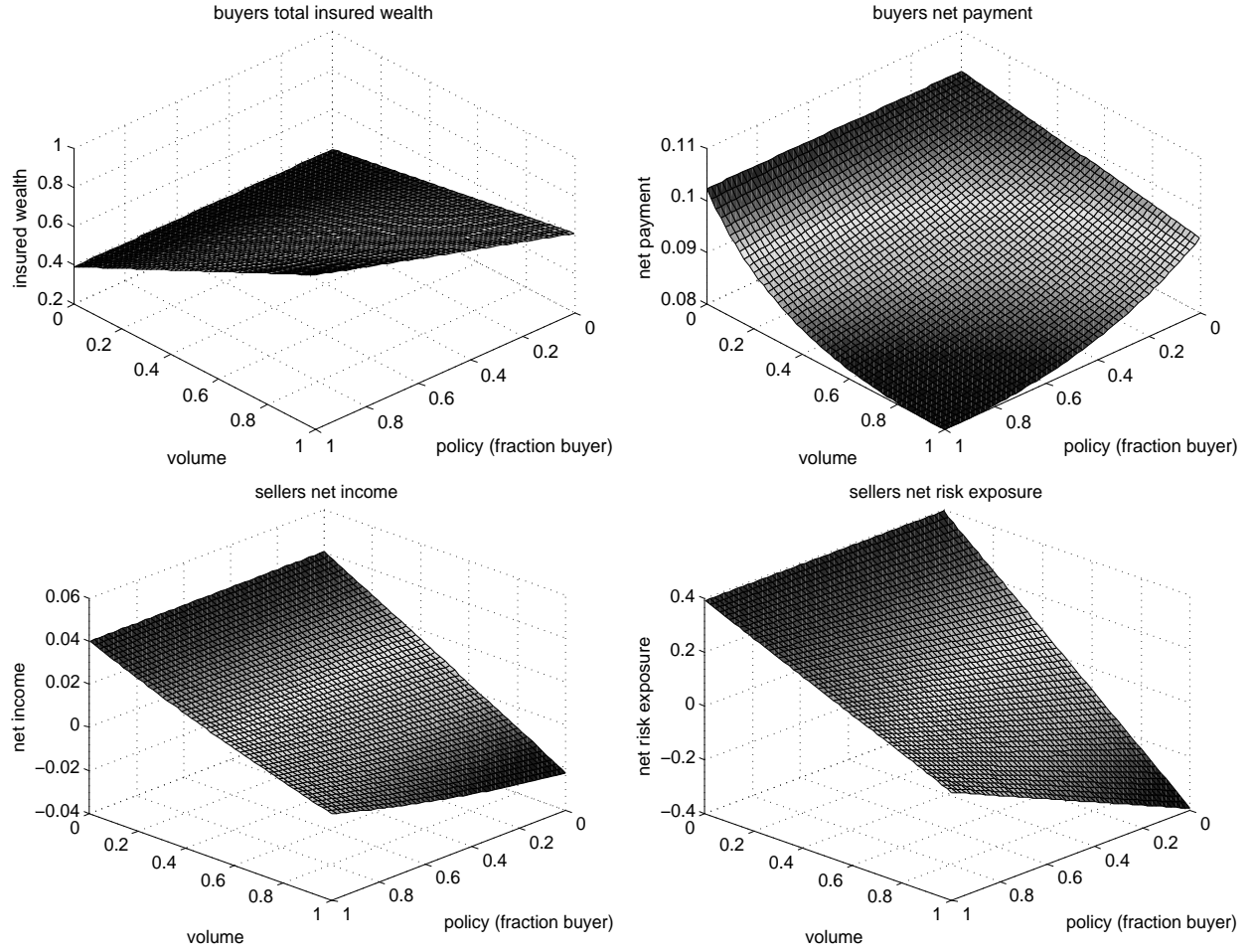


Figure 1: **Equilibrium Demand and Price.** The figures show properties of the equilibrium demand and loading in the private insurance market, depending on the size of the disaster relief fund  $\nu$  – volume, and the fraction of the fund paid for and received by the buyer  $\kappa$  – policy (fraction buyer). Exogenous parameters are: 8% unconditional expected loss, 20% unconditional standard deviation, buyer’s risk tolerance = 2, seller’s risk tolerance = 2.





**Figure 2: Net Position of Buyer and Seller.** The figures show the properties of the buyer's and seller's net position, depending on the size of the disaster relief fund  $\nu$  – volume, and the fraction of the fund paid for and received by the buyer  $\kappa$  – policy (fraction buyer). The top left graph shows the buyer's total insured wealth,  $q + \nu_b$ , and the top right graph the total price paid per unit of insured wealth,  $(q(\bar{l}\lambda + \Delta p) + C_b\nu)/(q + \nu_b)$ . The lower left graph shows the seller's net income,  $(q(\bar{l}\lambda + \Delta p) - C_s\nu)$ , the lower right graph shows the net cat risk exposure given by  $q - \nu_s$ . Exogenous parameters are: 8% unconditional expected loss, 20% unconditional standard deviation, buyer's risk tolerance = 2, seller's risk tolerance = 2.

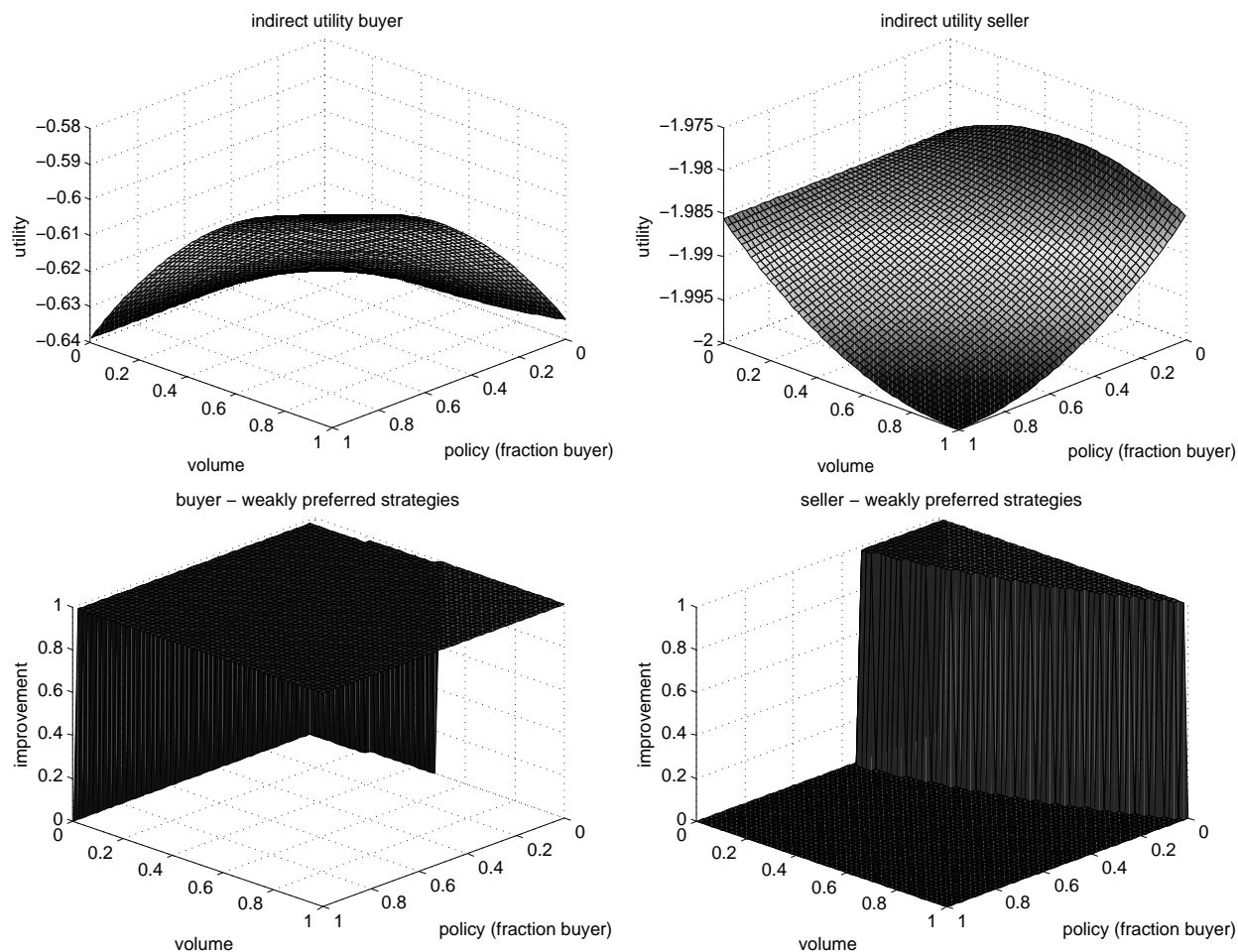
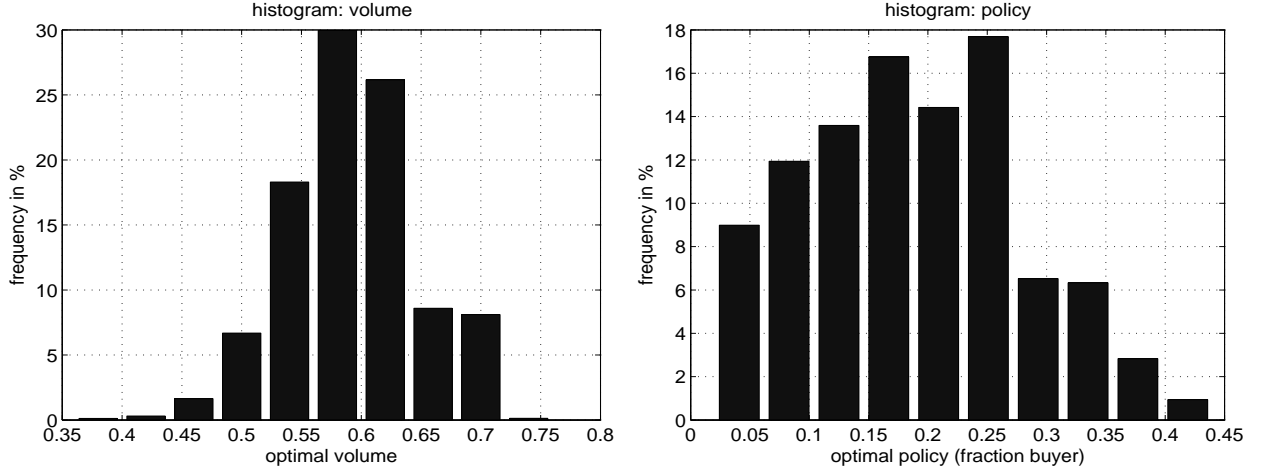


Figure 3: **Expected Utilities.** The figures show the properties of buyer's and seller's indirect utilities, depending on the size of the disaster relief fund  $\nu$  – volume, and the fraction of the fund paid for and received by the buyer  $\kappa$  – policy (fraction buyer). Upper graphs show the levels of indirect utilities, lower graphs show the weakly preferred strategies. Exogenous parameters are: 8% unconditional expected loss, 20% unconditional standard deviation, buyer's risk tolerance = 2, seller's risk tolerance = 2.

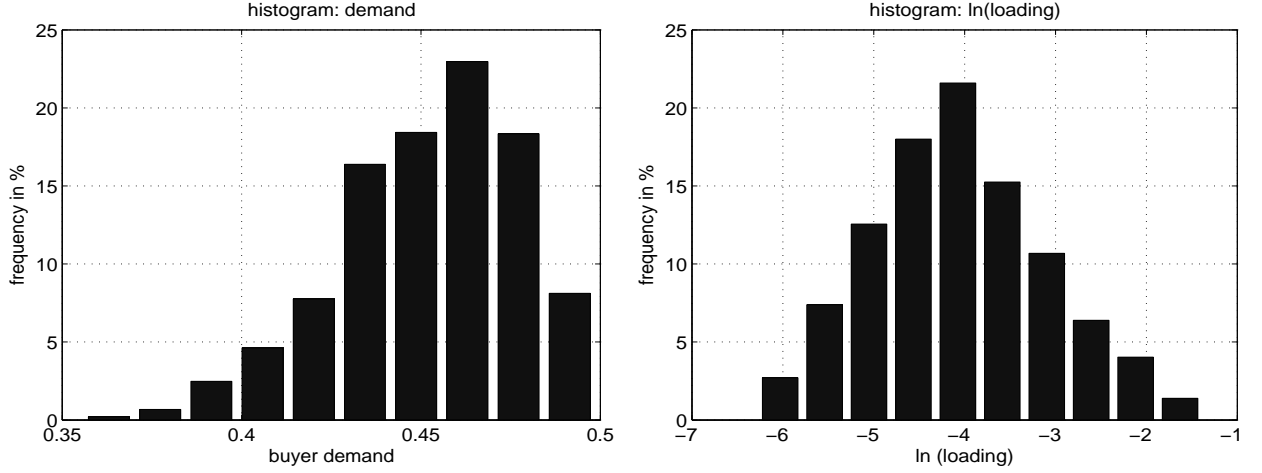


$$volume = 0.5089 + 0.0529 * AB - 0.0282 * AS + 0.0573 * EXPL + 0.1843 * SDEV \quad (C1)$$

$$policy = 0.0742 + 0.1258 * AB - 0.0705 * AS + 0.1147 * EXPL - 0.0276 * SDEV \quad (C2)$$

$$volume = 0.5328 + 0.3551 * policy \quad (C3)$$

Figure 4: **Disaster Relief Policies.** The graphs show histograms of the optimal disaster fund policies. The objective is to maximize buyer utility, conditional on being a weakly preferred strategy by the seller. The left graph shows the size of the fund - optimal volume, the right graph shows the fraction of the fund paid for and received by the buyer - optimal policy (fraction buyer). Results are based on monte-carlo simulations generating uniform distributions for the following sets of exogenous parameters: Unconditional expected loss - EXPL - between 2% and 14%, unconditional standard deviation - SDEV - between 10% and 30%, buyer's risk tolerance - AB - between 1 and 3, seller's risk tolerance - AS - between 1 and 3. The equations show the corresponding cross-sectional results of a regression analysis.



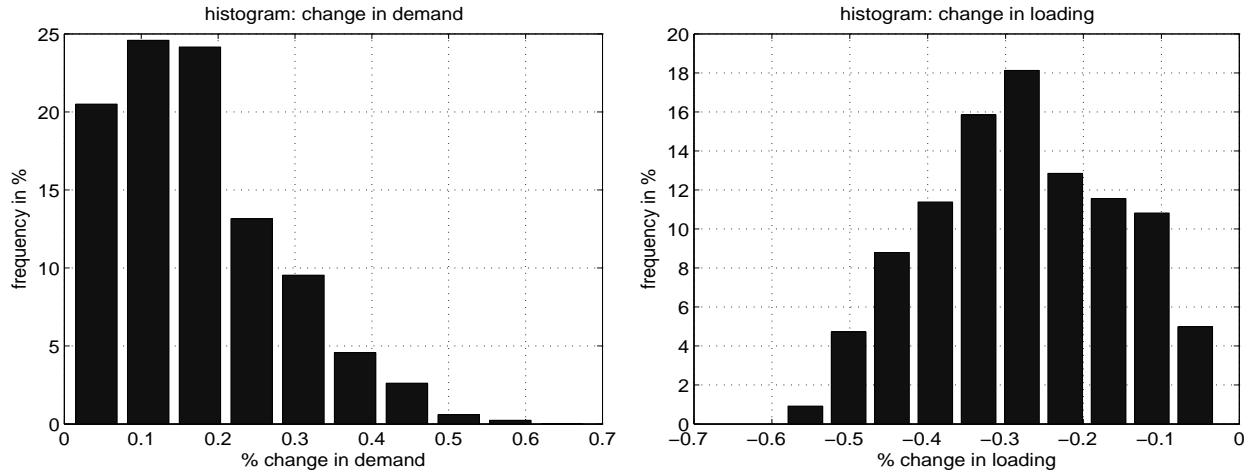
$$demand = 0.476 - 0.034 * AB + 0.020 * AS - 0.031 * EXPL + 0.035 * SDEV \quad (C4)$$

$$\ln(loading) = -3.634 - 1.239 * AB + 0.068 * AS - 1.393 * EXPL + 10.180 * SDEV \quad (C5)$$

$$demand = 0.448 + 0.108 * volume - 0.322 * policy \quad (C6)$$

$$\ln(loading) = -3.341 + 1.344 * volume - 8.066 * policy \quad (C7)$$

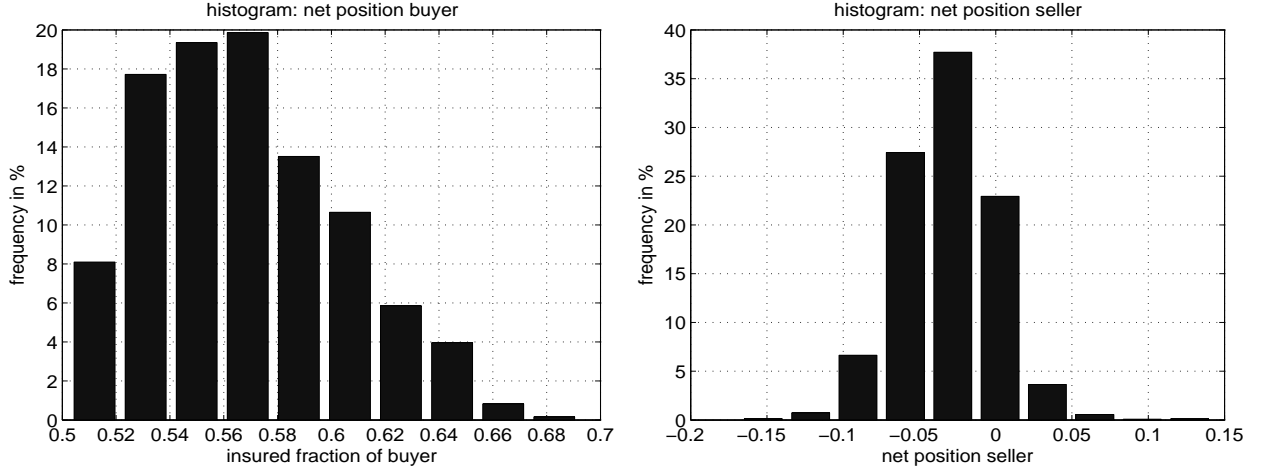
Figure 5: **Demand and Loading.** The graphs show histograms of the buyer's optimal demand - buyer demand, and the loading in the primary insurance market -  $\ln(\text{loading})$ , associated with the optimal disaster fund policies. Results are based on monte-carlo simulations generating uniform distributions for the following sets of exogenous parameters: Unconditional expected loss - EXPL - between 2% and 14%, unconditional standard deviation - SDEV - between 10% and 30%, buyer's risk tolerance - AB - between 1 and 3, seller's risk tolerance - AS - between 1 and 3. The equations show the corresponding cross-sectional results of a regression analysis.



$$\% \text{ change in demand} = -0.506 + 0.875 * \text{volume} + 0.819 * \text{policy} \quad (\text{C8})$$

$$\% \text{ change in loading} = 0.320 - 0.650 * \text{volume} - 1.112 * \text{policy} \quad (\text{C9})$$

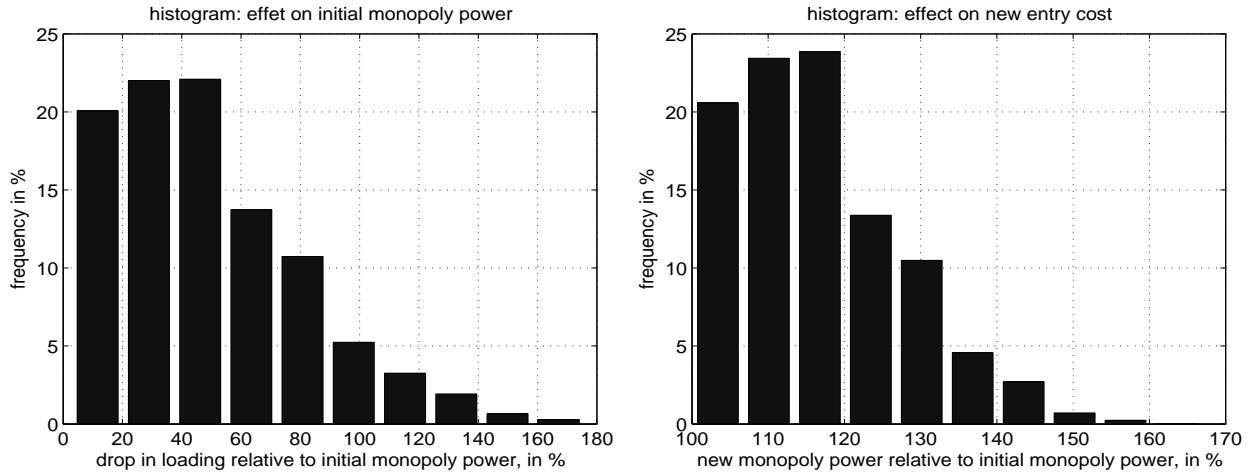
**Figure 6: Demand and Loading relative to No Disaster Fund.** The graphs show histograms of the percentage change in demand and loading associated with optimal disaster fund policies, relative to not having access to a disaster fund. Results are based on monte-carlo simulations generating uniform distributions for the following sets of exogenous parameters: Unconditional expected loss - EXPL - between 2% and 14%, unconditional standard deviation - SDEV - between 10% and 30%, buyer's risk tolerance - AB - between 1 and 3, seller's risk tolerance - AS - between 1 and 3. The equations show the corresponding cross-sectional results of a regression analysis.



$$net\ buyer = 0.511 + 0.051 * AB - 0.030 * AS + 0.045 * EXPL + 0.053 * SDEV \quad (C10)$$

$$net\ seller = 0.002 - 0.002 * AB - 0.002 * AS - 0.013 * EXPL - 0.132 * SDEV \quad (C11)$$

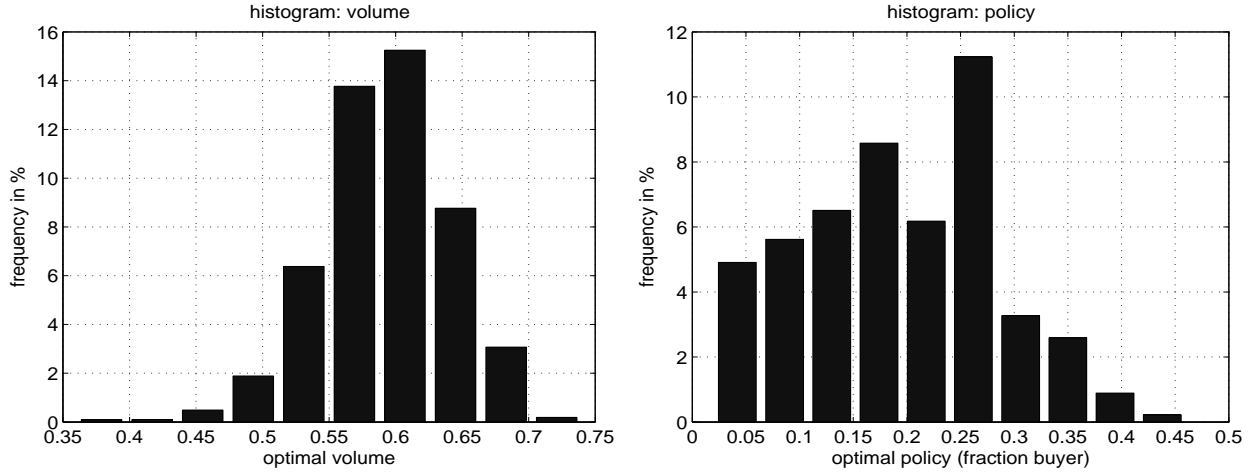
**Figure 7: Net Position of Buyer and Seller.** The graphs show histograms of the net position of buyer and seller associated with optimal disaster fund policies. The net position of the buyer is the demand in the private insurance market plus the fraction of disaster payment received. The net position of the seller is the amount sold in the private market net the fraction of the disaster payment received. Results are based on monte-carlo simulations generating uniform distributions for the following sets of exogenous parameters: Unconditional expected loss - *EXPL* - between 2% and 14%, unconditional standard deviation - *SDEV* - between 10% and 30%, buyer's risk tolerance - *AB* - between 1 and 3, seller's risk tolerance - *AS* - between 1 and 3. The equations show the corresponding cross-sectional results of a regression analysis.



$$mon\ power = 8.474 + 44.310 * AB - 27.270 * AS + 37.703 * EXPL + 17.912 * SDEV \quad (C12)$$

$$new\ entry\ cost = -96.506 + 148.396 * volume + 300.650 * policy \quad (C13)$$

Figure 8: **Effects on Competition.** The graphs show the effects on monopoly power, measured in terms of two diagnostics. The left graph shows how the drop in loading due to the existence of the disaster fund compares to the initial monopoly power without a disaster fund. The right graph shows the distance of loading relative to smallest accepted loading by the seller, with versus without the existence of the disaster fund. Results are based on monte-carlo simulations generating uniform distributions for the following sets of exogenous parameters: Unconditional expected loss - EXPL - between 2% and 14%, unconditional standard deviation - SDEV - between 10% and 30%, buyer's risk tolerance - AB - between 1 and 3, seller's risk tolerance - AS - between 1 and 3. The equations show the corresponding cross-sectional results of a regression analysis.

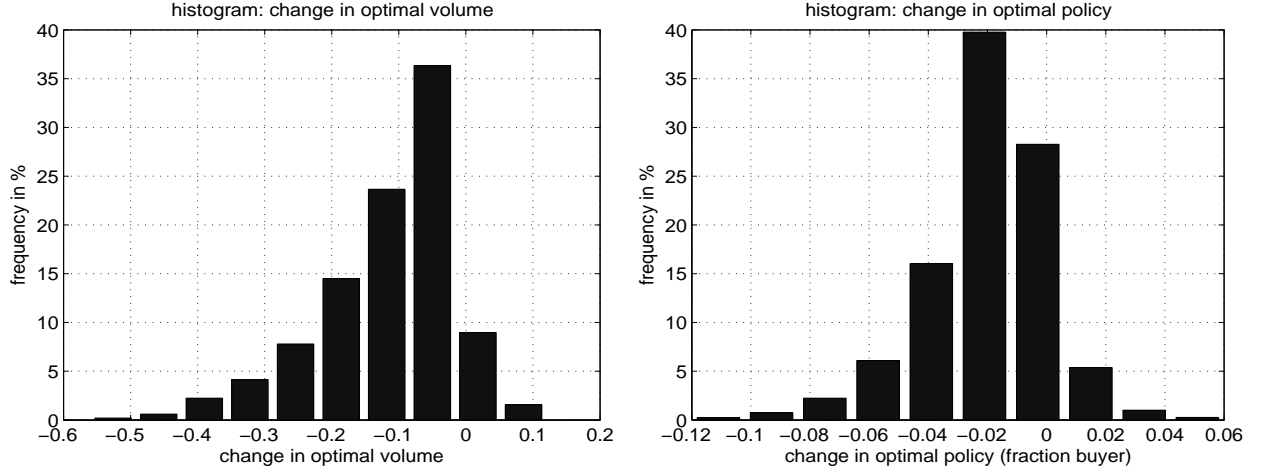


$$volume = 0.5723 + 0.0522 * AB - 0.0298 * AS - 0.1200 * LAMBDA \quad (C14)$$

$$policy = 0.0658 + 0.1304 * AB - 0.0705 * AS + 0.0179 * LAMBDA \quad (C15)$$

**Figure 9: Disaster Relief Policies - Effect of Lambda.** The objective is to maximize buyer utility, conditional on being a weakly preferred strategy by the seller. The left graph shows the size of the fund - optimal volume, the right graph shows the fraction of the fund paid for and received by the buyer - optimal policy (fraction buyer). The unconditional expected loss is given by 8%, arrival intensities - LAMBDA - are drawn from a uniform distribution between .08 and .30. Further exogenous parameters are the buyer's risk tolerance - AB - between 1 and 3, seller's risk tolerance - AS - between 1 and 3. Based on the inferred conditional expected losses between 27% and 100%, the resulting unconditional standard deviation ranges between 12% and 27%.





$$\text{change in volume} = -0.2131 - 0.0496 * AB - 0.0307 * AS + 1.2671 * SDEV \quad (C16)$$

$$\text{change in policy} = -0.0259 - 0.0089 * AB - 0.0078 * AS + 0.2033 * SDEV \quad (C17)$$

Figure 10: **Disaster Relief Policies - Effect of Small Loading.** These results are obtained from a perturbation exercise, in which the disaster fund is not only funded at expected losses, but also a small loading factor of .001. The left graph shows the effect on the size of the disaster fund - optimal volume, the right graph shows the effect on the fraction of the fund paid for and received by the buyer - optimal policy (fraction buyer). Results are based on monte-carlo simulations generating uniform distributions for the following sets of exogenous parameters: Unconditional expected loss - EXPL - between 2% and 14%, unconditional standard deviation - SDEV - between 10% and 30%, buyer's risk tolerance - AB - between 1 and 3, seller's risk tolerance - AS - between 1 and 3. The equations show the corresponding cross-sectional results of a regression analysis.

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