Vulnerable Banks*

Robin Greenwood
Harvard University and NBER

Augustin Landier
University of Toulouse

David Thesmar
HEC Paris and CEPR

First draft: October 2011
Current draft: June 2012

Abstract

When a bank experiences a negative shock to its equity, one way to return to target leverage is to sell assets. If asset sales occur at depressed prices, then one bank's sales may impact other banks with common exposures, resulting in contagion. We propose a simple framework that accounts for how this effect adds up across the banking sector. The framework explains how the distribution of bank leverage and risk exposures contributes to a form of systemic risk. We compute bank exposures to system-wide deleveraging, as well as the spillover of a single bank's deleveraging onto other banks. We use the model to evaluate a variety of crisis interventions, such as mergers of good and bad banks and equity injections. In our model, "microprudential" interventions, which target the solvency of individual banks, tend to be much less effective than "macroprudential" policies which aim to minimize spillovers across firms. We apply the framework to European banks vulnerable to sovereign risk in 2010 and 2011, and US banks between 2001 and 2010.

* We are grateful to Laurent Clerc, Linda Goldberg, Sam Hanson, Thomas Philippon, Carmen Reinhart, Andrei Shleifer, Jeremy Stein, Adi Sunderam, and seminar participants at the Federal Reserve Bank of New York, TSE-Banque de France conference in Paris, Harvard, Sciences-Po and Zûrich for their input.

I. Introduction

Financial stress experienced by financial institutions can contaminate others and spiral into a shock that threatens the entire financial system: this is systemic risk. The measurement of systemic risk has been high on financial regulators' priority list since the 2008 collapse of Lehman Brothers, which triggered widespread financial distress among large US financial institutions. The recent sovereign debt crisis and corresponding concerns about the solvency of European banks system have only made the need to measure system-wide stability more acute.

There are at least two ways in which linkages between financial institutions can create contagion. The first relies on contractual dependencies: when two banks write a financial contract such as a swap agreement, a negative shock to one bank can transmit to the other party as soon as one of the banks is unable to honor the contract (e.g., Allen and Babus 2009, Gorton and Metrick 2010, Giglio, 2011). Such bilateral links can create a channel for the propagation of financial distress, because the creditor bank may in turn default on its obligations to third parties (Duffie 2010, Kallestrup et al., 2011).[1]

A second type of linkage comes from fire-sale spillovers: when a bank is forced to sell illiquid assets, it depresses prices, which in turn can prompt financial distress at other banks that hold the same assets. Liquidation spirals of this sort have been explored in an extensive theoretical literature.[2] In a system of greater complexity, such spirals are believed by numerous economists and policy-makers to have become an important contributor to systemic risk over recent years.

This paper proposes a parsimonious model of this fire-sales channel of systemic risk that can be easily estimated with available data. The model takes as given (1) the asset holdings of each

[1] Kalemli-Ozcan(2011) investigate the impact of inter-bank linkages on business cycle synchronization.

[2] See for instance Shleifer and Vishny (1992, 2010), Gromb and Vayanos (2007), Brunnermeier and Pedersen (2009), Allen, Babus, and Carletti (2011), Wagner (2011).

financial institution, (2) an adjustment rule applied by institutions when they are hit by adverse shocks and (3) the liquidity of these assets on the secondary market (i.e., the ability of banks to sell these assets quickly with little discount). In our model, all three ingredients are exogenous, and thus we make no claims as to the optimality of bank behavior ex ante, appealing mostly to prior work to show that our assumptions are consistent with observed bank behavior. Nevertheless, combining these ingredients and adding up spillovers across banks yields a set of rich insights on how bank deleveraging may unfold following an initial shock.

One appealing feature of the model is that we can distinguish between a bank's *contribution* to financial sector fragility (which we call its "systemicness"), and a bank's *vulnerabilty* to deleveraging by other banks. For example, a small but highly levered bank may be quite vulnerable to financial sector deleveraging, but is unlikely to be systemic because its own asset sales will be modest in size, thus not triggering much in the way of spillovers.

The model delivers a number of intuitive properties concerning how the distribution of leverage and risk exposures across banks determines systemic risk. For instance, a negative return shock experienced by an asset held by relatively levered institutions has a larger aggregate impact than if the same asset was held by the less levered institutions. More generally, we show that the banking system is less stable with respect to net worth shocks when asset classes that are large in dollar terms are also held by the most levered banks. Assets that are both volatile and illiquid should be dispersed across banks, since shocks generate less price impact in a deleveraging cycle. In contrast, if illiquid assets have low price volatility, then it is better to isolate these assets in separate banks, so that they are not contaminated by other assets, which in turn are subject to larger shocks.

Though stylized, our framework can be used to simulate the outcome of various policies. To be clear, our model takes as given a given bank's behavior during deleveraging, and therefore there is no presumption that capital structure is optimal in an ex ante sense. Despite this limitation, the model is useful for understanding how spillovers play once a deleveraging cycle is entered, at which point banks are up against a leverage constraint. For example, we can evaluate the overall impact of the failure of a given bank on each other member of the financial system. Moreover, we can use the model to simulate the outcome of various policy interventions, such as a forced bank merger between vulnerable banks. Such a policy may affect systemic risk because it redistributes existing assets held by large intermediaries to other intermediaries, which may have different exposures to shocks, different sizes, or differ in their leverage ratios. We also explore the gains from equity that is optimally distributed around the most systemic banks, i.e., debt-equity swaps targeted to minimize the aggregate impact of deleveraging. In the context of this exercise, it should not be surprising that "microprudential" stabilization policies, which aim to fix insolvency at individual banks, tend to be inferior to "macroprudential" policies, which target the cross-bank spillovers directly. This is because optimal injections should not target banks that are directly exposed to shocks, but banks whose liquidations have the largest impact on other banks.

We apply the model in two empirical settings. First, we calibrate the model on European banks during the 2010-2011 sovereign debt crisis. For a large set of these banks, we have measures of sovereign bond exposures derived from the European Banking Authority's (EBA) July 2011 stress tests. We then use these exposures to estimate the potential spillovers which could occur during bank deleveraging precipitated by sovereign downgrades or defaults. Our analysis uncovers some interesting and worrisome linkages. For example, only a few banks have direct exposure to a Greek sovereign default. However, a much larger group of banks are indirectly exposed, because they hold assets that are held by banks that are directly exposed to Greek sovereign bonds. In the extreme event of a bank failure of a directly exposed bank, the indirectly exposed banks would suffer portfolio losses as well. Using the risk exposures as inputs, we document a correlation between our estimates of vulnerability and equity drawdowns experienced by European banks in 2010 and 2011. We then use our data to evaluate various policy interventions. We find that size caps, or forced mergers among the most exposed entities, do not reduce systemic risk very much. However, we show that modest equity injections, if distributed appropriately between the most systemic banks, can cut the vulnerability of the banking sector to deleveraging by more than half.

We then apply our framework to US financial institutions between 2007 and 2009. We perform this second set of empirical analysis to show how the model can be useful even when we do not have direct measures of banks' financial holdings. Much like Adrian and Brunnermeier (2010) and Acharya et al (2010), we now estimate bank's exposures using their equity returns. We then use these exposures, which we obtain on a rolling basis, to calculate the financial sector's exposure to deleveraging. We show that the model captures the pre-Lehman build-up in financial instability among banks, as well as helping explain bank stock returns during the financial crisis. These results obtain in spite of coarse estimates of each institution's holdings: a regulator in possession of detailed information on bank holdings could in principle do better.

The remainder of the paper is organized as follows. We first develop the model, solve it, and build intuition for financial sector stability under different configurations of leverage and risk exposure across the banks. In Section III, we explain how our approach fits, and contributes to, a growing literature on systemic risk. In Section IV, we use commercial bank exposures provided by the EBA's July 2011 stress tests to compute the vulnerability of European banks to sovereign defaults. In Section V, we test the basic framework on US financial institutions, where we rely on historical equity returns to back out exposures. The final section concludes.

II. A Hydraulic Model of Bank Deleveraging

We start by describing the framework. We then use it to derive easy-to-implement measures of systemic risk, at the bank and aggregate levels.

A. Setup

There are two periods t=1,2, and N banks. Each bank n is financed with a mix of debt d_{nt} and equity e_{nt}. A_t is the $N\times 1$ vector of banks' assets so that each term $a_{nt} = e_{nt} + d_{nt}$ at date t. B is the $N\times N$ diagonal matrix of leverage ratios, such that each diagonal term $b_n = d_{nt}/e_{nt}$.

Each bank n holds a portfolio of K assets: m_{nk} is the weight of asset k in bank n's portfolio. M is the $N\times K$ matrix of these weights. In each period, the vector of banks' unlevered returns is given by:

$$R_t = MF_t, \tag{1}$$

where the $K\times 1$ vector F_t denotes asset returns.

Assumption 1: Asset trading in response to bank return shock

Suppose banks receive an exogenous shock R_1 to their assets at t=1. Because banks are levered, these shocks move banks away from their current leverage. We assume that banks respond by scaling up or down their total assets in period 2 so as to maintain a fixed target leverage. Such leverage-targeting is in line with empirical evidence in Adrian and Shin (2010), who show that banks manage leverage to offset shocks to asset values.[3] Adrian and Shin's evidence implicitly suggests

[3] They provide evidence that commercial banks target a constant leverage ratio, while investment banks have procyclical leverage, which means that their leverage adjustments more than offset the changes in leverage induced by shocks to asset values.

that banks do not raise equity in response to a negative shock.[4] However, the analysis that follows does not change much if we instead assume that banks return to target leverage using a combination of asset sales and equity issues in fixed proportion.

If banks target leverage ratios given by the matrix B, then the $N\times 1$ vector of dollar net asset increases is simply A_1BR_1. When $R_1<0$, banks with negative asset returns have to sell assets to deleverage. When $R_1>0$, banks with positive returns need to borrow more to preserve leverage. The intuition of this formula is simple: suppose a bank with equity of 1 and debt of 9 experiences a 10% return on its assets, bringing its equity to 2. The bank will have to borrow an additional 9 and buy assets to return to the prior leverage of 9-to-1.[5]

If some elements of R_1 are negative and very large, then it is possible that the A_1BR_1 vector may have some negative elements that are bigger in absolute value than banks' assets. This happens if the initial shock is large enough to wipe out all of the equity of the bank, in which case no amount of asset sales will return the bank to target leverage. To prevent this from happening, we can modify the vector of net asset increases by replacing it by $A_1.\max(BR_1,-1-R_1)$, where "max" is the point-wise maximum matrix operator, defined by $\max(X,Y)=(\max(X_n,Y_n))$. In Section IV we use this modified formula, because the shocks we consider in Europe are large enough to wipe out some banks. But to simplify the exposition that follows, for now we keep the simpler linear formula.

Assumption 2: Target exposures remain fixed in percentage terms

Second, we must describe how banks sell individual assets to return to target leverage. We make the simplest assumption that banks sell assets so as to keep their exposures constant in a

[4] In situations where debt overhang is severe, issuing equity dilutes existing shareholders as the gains from the reduction in risk accrue disproportionately to debt holders.
[5] Essentially we are treating banks as similar to leveraged exchange traded funds (ETFs), which must readjust to their target leverage at the close of trading each day. See Greenlaw, Hatzius, Kashyap, and Shin (2008) and Adrian and Shin (2009) for further discussion of this point and related evidence.

proportional sense. More formally, this means that they sell assets in such a way as to hold the M matrix constant between dates 1 and 2. This assumption has been widely used in the mutual fund literature: investor flows have been shown to cause mutual funds to scale up and down their portfolios, but otherwise keep their portfolio weights constant (see Coval and Stafford, 2007, Greenwood and Thesmar, 2011, and Lou, 2011). In practice, banks may first sell their liquid assets. The constant portfolio assumption simplifies the algebra and the exposition of the model, but it is easy to modify the framework to account for more sophisticated liquidation rules, which we develop in Section IV *F*.

Let ϕ be the $K\times 1$ vector of net asset (dollar) purchases by all banks in period 2. If banks keep their portfolios constant, then:

$$\phi = M' A_1 B R_1. \tag{2}$$

To see the intuition, consider a bank with holdings of 10 percent cash, 20 percent in stocks and 70 percent in mortgage backed securities. If the bank scales down its portfolio by ten units, it will sell 2 units of stocks, 7 units of mortgage backed securities, and take its cash down by 1. Equation (2) describes this in matrix form, summed over all banks: for each bank n facing a shock R_{1n}, total net asset increase will be given by $a_n b_n R_{1n}$. Net purchases of asset k by the bank will be proportional to its holdings of asset k, i.e., $m_{nk} a_n b_n R_{1n}$. Equation (2) sums this expression across all n banks.

Assumption 3: Fire sales generate price impact

Third, we assume that asset sales in the second period ϕ generate price impact according to a linear model:

$$F_2 = L\phi, \tag{3}$$

where L is a matrix of price impact ratios, expressed in units of returns per dollar of net purchase.[6] We start by assuming that L is diagonal, meaning that fire sales in one asset do not directly affect prices in other assets.[7]

We combine equations (1), (2) and (3) to calculate the effect of bank unlevered asset returns in $t=1$ on returns in $t=2$:

$$R_2 = MF_2 = ML\phi = (MLM'BA_1)R_1. \tag{4}$$

In principle, one can iterate for multiple rounds of deleveraging, and thus incorporate more periods into the analysis, following an initial shock through further multiplying by the transition matrix $MLM'BA_1$. For simplicity, we restrict our attention to the first round.

B. *Measuring Aggregate Exposures to Deleveraging ("Aggregate Vulnerability")*

We start with a negative shock $-F_1=(-f_1,\ldots,-f_n)$ to asset returns: this translates into dollar shocks to banks' assets given by A_1MF_1. The aggregate *direct* effect on all bank assets the quantity is then $1'A_1MF_1$, where 1 is the $N\times 1$ vector of ones. This direct effect does not involve any contagion between banks, it is simply the change in asset value.

Following equation (4), To compute the dollar effect of shock F_1 on bank assets through fire sales, we pre-multiply $MLM'BA_1MF_1$ by $1'A_1$. We normalize this by total bank equity pre-delevering E_1 and define "aggregate vulnerability" as:

$$AV = \frac{1'A_1MLM'BA_1MF_1}{E_1}. \tag{5}$$

[6] For instance, Pulvino (1998) estimates the discount associated with fire sales of commercial aircraft by distressed airlines. In equity markets, Coval and Stafford (2007) estimate the L coefficient using forced purchases and sales of stock by mutual funds (see also Ellul et al, 2011, and Jotikasthira et al, 2011 who use similar methodologies in other asset markets). Bank loans can also be sold on fairly liquid markets (Drucker and Puri, 2008).

[7] Greenwood (2005) develops a model in which price impact spreads across similar assets. to the extent that off-diagonal elements are positive, this would further amplify the effects discussed below.

AV measures the percentage of aggregate bank equity that would be wiped out by bank deleveraging if there was a shock F_1 to asset returns. As a reminder, this formula omits the *direct* impact of the shock on net worth, emphasizing only the spillovers across banks. If all assets are perfectly liquid (i.e., all elements of the L matrix are zero), then AV=0: there is no contagion across banks because delivering does not involve price impact, even though there is still a direct effect of the shock on banks asset values given by $1'A_1MF_1$.

To understand the intuition behind Eq. (5), using $-R_1=-MF_1=(-r_{1t},\ldots,-r_{nt})'$, we can rearrange terms slightly and expand:

$$AV \times E_1 = \sum_n \gamma_n b_n a_{n1} r_{n1}, \tag{6}$$

where $\gamma_n = \sum_k \left(\sum_m a_m m_{mk} \right) l_k m_{nk}$ measures the "connectedness" of bank n. This is the extent to which bank n owns large ($s_k = \sum_n a_n m_{nk}$ large) or illiquid (l_k large) asset classes. Where this is the case, one dollar of fire sales by bank n will lead to a larger amount of the banking system's holdings, since it will reduce by more the price of larger asset classes.

Equation (6) shows that the systemic risk is large when large banks (banks with large a_{n1}) are levered (large b_{n1}), exposed to the shock in question (r_{n1}), or connected (large γ_n). These properties are intuitive: if large banks are levered and/or exposed, a given shock will trigger larger asset sales. In addition, if exposed banks hold assets that are illiquid and/or widely held, then price impact is large and the overall system is more vulnerable.

C. *Contribution of each Bank to Deleveraging: "Systemicness"*

We can calculate the contribution that each bank has -- through contagion -- on the aggregate vulnerability of the banking system. To do this, we again focus on the impact of a shock F_1, but assume it only affects bank n. In this case, it is easy to see that the impact coming from the liquidations of bank n on the aggregate of the banking system is:

$$S(n) = \frac{1' A_1 MLM' BA_1 \delta_n \delta'_n \, MF_1}{E_1} \tag{7}$$

where δ_n is the $N\times 1$ vector with all zeros except for the n^{th} element, which is equal to 1. We call $S(n)$ the "systemicness" of bank n. Systemicness can be interpreted as the contribution of bank n to aggregate vulnerability, as $AV = \sum_n S(n)$.

As we did for aggregate vulnerability, we can develop intuition by expanding terms in equation (7):

$$S(n) = \gamma_n \times \left(\frac{a_n}{E_1}\right) \times b_n \times r_{n1}, \tag{8}$$

which is the bank-level equivalent of Equation (6). Thus, a bank is more systemic if:

- *It is more levered* (*b_n is bigger*): a shock to a more levered bank is going to induce it to sell more, which generates more price-impact.
- *It is bigger* (*a_n/E_1 is bigger*): a given shock on a larger bank leads to more fire sales, which in turn leads to a large price impact.
- *It receives a bigger shock r_{n1}*
- *It is more connected* (*γ_n is bigger*): the bank owns assets that are both illiquid and widely held by other banks.

D. Impact of Deleveraging on each Bank: Indirect Vulnerability

We define a bank's "indirect vulnerability" with respect to shock F_1 as the impact of the shock on its equity through the deleveraging of other banks:

$$IV(n) = \frac{\delta'_n \, A_1 MLM' BA_1 MF_1}{e_{n1}}. \tag{9}$$

$IV(n)$ measures the fraction of equity of bank n that disappears when other banks deleverage following shock F_1. It differs from *direct* vulnerability, which measures the direct exposure of bank n's assets to shock F_1:

$$DV(n) = \frac{\delta'_n A_1 M F_1}{e_{n1}}. \tag{10}$$

In our empirical applications, we will systematically contrast the two measures: IV involves the deleveraging spiral, while DV does not.

To understand the intuition behind $IV(n)$, we can expand terms in equation (9):

$$IV(n) = \underbrace{\left(1+b_n\right)}_{\text{leverage}} \times \sum_k \left[\underbrace{l_k m_{nk}}_{\substack{\text{illiquidity- weighted} \\ \text{exposure to asset k}}} \times \underbrace{\left(\sum_{n'} m_{n'k} a_{n'} b_{n'} r_{n'} \right)}_{\text{fire sales of asset k}} \right] \tag{11}$$

The first term stands for the pure leverage effect: a given asset shock has a bigger impact on equity if the bank is more levered. The second term measures the importance of connections between banks. $\sum_k \gamma_k a_k \Delta r_k$. It is large when the bank is exposed to assets that are illiquid and exposed to heavy fire sales.

E. Indirect Vulnerability to a specific bank

Suppose one is interested in the impact of a single bank deleveraging (for example, if it were to fail and its assets were liquidated). In this case, we can compute IV in the special case where the vector of banks' returns R_1=-$\sigma.\delta_m$, i.e. assuming that bank m (and only bank m) will deleverage following a shock σ to it assets. Then, following equation (9), the indirect vulnerability of bank n to this shock is:

$$IV(n,m) = \sigma \frac{\delta'_n A_1 M L M' B A_1 \delta_m}{e_{n1}}. \tag{12}$$

This measure captures the interdependence through deleveraging of banks n and m. $IV(n,m)$ is large when sender bank m is large and levered, when receiver bank n is levered, and more interestingly when the term $\delta_n' MLM' \delta_m$ is big, i.e., when n and m own similar illiquid assets.

F. Theoretical Properties

i. Heterogeneity and Systemic Risk

One implication of equation (6) is that making the banks more similar may reduce spillovers, and thus AV. This contrasts with much of the existing literature on systemic risk, which assumes that systemic risk is high when banks have correlated stock returns. The economic intuition for this comes from two opposing effects. First, because banks liquidate all assets they hold when they are shocked, shocks to liquid assets trigger fire sales of illiquid assets when banks own both types. This can make it stabilizing to ring-fence the illiquid assets into specific banks. There is, however, also an effect that makes diversification desirable: when all banks own all assets, any shock to asset prices will spread the fire sales across all asset markets, which tends to reduce the total price impact. The diversification effect dominates when illiquid (high l_k) assets receive stronger shocks (high f_k): diversified (correlated) banks are better, because they can react to these shocks by partly selling liquid assets which reduces global price impact. But when liquid (low l_k) assets receive bigger shocks (high f_k), the contagion effect is more important. In this case, stability can be increased by isolating the illiquid assets into specific banks.

To illustrate this intuition more formally, consider the case of N assets and N banks of identical size a and leverage b. Suppose that assets are equally spread across banks (heterogeneity), $M=Id$. In this case, equation (6) can be rewritten as $AV = a^2 b \sum_{i=1}^{N} l_i f_i$. In contrast, if each asset is exclusively held by one bank dedicated to that asset (homogeneity), we have $M=(11')/N$ (this is a matrix where all coefficients are equal to $1/N$) and $AV = a^2 b \sum_{i=1}^{N} \bar{l} f_i$ where $\bar{l}$=$(\sum_{i=1}^{N} l_i)/N$ is the average liquidity of assets. Thus homogeneity leads to lower AV than heterogeneity if $\sum_{i=1}^{N} (\bar{l} - l_i) f_i > 0$, i.e. when assets with large shocks tend to be more illiquid.

ii. Absence of "Too Big to Fail" effect

Another somewhat surprising property of our framework is that AV is not directly impacted by the size of banks. For instance, we can prove that slicing a bank into *n* smaller banks, with the same asset mix and leverage as the original bank, leaves AV unchanged (see appendix). This is because each of these new banks reacts to shocks exactly as the original bank, scaled by the ratio of their sizes. Thus, the combined impact on the rest of the system is exactly identical to that of the original banks. Conversely, merging banks with same asset mix and leverage also leaves AV unchanged.

III. Relation to Literature

The tradition in recent papers on systemic risk has been to infer bank linkages from correlations in market prices. A first set of papers seeks to estimate risk directly from bond or CDS (see for instance Ang and Longstaff (2011)). Giglio (2011), for example, uses the difference between bond and CDS spreads to estimate the joint probability of failure of large banks who are sellers of protection. A second set of papers measures systemic risk through comovement in the equity returns of financial intermediaries (Adrian and Brunnermeier (2010), Acharya, Pedersen, Philippon and Richardson (2010), Billio et al (2010)).

Our framework departs from this literature by making simple assumptions about how funding shocks propagate across banks. To do so comes at some cost—we adopt a narrow definition of systemic risk based on banks' common exposures. On the other hand, the benefits are that our model-based approach can be used to do policy analysis.

The structure of our model is similar to Acemoglu, Ozdaglar and Tahbaz-Salehi (2010), who study the propagation of shocks in the real economy. They derive conditions under which aggregate volatility remains high even when the network is large. Assuming their asymptotic approximation is correct for a large universe of banks, some of their insights could conceivably be applied here.

An important feature of our model relative to existing work is that it distinguishes between a bank's *contribution* to the risk of aggregate deleveraging ("systemicness"), and a bank's *sensitivity* to deleveraging by other banks ("indirect vulnerability"). Adrian and Brunnermeier (2010) define and estimate the "CoVaR" of institution n as the Value at Risk of the *whole financial sector* conditional on bank n being in distress. In our model, "systemicness" $S(n)$ is the equivalent of their CoVaR measure; the main difference is that, while CoVaR is estimated using comovement in stock returns, we put structure on the propagation mechanism, which could result in patterns of return comovement that differ from that observed during ordinary times. On the other hand, Acharya et al. (2010) propose a measure closer to "indirect vulnerability" $IV(n)$. For each bank, they estimate average returns during the 5% worst days of market conditions. They combine this estimate with bank leverage to compute the "marginal expected shortfall (MES)," which captures how much capital a bank must raise when faced with adverse market conditions. Finally, Billio, Getmansky, Lo, and Pelizzon (2010) measure systemic risk using bilateral time-series dependencies between firms (see also Diebold and Yilmaz, 2011): Our cross-bank indirect vulnerability measure $IV(n,m)$ provide a possible foundation for some of these connections.

Last, our analysis is closely related to policy proposals recently put forth by Duffie (2011) and Brunnermeier, Gorton, and Krishnamurthy (2011). Duffie (2011) proposes that a core group of large financial firms report for a list of stressful scenarios their gains or losses together with the large counterparties with whom the gain or loss for that scenario is the largest. Brunnermeier, Gorton, and Krishnamurthy (2011) suggest eliciting firms' sensitivities to different risk factors and scenarios. Our paper is an attempt to model these sensitivities, and to quantify how these stress scenarios could play out across the broader financial sector.

IV. The Vulnerability of European Banks

Europe is a natural testing ground for the model because detailed holdings data per bank are available through the European Banking Authority (EBA) as a result of the 2011 stress tests. Given the role that sovereign debt has played in the European banking crises, we focus our analysis on banks' sovereign bond holdings, and specifically writedowns of Greek, Irish, Italian, Portugese, and Spanish debt, which we henceforth abbreviate as GIIPS.

A. Data

We use the results of the European stress tests published in July 2011 on the EBA website, which provides harmonized balance sheet composition for the 90 largest banks in the EU27 countries. The complete list of banks is in the Appendix.

Matrix A_1: The matrix of assets is obtained directly from the EBA data by summing over all banking exposures to loans of each bank n. Diagonal elements a_{nn} are the "total exposure" in euros of bank n. The average exposure is €260 billion. The biggest bank is HSBC (€1440bn), the smallest one is Caixa d'Estalvis de Pollensa (€338 million).

Matrix *M*: To calculate the exposure matrix M, we collapse the EBA data into 42 asset classes: sovereign debt of each of the 27 EU countries plus 10 others, commercial real estate, mortgages, corporate loans, retail SME and retail revolving credit lines. The M matrix is thus a 90 x 42 matrix, where m_{nk} is the fraction of exposure to asset k of bank m. Aggregate exposure to commercial real estate across the 90 banks is €1.2 tn (5% of banking sector assets); small business lending is €744 bn (3.2%); mortgages are €4.7 tn (20%); and corporate loans are €6.7 tn (29%). Sovereign bonds account for €2.3 tn (13%).

Matrix *B*: The leverage matrix B is the diagonal matrix of debt-to-equity ratio. We use book leverage because (1) the EU data does not lend itself to the use of market leverage (half of the 90 banks are not listed, and EBA exposure data are mostly not marked-to-market), and (2) measures of risk weighted leverage are strongly affected by regulatory arbitrage (Acharya, Schnabl and Suarez, 2011). To obtain each element b_{nn}, we divide total exposure (the a_{nn} element of A) *minus* book equity by book equity. Because some EU banks are very levered, this number has a few outliers (540 for Allied Irish Banks, 228 for the Agricultural Bank of Greece). Because we do not want our results to be driven by these outliers, we cap target leverage b_{nn} at 30: this cap is imposed on 20 banks.

Matrix *L*: We assume $L=10^{-13}$ x Id, where Id is a 42 x 42 diagonal matrix of ones. We therefore assume that all 42 assets have the same price impact. 10^{-13} means that €10bn of trading imbalances lead to a price change by 10bp. This is in the neighborhood of recent empirical estimates of price impact in the bond market, but probably an underestimate for some other asset classes.

Shock *F*₁: We study a 50% write-off of all GIIPS debt. Hence, the shock vector F_1 is equal to zero for all 42 assets, except for the five GIIPS sovereign debts, for which we assume a return of -50%. Given banks' exposures, the *direct* effect of this shock on aggregate bank equity is given by -$1'A_1MF_1$, which is equal to 381bn €, or 40.1% of aggregate bank equity.

B. Validation using stock returns during the sovereign debt crisis

We first validate our deleveraging model using past data on bank returns during the crisis. Between the Between Dec 31, 2009 and September 16, 2011, European bank stocks (the subset of our sample which is publicly traded) fell by an average of 54%. In this Section, we ask if this meltdown comes from market perception of direct exposures $DV(n)$ and indirect vulnerabilities $IV(n)$ to losses on GIIPS sovereign debt. If the market prices bank interdependence via deleveraging, $IV(n)$ should explain the cross-section of bank returns during the crisis, even controlling for $DV(n)$.

To calculate $DV(n)$ we use equation (10). To compute $IV(n)$, we use a modified version of equation (9), where we account for the fact the fire sales cannot exceed the total assets of a bank (see Section II.A.). This adjustment is necessary as some banks are severely hit by the large shock we assume, so as to entirely wipe out their equity. This leads to the following definition of $IV(n)$:

$$IV(n) = \frac{\delta'_n A_1 MLM' \max\left(BA_1 MF_1, A_1(I - MF_1)\right)}{e_n},$$

where $max(X,Y)$ is the element-by-element max operator. In this definition, we plug in the above matrices and the GIIPS shock vector F_1.

Table 1 lists the top 10 banks, sorted according to $IV(n)$. To see how $IV(n)$ differs from more direct exposures $DV(n)$, we also report direct vulnerability, along with each bank's leverage. Rankings in terms of indirect and direct effect are far from being perfectly correlated: the Spearman rank correlation between DV and IV with respect to a GIIPS shock is 0.17, and is not significantly different from 0 at the 5% level. On average, the direct impact of a full-blown GIIPS crisis would be to wipe out 1.11 times the equity for the average bank. To this direct effect, the impact of the subsequent deleveraging would further wipe out some 302% of the equity of the average bank. As a reminder, all estimates of the impact of deleveraging are contingent on our price impact estimate discussed earlier.

We then regress cumulative returns over 2010 and September 2011 of each bank on indirect vulnerability, controlling for direct vulnerability, bank size (as measured by log of bank total exposure $log(a_{nn})$) and leverage. These controls ensure that vulnerability to the deleveraging process $IV(n)$ adds explanatory power beyond a bank's direct exposure. Table 2 shows these results.

The first three columns are simple OLS regressions. Out of 90 banks covered by the stress tests, only 51 are publicly listed, and we have complete returns data for 49 of them. To reduce sensitivity to outliers, we also report median regression results in columns 4-6. Both sets of results confirm that the differences in indirect vulnerabilities explain part of the cross-section of bank returns during the crisis. In OLS results, the R^2 of indirect vulnerability alone is 9%, compared with 14% when direct exposure is also included. The bank size control does not affect the estimated impact of $IV(n)$ on returns. The direct and indirect vulnerabilities have the same explanatory power on the cross-section of bank returns. For two banks that are one sample standard deviation apart in terms of $IV(n)$, cumulative returns drop by 5 percentage points more in the bank most exposed to sector-wide deleveraging.

C. Systemicness

In this Section, we briefly discuss the properties of our systemicness measure $S(n)$ on European Data. As for vulnerability, we need to amend equations (7) and (8) to ensure that bank-level total fire sales are less than total assets (see Section II.A).

$$S\left(n\right)=\frac{1'A_1MLM'\delta_n\delta'_n A_1\max\left(BMF_1,(I-MF_1)\right)}{E_1}$$
$$=\gamma_n\times\left(\frac{a_{nn}}{E_1}\right)\times\max\left(b_{nn}\delta'_n MF_1,1-\delta'_n MF_1\right).$$

which shows that the systemicness of bank n can be decomposed into the product of three scalars: γ_n, which captures the impact of bank n on other banks through deleveraging, a_{nn}/E_1, which captures the relative size of bank n, and $\max\left(b_{nn}\delta'_n MF_1,1-\delta'_n MF_1\right)$, which reflects the size of fire sales by bank n.

Table 3 reports the systemicness ranking for the 10 most systemic banks in Europe, along with the three components of the decomposition above. Unsurprisingly, in the overall sample, systemicness is correlated with size (spearman correlation of .52, statistically significant at 1%), but this correlation is far from perfect, as can be seen among the 10 most systemic banks. For example, HSBC, the largest EU bank, does not appear in this ranking. BNP Paribas, which is the second largest, is only the fifth most systemic bank. Size does not explain everything because there is substantial heterogeneity across banks in terms of necessary fire sales. Bankia, which is relatively small, is among the most systemic banks because fire sales would be enormous (92% of its assets), and it is highly connected with the rest of the financial system through its asset holdings (its linkage component equals 0.42). Assuming, for instance, that Bankia had an average linkage level (0.30 instead of 0.42), its systemicness would be equal to 0.29x0.95x0.30=0.08, which would make it the 8th most systemic bank instead of the 6th.

The sum of systemicness across all 90 banks is equal to 2.45, which means that through the deleveraging process, our model predicts that 245% of aggregate bank equity would be wiped out. This is sizeable, since the direct impact of the GIIPS writedown total 40.1% of EU bank equity. The deleveraging effect is therefore 6 times larger than the direct shock. In what follows, we focus on deleveraging.[8]

D. Policy simulations

In this section, we use our model to evaluate a number of different policies which have the potential to reduce spillovers from fire sales when banks are deleveraging. As a reminder, the model does not take a position on whether banks are behaving optimally, and assumes that all banks face currently binding leverage constraints, meaning that they adjust immediately to reach new target leverage. Thus, the interventions that follow should be interpreted as potential ex post interventions that could be used in a moment of crisis. The results of the experiments are reported in Table 4. For each policy intervention, we calculate the aggregate vulnerability to the 50% write-down on all GIIPS debt.[9]

Limiting Bank Size: We start by considering the effect of a cap on bank size, holding constant leverage. We do this as follows. Suppose a bank n holds $a_n m_{nk}$ euros of asset k. If assets $a_n > c$, where c is the cap, we set the bank's assets to c, and redistribute residual asset holdings $(a_n - c)m_{nk}$ equally among non-capped banks. This procedure does not affect the portfolio structure of the capped bank, but does affect the portfolios of the other banks, which become richer in the assets held by the capped bank. After one iteration, some previously uncapped banks end up with size greater than c. We iterate this process until all banks are below or at the size cap.

[8] To properly calibrate this effect, we would need to amend our exercise in two directions: change the L matrix so as to account for the fact that assets are less liquid, and change the liquidation rule of banks so as to account for the fact that banks fire-sell liquid assets more. The first change would make estimates of systemic risk bigger, while the second one (making banks smarter) would reduce it.

[9] Similar qualitative insights obtain using alternative, "less extreme" shocks, such as a 50% write-down on Greek debt only, or a 50% write-down on Greece, Ireland and Portugal.

In calculating the new Aggregate Vulnerability, we keep leverage constant. This means we are implicitly assuming that receiving banks can issue enough equity to absorb the new assets, while capped banks reduce their equity when they downsize. The intention is to isolate the effect of size capping separately from deleveraging.

We report the results of this experiment for caps of €500 bn, €900 bn and €1300 bn euro in the first three rows of Table 4. The table shows that capping at €500 bn requires us to redistribute assets out of 17 banks; only two banks would be downsized if we set the cap to be €1300 bn. The main lesson from this analysis is that the overall impact of size caps on aggregate vulnerability is small, and, if anything, tends to increase AV.

The intuition for this can be understood by using the definition of AV and taking the difference before and after the policy has been implemented:

$$\begin{aligned}\Delta AV &= \sum_n \Delta\left(\gamma_n \frac{a_{n1}}{E_1} r_{n1}\right) \\ &= \underbrace{\sum_n \overline{\gamma_n r_{n1}} \Delta \frac{a_{n1}}{E_1}}_{\text{size reallocation}} + \underbrace{\sum_n \overline{\frac{a_{n1}}{E_1}}\, \overline{r_{n1}} \Delta \gamma_n}_{\text{connection reallocation}} + \underbrace{\sum_n \overline{\frac{a_{n1}}{E_1}}\, \overline{\gamma_{n1}} \Delta r_{n1}}_{\text{exposure reallocation}}\end{aligned} \tag{13}$$

where Δx measures the change in x between before and after the policy, and $\bar{x}$ measures the average of x between before and after the policy. r_{n1} is the adjusted levered exposure given by $\max(b_{nn}\delta_n MF_1, 1-\delta_n MF_1)$. AV changes because the size cap reallocates assets across banks. The overall effect can be decomposed into three pieces. First, there is a size reallocation effect, in which AV is increased if banks that are more connected or more exposed/levered receive more assets. Second is a "connection reallocation" effect, in which AV increases when large, exposed/levered banks become more connected. The third effect is "exposure reallocation", which increases systemic risk if it makes large connected banks more exposed.

We report this decomposition in Table 4, next to the size cap simulation. The net increase in systemic risk is driven by two opposing forces. These two forces are the strongest for the most drastic cap (€500 bn), so we focus on this one. On the one hand, average (size- and connectedness-weighted) exposure decreases, which reduces systemic risk. This happens because large banks tend to be significantly less exposed: GIIPS debt accounts for 3.2% of their assets, against 5.8% for banks below €500 bn.[10] As a result, the average large banks has less GIIPS exposure: the transfer of one euro from large to small banks will reduce the average exposure of smaller banks, while keeping the average exposure of larger banks constant. Through this effect, the €500bn cap policy reduces exposure at smaller banks by 10.5 percentage points, on average. This "risk dilution effect" (further amplified by the fact that the smaller banks get relatively larger) decreases AV.

On the other hand, AV goes up because more exposed banks (which happen to be the smaller banks) receive more assets. Through this "contamination effect", safe assets which were previously held by relatively sheltered institutions are now held by more exposed banks, increasing AV. Overall, in the €500 bn cap policy, the contamination effect dominates the risk dilution effect.

GIIPS debt re-nationalization: We also look at the effect of reallocating GIIPS sovereign debt to banks in their home country. This exercise is motivated by two facts. First, between July and December 2011, under pressure of markets and regulators, GIIPS-based banks increased their holdings of GIIPS debt by about 1%, while non GIIPS-based banks reduced them by about 22%. Second, between December 2011 and January 2012, while the ECB lent about €500 bn to euro-area banks, Spanish banks bought about 23bn euro of government debt and Italian banks some €20 bn. A

[10] This difference also holds for levered exposure r. A 50% GIIPS debt write-down would wipe out 35% of the book equity of large banks on average, against 46% for banks below the €500 bn threshold.

partially intended consequence of prudential and monetary policies over the fall of 2011 has thus been to re-nationalize GIIPS debt.

We thus implement the reallocation of 20% of aggregate holdings of each sovereign back to the balance sheets of banks of its own country. First, for each sovereign k, we aggregate euro holdings by all banks according to $s_k = \sum_n m_{nk} a_n$. For each bank n outside country k, we then remove $20\% \times s_k \times \frac{a_n m_{nk}}{\sum_{m \in foreign} a_m m_{mk}}$ euro of sovereign k from its balance sheet. Then, for each domestic bank n' in country k, we inject the holdings in proportion of its holdings of the sovereign among banks of country k: $20\% \times s_k \times \frac{a_{n'} m_{n'k}}{\sum_{m \in domestic} a_m m_{mk}}$. This reallocation never leads to negative holdings as long as foreign banks own at least 20% of the aggregate holdings of sovereign k, which is the case in our simulation.[11]

Table 4 reports the results of this simulation. We find that it *reduces* systemic risk by about 8%, an effect larger than the €500 bn size cap. This effect is large: the amount of sovereign debt reallocated in the process is only €96 bn, while the €500 bn size cap reallocates trillion of euro of assets.

What drives the reduction in AV? We can break down the overall impact into three components. Most of the effect comes through the aggregate reduction in exposure. When reallocating GIIPS debt, we are reducing GIIPS exposure of non-GIIPS banks (on average, by 0.2% of total assets), while increasing the exposure of most GIIPS banks (on average, by some 0.03% of

[11] The only country in our sample where domestic banks own more than 80% of the aggregate bank holdings is the UK (81.6%).

their total assets).[12] Given that GIIPS banks are on average less levered than non-GIIPS banks (with a debt-to-equity ratio of 21 against 23), this implements an overall reduction in fire sales and hence *AV*.

Euro-bonds: Our next intervention replicates the effect of substituting all the different sovereign bonds in Europe for one debt security that has the same payoff. The intuition behind the experiment is to break the loop between banks and their sovereigns (Acharya, Dreschler and Schnabl, 2010). Some recent proposals have suggested replacing part of individual sovereign bonds in the eurozone with the equivalent amount of a euro-level sovereign bond.[13] According to these authors, this would make banks less sensitive to their own sovereign default.

Suppose we could substitute the sovereign portfolio of each bank with a new portfolio of sovereigns which has (1) the same size and (2) weights that are the same across banks. Each bank thus receives an identical portfolio. More precisely, we change the exposure m_{nk} into $sharesov_k \times \%sov_n$ where $sharesov_k$ is the share of sovereign k in aggregate sovereign holdings, while $\%sov_n$ is the share of sovereign holdings in bank n's portfolio. This reshuffling of bonds across banks preserves each bank's total sovereign exposure, and aggregate exposure (holdings) to each sovereign. But it makes banks more similar in terms of individual country exposure. In the context of our model, it is as if all banks are holding Eurobonds.

Table 4 shows that this policy involves a considerable reshuffling of assets across banks: some 1.6tn euro of bonds change owners. It also increases *AV*. As in the previous experiment, the reason is that exposure is reallocated to firms that are more levered, so that only the "exposure

---

[12] Some GIIPS banks experience a decrease in exposure. This happens because these banks own a lot of GIIPS debt but relatively little of their own sovereign (for instance most Italian banks own much a lot of non-Italian debt, and relatively less Italian debt). As a result, the policy reduces overall exposure to GIIPS for these banks.
[13] See Delpla and Von Weizacker (2010), Brunnermeier et al (2011), Hellwig and Philippon (2011) among others.

change" components appears. The intuition is that non-GIIPS banks are both less exposed but more levered in the data. The eurobond experiment transfers GIIPS debt from GIIPS banks to non-GIIPS banks, and therefore increases exposure of the most levered banks.

Ring-fencing risky assets: Perhaps more targeted policies can make the most systemic banks safer? To understand the effect of a merger, let us assume that banks indexed by n are merged together into a bank denoted by *. Noting that the merger preserves the quantity of each holding, it is straightforward to show that:

$$\Delta AV = \sum_{n \text{ merged}} \frac{a_m}{E} \gamma_m \left(r^* - r_m\right) \tag{14}$$

The interpretation of this equation is simple: if banks that are larger or more connected have an exposure lower than the merged entity, the merger increases systemic risk. The intuition is that the merger creates contagion: banks who were relatively large and connected, but less exposed, were protected against the shock. By being merged into an entity with larger exposure, these assets become vulnerable to fire sales, increasing AV.

Suppose now that the regulator merges the most exposed banks into a single large bank. For each bank, we define as 'exposure' the fraction of bank equity that would be lost directly in a 50% write-down of GIIPS debt. We then study three scenarios: merge all banks with exposure above 50%, above 100% and above 150% of their own equity. This means merging respectively 47, 20 and 9 banks.

Table 4 shows that the effect of the bank mergers is nearly zero. The reason is that the policy regroups banks that have very similar exposure-to-equity r_{n1}. And, as equation (14) demonstrates, the expected change in AV is small when expected leverage adjusted-exposure r_{n1} is the same across merged firms. In this case, ring-fencing does not reduce systemic risk: the policy simply transforms several similar small banks into one big bank with the same exposure.

Merging exposed banks with unexposed ones: Suppose we merge the 20 most exposed banks with the banks that are unexposed to the GIIPS write-down (6 of the 90 banks are unexposed). To isolate the impact of merging the two groups, we first merge the exposed banks together, then merge the unexposed banks together, and then finally perform the full merger. Merging unexposed banks does not change *AV*, because of the effect discussed in the previous experiment: they are identical with respect to the shock. For the same reason, merging exposed banks does not change things much either. Merging the two groups into one bank does, however, increase systemic risk by 20% of aggregate equity. The intuition is that the assets of unexposed banks, which were previously not sold in response to the shock, become contaminated by the poor performance of GIIPS debt. This is because, in the data, the measure of connectedness of bank *n*, $\sum_k m_{nk} s_k$, is larger for initially unexposed banks than for exposed banks. As a result, merging the two categories of banks exposes the connected balance sheet of unexposed banks to the GIIPS shock.[14]

[14] This effect of increasing AV after merger shows up even in simulations where we assume that all banks have the same leverage b_n and the same size a_n. If in equation (13) we set $a_n=a^*$ and $b_n=b^*$, we obtain:

$$\Delta AV \times E_1 = la^* b^* \sum_{i \text{ merged}} \left[((-r_1^*) - (-r_{i1})) \times \sum_k m_{ik} s_k \right]$$

where $r^* = (1/N_{\text{merged}}) \cdot \sum_{i \text{ merged}} r_{i1}$. It appears from this expression that the increase in AV is positive if banks with above average exposure $-r_{i1}$ have below average connectedness $\sum_k m_{ik} s_k$. This is the case in the data, where exposed banks have a connectedness level 13% below unexposed banks.

Leverage cap: We next study the impact of capping leverage. Here, the policy is much simpler: if x is the cap, then, for all banks with leverage above x, we set $D/E = x$. We implicitly assume these banks can raise equity to reach the maximum leverage, but do not change their sizes. Economically in our model, such a policy reduces the need for banks to fire-sell assets, so it unambiguously reduces AV. From Equation (6) we see that:

$$\Delta AV \times E_1 = l \sum_n \left[\Delta b_n \times \underbrace{a_{n1}}_{\text{n is large}} \times \underbrace{(-r_{n1})}_{\text{n is large}} \times \underbrace{\left(\sum_k m_{nk} s_k \right)}_{\text{n holds large asset classes}} \right] \text{, with } s_{\text{k}} = \sum_{n'} m_{n'k} a_{n'1}$$

The policy is more effective when targeted banks are either (1) bigger, (2) more exposed, or (3) hold large asset classes.

We try three different caps (knowing we capped leverage to 30 in the data): 15, 20 and 25. We calculate the amount of equity capped banks need to raise to reach this cap: for instance capping leverage at 15 (25^{th} percentile) requires banks to raise a staggering of €480 bn. The table shows that, to obtain a significant reduction in systemic risk, the regulator would need to set a very drastic cap. For instance, capping leverage at 25 (this is leverage at the 63^{rd} percentile bank) only reduces vulnerability to a GIIPS shock from 245 to 238% of aggregate equity. The impact of reducing leverage to 20 is much larger.

E. *Optimizing capital injection*

The policy interventions discussed above are disappointing in that they suggest that capping leverage yields only modest improvements in AV, and that other policies have ambiguous, or even adverse, impacts on AV. In a moment of crisis, what tools can reduce contagion at minimal expense to the regulator? In this last exercise, we explore the power of an optimal targeted policy. Recall from Eq. (8) that aggregate vulnerability to a shock vector S can be written as a weighted average of the debt-to-equity ratios b_n's. The weights measure the extent to which the leverage of a particular bank n is bad for aggregate vulnerability. This happens when the bank is large, the bank is exposed to shocks, and linkages are strong.

Suppose the regulator has a given amount of cash F available to invest in bank equity, and cares only about reducing spillovers between banks in a deleveraging cycle. Equity injection into bank n is given by the vector $f = (f_1,\ldots,f_n)$, so that $1'f = F$. When a bank receives f_n euros of fresh equity, we assume the entire amount is used to repay existing debt, so that its debt to equity ratio becomes $(E_i - f_n)/(E_n + f_n)$.

We minimize Eq. (8) subject to the constraints that $1'f = F$ and $(E_i - f_n)/(E_i + f_n)$. We also impose the constraint that the regulator cannot withdraw cash from equity-rich banks, so that $f_n > 0$ for all i.

Optimizing equity injection across banks allows us to reduce aggregate vulnerability a lot more than any of the policy experiments we considered in Table 4. We can see this result visually in Figure 2, where we report the optimal AV obtained for various levels of aggregate investment F. Panel A shows the aggregate vulnerability to a GIP shock, while Panel B shows aggregate vulnerability to a GIIPS shock (both assuming a 50% write-down). Data from panel A shows a reduction by a third in systemic risk: AV declines from 47% to 31% using only €50 bn of equity.

The marginal impact of additional euros of equity injections decreases: €200 bn leads to an AV of 23%; €500 bn leads to an AV of 18%. The effect on aggregate vulnerability to GIIPS is smaller in relative terms, and decreases more slowly, as more banks are exposed to GIIPS debt than to GIP debt. €50 bn only buy a reduction from 285% to 240% of aggregate equity. Still, the effect is large compared to previous policies considered in this paper.

Table 5 then reports the optimal equity injections for each bank. Here, we use the scenario in which the regulator invests €200 bn, and seeks to minimize aggregate vulnerability to a 50% write-down on GIIPS debt. Table 10 only reports the 20 largest banks, ranked by the size of their equity injection. This list consists mostly of Italian, Spanish and Greek banks. These banks are not the largest, but the most exposed to the write-down.

By construction, optimal injection has a very strong correlation with systemicness (.91). Correlation with the four components of systemicness is lower: .16 (leverage), .16 (Size), 38 (direct exposure), .21 (linkage). This shows that when deciding to inject fresh capital into banks, the regulator should consider all components of systemicness to minimize taxpayers' investment.

F. Considering different liquidation rules

Earlier we suggested that the model could be adjusted for different liquidation rules. A natural one to consider is one in which banks first sell off their most liquid assets. Here we focus on an extreme case and show its impact on the empirical results.

Suppose that banks have the flexibility to sell their sovereign bonds, but that their other assets (primarily loans) are infinitely illiquid, meaning that their early disposal would yield zero proceeds. In this case, the banks would have to concentrate their liquidations of sovereign bonds alone. In this case, we can write down a modified version of the formula for aggregate vulnerability AV to a shock S:

$$AV = \frac{1' A_{t-1} MLM^{*\prime} M' BA_{t-1} MS}{E_{t-1}}, \tag{15}$$

where M^* is a weight matrix that accounts for the fact that non-sovereigns are not liquidated. Each element is given by: $m^*_{ik} = m_{ik} / \left(\sum_k m_{ik}\right)$. We only focus on factors k which corresponds to sovereign holdings. Hence, elements of M^* are bigger: banks will liquidate more sovereigns in response to an adverse shock to their balance sheets.

A striking feature of these simulations is that aggregate vulnerability is much lower under this alternative liquidation rule. The aggregate vulnerability of banks to a GIIPS write-down is now 23%, instead of 285%.

Changing the liquidation rule has two opposite effects. On the one hand, banks liquidate much more sovereign bonds, which has a stronger price impact on other banks. On the other hand, fire sales don't contaminate other assets, which in this case are the majority of assets held on bank balance sheets.

Table 6 reports values of AV for alternative liquidation rules. We progressively add other asset classes to the list of liquid assets. As can be seen from Table 6, as long as the list of liquid assets is small enough (i.e. corresponds to less than 41% of banks' assets), aggregate vulnerability is reduced by illiquidity of the other assets. The intuition is that illiquidity prevents banks from transmitting their shocks to otherwise immune banks. When, however, sellable assets take up a larger fraction of the balance sheet (in our simulations, this happens as soon as we include corporate loans), then the fire sale concentration effect starts dominating the "ring fencing" effect: because banks cannot liquidate everything, they sell more liquid assets, which increases the price impact and therefore contagion. Table 6 illustrates the ambiguity of alternative liquidation rules on AV.

V. Measuring Vulnerability of US Banks

In this section we use the model to measure the vulnerability of US banks between 2001 and 2010. We start by describing the sample and how we estimate the factor exposures. We then validate the model by looking at the build-up of systemic risk during the 2007 pre-crisis period. We also analyze the predicted effect of the Lehman Brothers failure on other banks. After these checks, we present three sets of outputs, including (a) the most vulnerable banks at various points in time, (b) the most systemic banks in terms of their contribution to potential deleveraging.

A. Data

We select the largest US-listed 100 financial firms by market capitalization in 2006 on the CRSP database. Financial firms have SIC codes between 6000 and 7000. The complete list is shown in the Appendix, and includes commercial banks, investment banks, insurance companies, and money managers. Citigroup and Bank of America are the largest firms in December 2006, but investment banks form the next group of large firms. For this sample, we collect weekly and monthly stock returns from January 2001 through March 2011. Because firms list, delist, and merge through the 2001-2011 period, the average number of firms with complete data at any point in time is 88. Finally, we merge financial firm stock returns data from year t with annual balance sheet data at the end of year t-1 from COMPUSTAT.

To compute the systemic risk measures, we need estimates of M, L, B, and A, which we obtain as follows.

$\boldsymbol{A_{t\text{-}1}}$: We compute market value of the firm's assets (i.e., enterprise value) on a weekly basis by adding book assets (Compustat item AT) and the market value of equity from CRSP, and subtracting book common equity (Compustat item CEQ). Because the accounting data refresh annually, this means that our estimates of enterprise value are increasingly stale as we approach the end of each calendar year. For fast growing firms, this introduces some lumpiness in our measures. We define debt as the difference between book assets and book equity and compute market leverage d_i/e_i by taking the ratio of debt to market equity.

B: We assume that target leverage is the same as lagged leverage. Equivalently, we assume that firms adjust their capital structures quickly in response to shocks. This assumption may be too extreme during deleveraging scenarios, particularly for the most levered firms. For example, consider how a bank with D/E = 19 might behave following a 2 percent drop in the value of its portfolio. Realized leverage increases to 31.7 (=19/(1-2%x20)). To return to target leverage of 19, the bank would have to sell 41% of the remaining assets in the portfolio. In practice, the bank may do this slowly, remaining over-levered in the short-run, and perhaps raising equity or lowering dividends.. In order to maintain realism and prevent our measures from blowing up, we cap target leverage at 20.

L: This diagonal matrix measures for each asset, the price impact in percentage terms of a one dollar liquidation. For non-financial equities, one can estimate this number following previous research on price impact in equity markets. For each stock, we compute individual Amihud (2002) price impact ratios based on the first 90 trading days of 2002, and then aggregate these to yield a a market-wide price impact of 6.24×10^{-12}. This means that to depress the market by one percent would require order flow of $16 billion, approximately 10% of weekly trading volume.[15]

The most challenging part of this exercise is determining how to compute liquidity ratios for factors other than equity. We suspect, for example, that a bank selling a specialized loan portfolio might incur a larger fire sale discount than a bank selling a portfolio of liquid S&P 500 stocks. But, absent other data on price impact, we take a conservative approach and assign these factors the same price impact parameter as that of equities. This has the effect of making *L* matrix proportional to the

[15] We compute the implied price impact of the complete stock market by aggregating the individual ratios according to the sum over all firms *i* of $w_i^2 Amihud_i^2$ where w_i is the weight of equity of stock *i* in the aggregate stock market.

identity matrix. While we view this simplification as unfortunate, we believe it to be conservative, and also somewhat unavoidable.

Factor Selection and the Portfolio Matrix M: The portfolio matrix M contains, for each bank i the weights m_{nk} of each asset k in the portfolio. Here we do not observe banks' portfolios directly, so we estimate M with a factor model. For each bank n, we run the following regression on a rolling basis:

$$R_{n,t} = \sum_k m_{nk} F_{kt} + \varepsilon_{n,t} \tag{16}$$

Each week, we run this regression over the past 104 weeks, thereby obtaining rolling estimates of M. Provided we have the full vector of asset returns $F_{k,t}$, the estimated m_{nk} is equal to the weight of each asset in the bank's portfolio. To be able make this inference, R_{nt} has to be obtained through unlevering the equity returns. Implicitly, we assume that: (1) we have the adequate set of factor returns to represent each bank's portfolio, (2) that holdings are fairly stable (i.e. did not move too much over the past 2 years), and (3) that the stock market has some understanding of each bank's exposure to each asset.

In selecting factors, we adopt the following principles. First, we were careful to select a series of factors which were not too collinear (for example, it would be challenging to estimate a bank's separate exposure to AA and A bonds from a stock return regression). Second, it is important to select factors which proxy for the returns of the underlying assets held by each institution.[16] Third, we sought a sufficiently large list of factors so as to be able to capture diversity in the holding of the different banks. These considerations in mind, the factors we use are based on the returns of (1) non-

[16] This led us to exclude, on principle, factors that were associated with bank equity returns but were unlikely related to the underlying assets held by the bank. For example, changes in the TED spread are significantly correlated with bank equity returns during the financial crisis, but are more likely related to the cost associated with the bank's liabilities rather than its assets.

financial firms in the S&P 500; (2) mortgage REITs; (3) 10-year nominal US Treasuries; (4) Commodities, proxied using the Goldman Sachs Commodity Index; and (5) High Yield Bonds based on the Morgan Stanley High Yield Bond Index.[17] Table 1 summarizes the five factors, both during the full sample and during the March 2007-June 2011 crisis subperiod. To reduce the impact of measurement error, we zero out elements of the M matrix for which the estimated coefficient has a t-statistic less than 1.5.

Since much of the cross-sectional variation between banks' contributions to systemic risk comes from their different risk exposures, we have verified that there is enough interesting variation across firms. A simple way to see this is to compute time-series average exposures for each of the banks, and then compare banks. State Street bank, for example, has sample average factor exposures of (0.12, 0.03, 0.02, 0.00, and 0.02) while Mellon Bank has exposures of (0.25, 0.01, 0.16, 0.00, and 0.14) The nature of the exposures differs across banks, with State Street having greater exposure to non-financial firm equity and Mellon Bank having higher exposure to mortgage REITs.

B. Aggregate Vulnerability to deleveraging in the US time-series

We start by performing a series of simple exercises to validate the empirical relevance of the model. We start by showing time-series measures of aggregate vulnerability AV, as well as the contributions (the systemicness $S(i)$) of a few important firms such as Lehman Brothers and Citigroup. We show that bank-specific vulnerabilities are useful for predicting the maximum

[17] Because these factors were chosen with hindsight bias, we perform a robustness test in which the factors are estimated directly from principle components of bank stock returns. The main drawback is that statistical factors are harder to interpret economically: factors are not "assets" so the elements of the M matrix cannot be interpreted as portfolio weights. This is why we rely primarily on the economic factors for most of our analysis, but show in the appendix that using the statistical factors estimated through PCA over 2001-2006 produces similar insights

drawdown of these firms during the 2007-2009 financial crisis. We then show that the model is quite useful for predicting how individual bank stocks respond to the failure of Lehman Brothers.

Figure 3 shows aggregate vulnerability AV, which recall is the total (i.e., systemwide) dollar price impact of deleveraging resulting from a one standard deviation shock to each of the five factors. The series starts low in early 2001, drops in mid 2005, and then rises quickly in 2007.

We remind the reader that while the magnitude of these results depends on the scaling matrix L, the time-series behavior is unlikely much affected. To the extent that we believe price impact went up during the crisis; or that price impact varies significantly across asset classes, the dollar magnitude is impacted.

Figure 4 plots time-series of contributions to vulnerability, ie., the systemicness $S(i)$ of six important banks in our sample: Wells Fargo, JP Morgan Chase, Bank of America, Citigroup, Lehman Brothers, and Goldman Sachs. The figure shows that many of these individual bank series share the common characteristic of systemicness $S(i)$ rising through the crisis to a peak in January 2009, subsequently falling as equity markets rebound and factor volatility drops.

Figure 5 shows that systemicness is related to size and leverage in the cross-section, but that each of these variables explains less than 60 percent of the variation: differential exposures in the M matrix explain the rest.

C. Bank Sensitivity to Deleveraging: Lehman bankruptcy

Eq. (12) shows how to compute the impact of a shock to the assets of bank i on any other bank j. In this section, we study the impact of the failure of Lehman Brothers on September 15, 2008. Before markets opened that day, Lehman Brothers announced that it would file for bankruptcy protection, citing debt of \$768 billion and assets with a market value of \$639 million. Although the company filed for reorganization under the US bankruptcy code, market participants could have reasonably expected substantial liquidations of its asset portfolio.

Taking the liquidation rule of our model literally, we would expect banks with high exposures to the same assets would experience reductions in their portfolio value as a fraction of equity. Since pre-failure, Lehman had market leverage of approximately 20-to-1, a -5% shock to its assets would result in complete liquidation of its portfolio.

We then compare this predicted equity shock to the actual return. This is shown graphically in Figure 6. As can be seen, there is a discernible positive correlation between the predicted return and the actual stock return on Monday September 15, 2008. We analyze returns over a short window because of significant financial news the next day: on September 16, 2008, the Federal Reserve Board authorized lending of up to $85 billion to insurance company AIG.

We would expect the relationship between indirect vulnerability $IV(n)$ and realized returns in Figure 6 to be quite noisy, as the Lehman failure was also a significant information event, both on the magnitude of losses faced by the banking sector, and on the willingness of the government to intervene to stem those losses. Table 8 shows the results of cross-sectional regressions of realized stock returns on September 15, 2008 on vulnerability to Lehman deleveraging. One possible concern with our vulnerability measure is that it does not add much information to size and leverage, since large banks, or levered banks are the most adversely affected the Lehman bankruptcy. We include bank leverage and bank size as controls in our regressions of Table 8.

D. Bank vulnerability and market performance during the crisis

Our firm-specific vulnerability measures $IV(i)$ might be useful for explaining the cross-section of returns following a systemwide deleveraging shock. To operationalize this, here we study the relationship between the maximum drawdown in stock returns experienced by each firm during the crisis. Maximum drawdown is the minimum cumulative rolling return from July 2007 through March 2011 (i.e., the cumulative return corresponding to the lowest price experienced during that period).

Figure 7 plots this relationship, revealing a negative correlation of -28%. The corresponding regression, also shown in the figure, yields a t-statistics of -3.88 on bank vulnerability. Interestingly, this result is not driven by leverage alone. In a multivariate regression of drawdowns on vulnerability and bank leverage, vulnerability retains a similar coefficient and a t-statistic of -3.12.

E. Outputs: Bank Contributions to Systemic Risk and Vulnerability

The most systemic banks are large levered financial institutions which tend to have similar sets of exposures. Table 9 lists the top 10 systemic banks in January 2007, January 2008, and January 2009. In the table we show the systemicness $S(n)$. In a separate column, we show $S(n)$ scaled by AV. This rescaled numbers tells us how important a given bank is in relative contribution to aggregate vulnerability. Of course, a bank may have a relatively large contribution to AV when the level of AV is low, in which case the scaling is less meaningful.

As can be seen, this exercise turns up the usual crowd of large levered financial institutions. In January 2007, AIG, JP Morgan, and Morgan Stanley are at the top of the list; by January 2009, the dollar impact of their deleveraging is much greater (JP Morgan rises from $1.4 billion to $16 billion), and the rankings change somewhat, with Wells Fargo, JP\ Morgan, and Bank of American topping the list.

A possible concern is that the rankings in Table 9 do not capture much more than the product of size and leverage. However, we find only a 0.7 correlation between $S(i)$ and the product of size and leverage in January 2009, and lower correlations still for the other two panels. We provide graphical evidence of such imperfect correlation in Figure 8, where we plot systemicness against leverage or bank size. While indeed systemicness appears correlated with both size and leverage, they are far from explaining the full cross section of our measure. For instance, BofA is the biggest bank but scores low on systemicness.

We now turn to bank vulnerability, which is the impact of a shock to all factors on each single bank. We can express this in dollar terms or normalize it as a percentage of bank's equity. Panel A of Table 10 shows dollar vulnerability in January 2007, January 2008, and January 2009. We show the top 10 most vulnerable banks, meaning the ten banks which would suffer the largest reduction in net worth if there were a simultaneous shock to each of the factors. According to this measure, AIG, JP Morgan, and Citigroup are the most vulnerable banks in early 2007; the rankings do not change much over time: by 2009, Wells Fargo, JP Morgan and Citigroup are the most vulnerable.

Panel B of Table 10 shows vulnerability for the same set of dates, except now we scale by each firm's equity value. Although AIG still appears among the top banks according to this scaling, the list otherwise looks quite different. For example, Radian Group, a highly levered bond insurer, shows up as the most vulnerable institution in both 2007 and early 2008. Although it is difficult to generalize as to which firm characteristics land them on this list, cursory inspection reveals a number of insurance companies specialized in insuring mortgage-related securities.

VI. Conclusions

During the financial crisis of 2007-2009, regulators in the United States and Europe have been frustrated at the difficulty of understanding the complete set of risk exposures of the largest and most levered financial institutions. Yet, at the time, it was unclear how such data might have been used to make the financial system safer. Our paper is an attempt to show how such information can be used in an analytically coherent way.

The key assumption in our model is that banks use asset liquidations to return to target leverage. We use this assumption to predict how individual banks will behave following shocks to their net worth, and how the resulting fire sales may spillover to other banks.

While the model is quite stylized, it generates a number of useful insights concerning the distribution of risks in the financial sector. For example, the model suggests that regulators should pay close attention to risks that are concentrated in the most levered banks. The model also suggests that policies which explicitly target bank solvency may be suboptimal from the perspective of controlling contagion.

We then apply the model to the largest financial institutions in the United States and Europe, and use it to evaluate a number of policy proposals to reduce systemic risk. When analyzing the European banks in 2011, we show how a policy of targeted equity injections, if distributed appropriately across the most systemic banks, can significantly reduce systemic risk.

References

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, 2010, Cascades in Networks and Aggregate Volatility, Working Paper.

Acharya, Viral, Lasse Heje Pedersen, Thomas Philippon, and Matt Richardson, 2010, Measuring Systemic Risk, Working Paper.

Acharya, Viral and Philipp Schnabl, 2010, Do Global Banks Spread Global Imbalances? Asset-Backed Commercial Paper during the Financial Crisis of 2007-09, *IMF Economic Review*, 58(1), 37-73

Acharya, Viral, Philipp Schnabl and Gustavo Suarez, 2011, "Securitization without Risk Transfer", *forthcoming Journal of Financial Economics*

Adamic, Lada, Celso Brunetti, Jeffrey H. Harris, and Andrei A. Kirilenko, 2010, Trading Networks, Working Paper.

Adrian, Tobias, and Brunnermeier, Markus, 2010, CoVar, Working Paper.

Adrian, Tobias, and Shin, Huyn, 2010, Liquidity and Leverage, *Journal of Financial Intermediation*.

Allen, Franklin, Ana Babus and Elena Carletti, 2010, Financial Connections and Systemic Risk, NBER Working Paper 16177.

Allen, Franklin, Ana Babus, and Elena Carletti,2 011, Asset Commonality, Debt Maturity and Systemic Risk, Working paper.

Ang, Andrew, and Longstaff, Francis, 2011, Systemic Sovereign Credit Risk: Lessons from the U.S. and Europe, working paper.

Amihud, Yakov,2002, Illiquidity and Stock Returns: Cross-Section and Time Series Effects, Journal of Financial Markets, 5, 31-56.Basel Committee on Banking Supervision, 2009, Principles for Sound Stress Testing Practices and Supervision, Bank for International Settlements.

Billio, Monica, Mila Getmansky, Andrew W. Lo, and Loriana Pelizzon, 2010, Measuring Systemic Risk in the Finance and Insurance Sectors, Working Paper.

Brunnermeier, Markus, 2009, Deciphering the 2007-2008 Liquidity and Credit Crunch, *Journal of Economic Perspectives* 23, 77-100.

Brunnermeier, Markus, Gary Gorton, and Arvind Krishnamurthy, 2011, Risk Topography, Working Paper.

Carvalho, Vasco M.,2010, Aggregate Fluctuations and the Network Structure of Intersectoral Trade, Working Paper.

Cecchetti, Stephen,2009, Crisis and Responses: The Federal Reserve in the Early Stages of the Financial Crisis, *Journal of Economic Perspectives* 23, 51-75.

Coval, Joshua, and Erik Stafford,,2007, Asset Fire Sales,and Purchases) in Equity Markets, *Journal of Financial Economics* 86,479-512.

Diebold, Francis X. and Kamil Yilmaz,2011, Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms, Working Paper.

Duffie, Darrell,2011, Systemic Risk Exposures: A 10-by-10-by-10 Approach, Working Paper.

Ellul, A., Jotikasthira, C., Lundblad, C.T.,2011, Regulatory pressure and fire sales in the corporate bond market, *Journal of Financial Economics*, 101,3, 596-620.

Giglio, Stefano, 2011, Credit Default Swap Spreads and Systemic Financial Risk, mimeo Chicago Booth.

Greenlaw, David, Jan Hatzius, Anil Kashyap, and Hyun Song Shin,2008, Leveraged Losses, Lessons from the Mortgage Market Meltdown, Proceedings of the U.S. Monetary Policy Forum 2008.

Greenwood, Robin and David Thesmar, 2011, Stock Price Fragility, *Journal of Financial Economics* 102, pp 471-490.

Gromb, Denis and Dimitri Vayanos, 2010, Limits of Arbitrage: The State of the Theory, *Annual Review of Financial Economics*.

Gromb, Denis and Dimitri Vayanos, 2007, Financially Constrained Arbitrage and Cross-market Contagion, Working Paper.

Hanson, Samuel, Anil Kashyap, and Jeremy Stein,2011, A Macroprudential Approach to Financial Regulation, *Journal of Economic Perspectives,* 25, 3-28.

He, Zhiguo, In Gu Khang and Arvind Krishnamurthy,2010, Balance Sheet Adjustment in the 2008 Crisis, *IMF Economic Review*, 58, 118 - 156.

Jotikasthira, C., Lundblad, C.T., Ramadorai, T., ,2011, Asset Fire Sales and Purchases and the International Transmission of Financial Shocks,Unpublished Manuscript.

Kallestrup, Rene, Lando, David and Murgoci, Agatha, 2011, Financial sector linkages and the dynamics of bank and sovereign credit spreads, Working Paper.

Kalemli-Ozcan, Sebnem, Papaioannou, Elias and Peydro, Jose-Luis, 2011, Financial Regulation, Financial Globalization and the Synchronization of Economic Activity, Working Paper.

Krishnamurthy, Arvind and Zhiguo He,2010, Intermediary Asset Pricing, Working Paper.

Lou, Dong, 2011, A Flow-based Explanation for Return Predictability, Working Paper.

Schwarcz, Steven, 2008, Systemic Risk, The Georgetown Law Journal 97, 193-249.

Shleifer, Andrei and Robert Vishny, 2011, Fire Sales in Finance and Macroeconomics, *Journal of Economic Perspectives*, 25, 29-48.

Shleifer, Andrei and Robert Vishny, 1992, Liquidation Values and Debt Capacity: A Market Equilibrium Approach, *Journal of Finance* 47, 1343-1366.

Stein, Jeremy C.,2011, Monetary Policy as Financial-Stability Regulation, forthcoming in the *Quarterly Journal of Economics*.

Wagner, Wolf, 2011, Systemic Liquidation Risk and the Diversity-Diversification Trade-Off ,, *Journal of Finance* 64, pp. 1141-1175.

Appendix A. European Banks Involved in the 2011 stress tests. The sample includes the banks included in the EBA stress tests and thus considered in our European analysis.

Publicly listed banks	Non-public banks
Irish Lf.& Perm.Ghg.	Banque Et Caisse D'epargne De L'etat
Bank Of Cyprus	Bayerische Landesbank
Marfin Popular Bank	Bpce
Otp Bank	Caixa D'estalvis De Catalunya, Tarragona..
Swedbank 'A'	Caixa D'estalvis Unio De Caixes De Manll..
Banco De Sabadell	Caixa De Aforros De Galicia, Vigo, Ouren..
Dnb Nor	Caixa Geral De Depîsitos, Sa
Efg Eurobank Ergasias	Caja De Ahorros Y M.P. De Gipuzkoa Y
Bank Of Piraeus	Caja De Ahorros Y M.P. De Zaragoza,
Bnp Paribas	Caja De Ahorros Y Pensiones De Barcelona
Abn Amro Holding	Caja Espa,,A De Inversiones, Salamanca Y ..
Ing Groep	Dekabank Deutsche Girozentrale, Frankfurt
Nordea Bank	Dz Bank Ag Dt. Zentral-
Banca Monte Dei Paschi	Effibank
Banco Popolare	Grupo Bbk
Banco Santander	Grupo Bmn
Banco Bpi	Grupo Caja3
Alpha Bank	Hsh Nordbank Ag, Hamburg
Societe Generale	Landesbank Baden
Banco Pastor	Monte De Piedad Y Caja De Ahorros
Banco Comr.Portugues 'R'	Norddeutsche Landesbank
Bankinter 'R'	Nova Ljubljanska Banka
Bbv.Argentaria	Nykredit
Espirito Santo Financial	Oesterreichische Volksbank Ag
Dexia	Powszechna Kasa Oszcz_Dno_Ci Bank
Erste Group Bank	Rabobank Nederland
Lloyds Banking Group	Raiffeisen Bank International
Barclays	Skandinaviska Enskilda Banken Ab
Royal Bank Of Sctl.Gp.	Westlb Ag, Dusseldorf
Commerzbank	Wgz Bank Ag Westdt. Geno. Zentralbk, Ddf
Allied Irish Banks	
Deutsche Bank	
Bank Of Ireland	
National Bk.Of Greece	
Kbc Group	
Hsbc Holdings	
Unicredit	
Intesa Sanpaolo	
Banco Popular Espanol	
Danske Bank	
Svenska Handbkn.'A'	
Landesbank Bl.Hldg.	
Agri.Bank Of Greece	
Credit Agricole	
Ubi Banca	
Hypo Real Estate Hldg	
Sns Reaal	
Tt Hellenic Postbank	
Caja De Ahorros Del Mediterraneo	
Bankia	
Banca Civica	

Appendix B. US Financial firms in sample. The sample includes the largest 100 financial firms by market capitalization in December 2006.

Name	MV Equity	Name	MV
Citigroup Inc	$273,691	C I G N A Corp	$13,495
Bank Of America Corp	239,758	Northern Trust Corp	13,273
American International Group Inc	186,296	Ameriprise Financial Inc	13,187
Jpmorgan Chase & Co	167,551	Marshall & Ilsley Corp New	12,590
Wells Fargo & Co New	120,049	Legg Mason Inc	12,491
Wachovia Corp 2Nd New	114,542	Sovereign Bancorp Inc	12,007
Morgan Stanley Dean Witter & Co	85,410	T Rowe Price Group Inc	11,597
Goldman Sachs Group Inc	84,890	C I T Group Inc New	11,059
Merrill Lynch & Co Inc	82,050	Aon Corp	10,944
American Express Co	73,094	C N A Financial Corp	10,924
U S Bancorp Del	63,617	Nymex Holdings Inc	10,788
Federal National Mortgage Assn	57,908	Synovus Financial Corp	10,019
Federal Home Loan Mortgage Corp	47,035	M B I A Inc	9,849
Berkshire Hathaway Inc Del	45,920	T D Ameritrade Holding Corp	9,709
Metlife Inc	44,861	E Trade Financial Corp	9,558
Washington Mutual Inc	42,725	Ambac Financial Group Inc	9,450
Lehman Brothers Holdings Inc	41,408	Comerica Inc	9,322
Prudential Financial Inc	40,955	Zions Bancorp	8,798
Allstate Corp	40,690	Unionbancal Corp	8,597
Travelers Companies Inc	37,047	C B O T Holdings Inc	8,004
Capital One Financial Corp	31,397	Coventry Health Care Inc	7,976
Suntrust Banks Inc	29,907	Cincinnati Financial Corp	7,839
Bank Of New York Mellon Corp	29,601	Compass Bancshares Inc	7,837
Hartford Financial Svcs Grp Inc	29,573	Hudson City Bancorp Inc	7,742
Franklin Resources Inc	27,932	C B Richard Ellis Group Inc	7,481
Countrywide Financial Corp	26,365	T D Banknorth Inc	7,374
Schwab Charles Corp New	24,469	Safeco Corp	7,222
B B & T Corp	23,763	Unum Group	7,118
National City Corp	23,092	American Capital Ltd	6,828
Fifth Third Bancorp	22,767	Assurant Inc	6,818
A F L A C Inc	22,747	Commerce Bancorp Inc Nj	6,614
Aetna Inc New	22,540	Berkley W R Corp	6,613
State Street Corp	22,395	Peoples United Financial Inc	6,345
Chubb Corp	21,780	Torchmark Corp	6,253
P N C Financial Services Grp Inc	21,754	Intercontinentalexchange Inc	6,198
S L M Corp	19,935	Mercantile Bankshares Corp	5,872
Bear Stearns Companies Inc	19,112	Health Net Inc	5,672
Lincoln National Corp In	18,418	Huntington Bancshares Inc	5,593
Progressive Corp Oh	18,221	Old Republic International Corp	5,366
Regions Financial Corp New	17,996	Fidelity National Finl Inc New	5,223
C M E Group Inc	17,746	First Horizon National Corp	5,200
Blackrock Inc	17,686	M G I C Investment Corp Wis	5,192
Mellon Financial Corp	17,504	First Marblehead Corp	5,159
Western Union Co	17,184	Popular Inc	5,003
Marsh & Mclennan Cos Inc	16,897	Edwards A G Inc	4,777
Principal Financial Group Inc	15,835	New York Community Bancorp Inc	4,752
Genworth Financial Inc	15,470	Markel Corp	4,639
Keycorp New	15,272	Associated Banc Corp	4,495
N Y S E Euronext	15,186	Radian Group Inc	4,344
M & T Bank Corp	13,519	Janus Cap Group Inc	4,279

Figure 1. Bank mergers and aggregate vulnerability. This figure shows what happens when two banks with different leverage marge. The merged bank has less than or equal leverage to the asset-weighted leverage of the two merging banks.

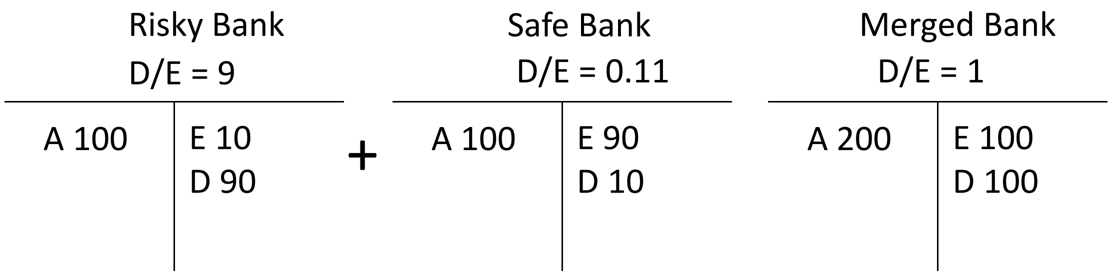
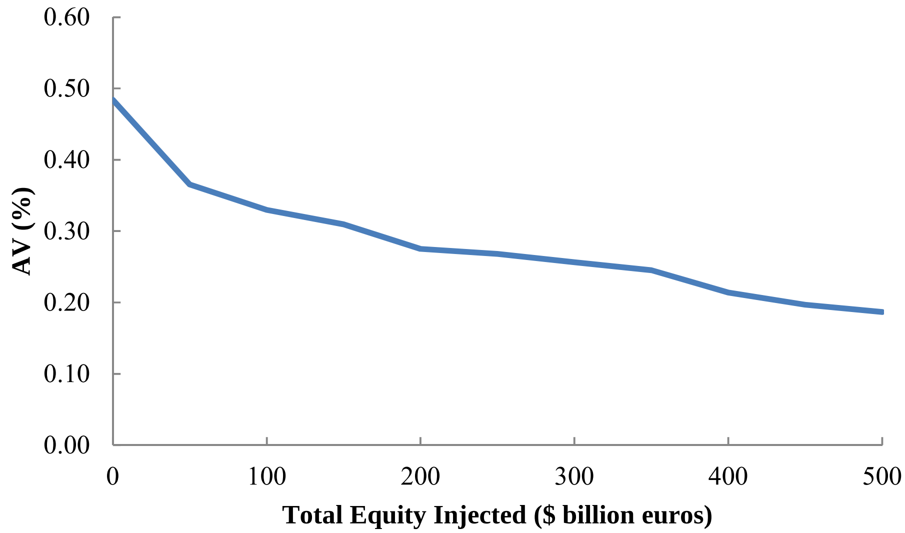


Figure 2. Optimal Aggregate Vulnerability, as a Function of Aggregate Equity Injected (in bn euros). This figure reports the optimal AV to a 50% write-off on GIP debt (Panel A), GIIPS debt (Panel B). Such optimal AV is obtained assuming the social planner can freely allocate €200 bn of equity into banks, keeping their sizes constant, so the equity injection serves to reduce debt. In Panel A, for 0bn, we obtain AV of 0.47. This means that, absent a capital injection, a 50% write-off on GIP debt would reduce aggregate bank equity by 47%.

Panel A: Aggregate vulnerability to a 50% write-off to GIP debt (per euro of aggregate equity)



Panel B: Aggregate vulnerability to a 50% write-off to GIIPS debt (per euro of aggregate equity)

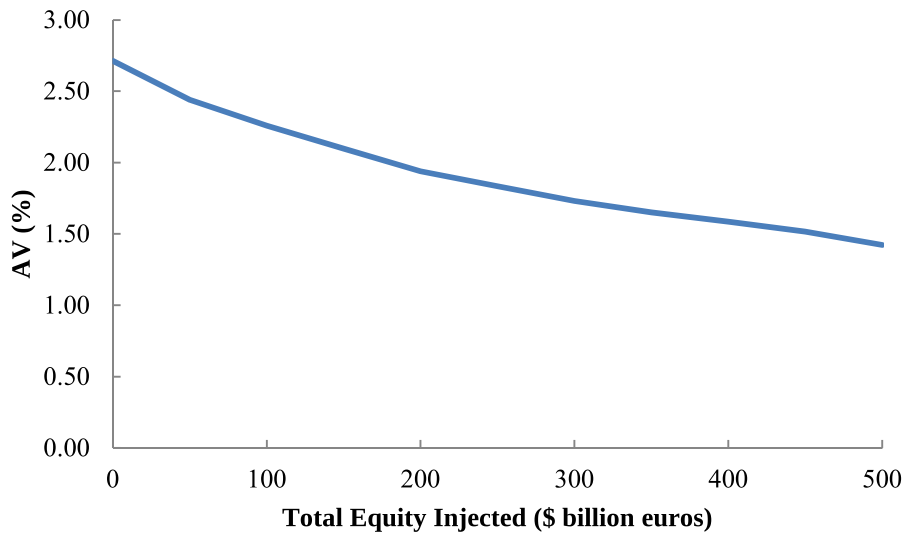


Figure 3. Aggregate vulnerability, United States financial institutions. Aggregate vulnerability AV is defined according to Eq. (6) in the text. The sample includes the top-100 US financial firms listed on CRSP in 2006.

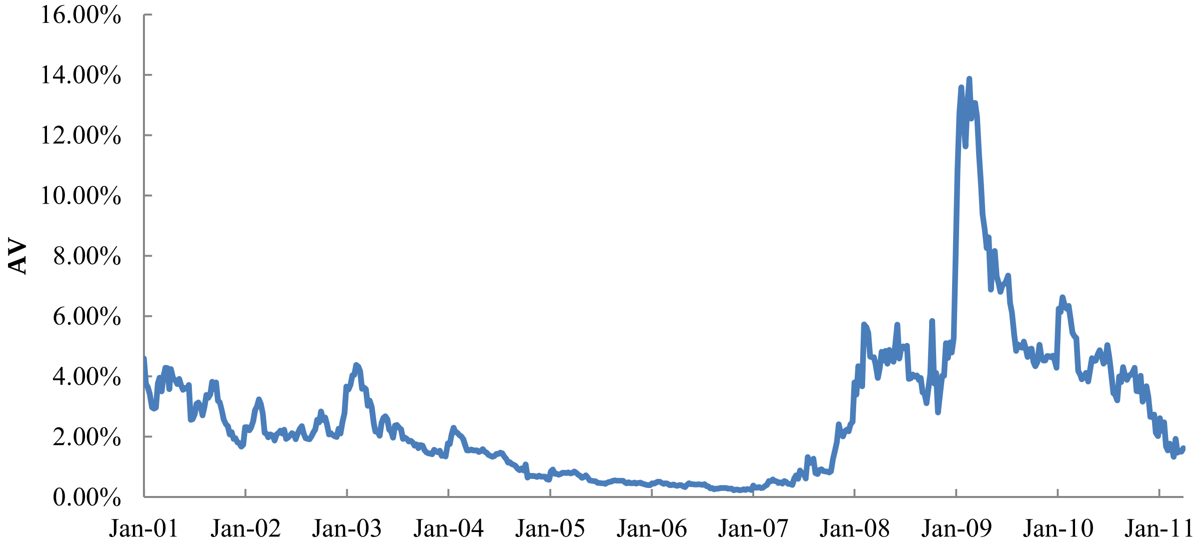


Figure 4. Contributions to time series vulnerability from various financial institutions. Vulnerability of bank i, $V(i)$, is expressed as a percentage of the bank's total equity value of all financial institutions, as in Equation (11) in text. The figure shows a few of the most important banks.

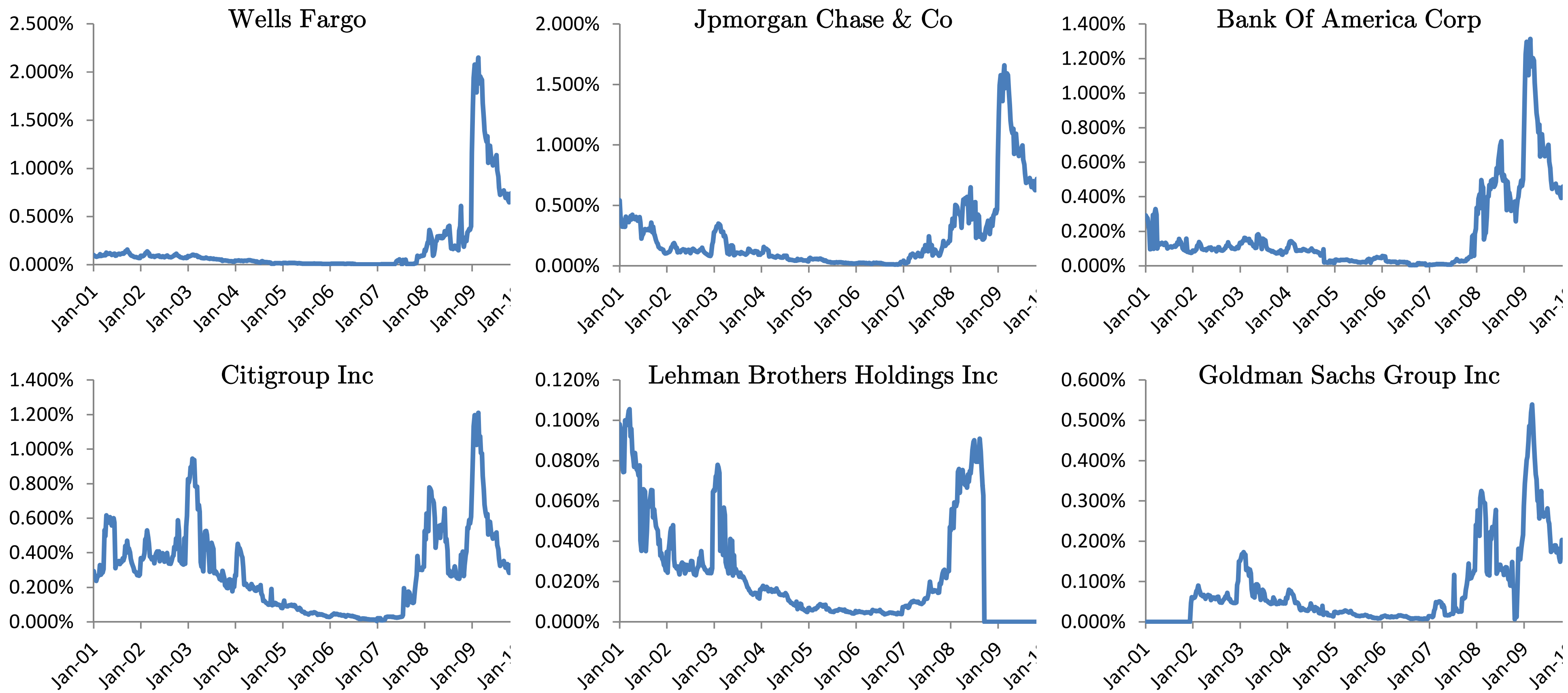
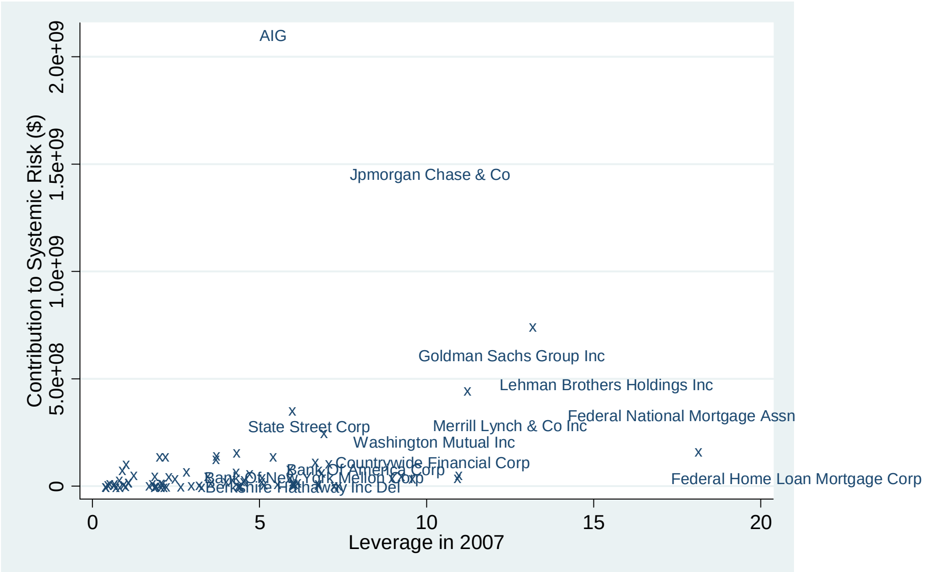


Figure 5. What drives individual banks' systemicness? We plot systemicness S(i) (in January 2008) against leverage (Panel A), and against Size (Panel B).

Panel A. Leverage vs. Systemicness



Panel B. Size vs. Systemicness

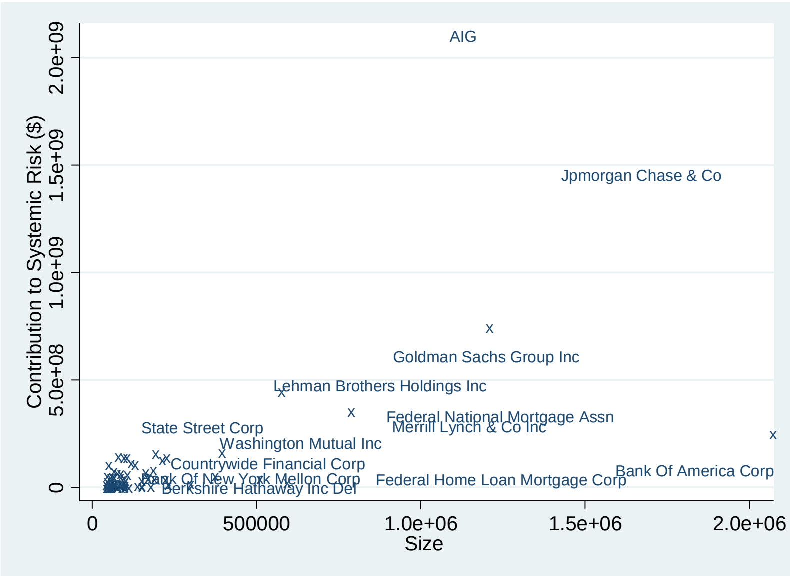


Figure 6. Bank Stocks vulnerability to Lehman Brothers collapse. Vulnerability *V*(*i*,Lehman) is the dollar price impact of predicted deleveraging driven by an expected liquidation of Lehman Brothers holdings on September 15, 2008.

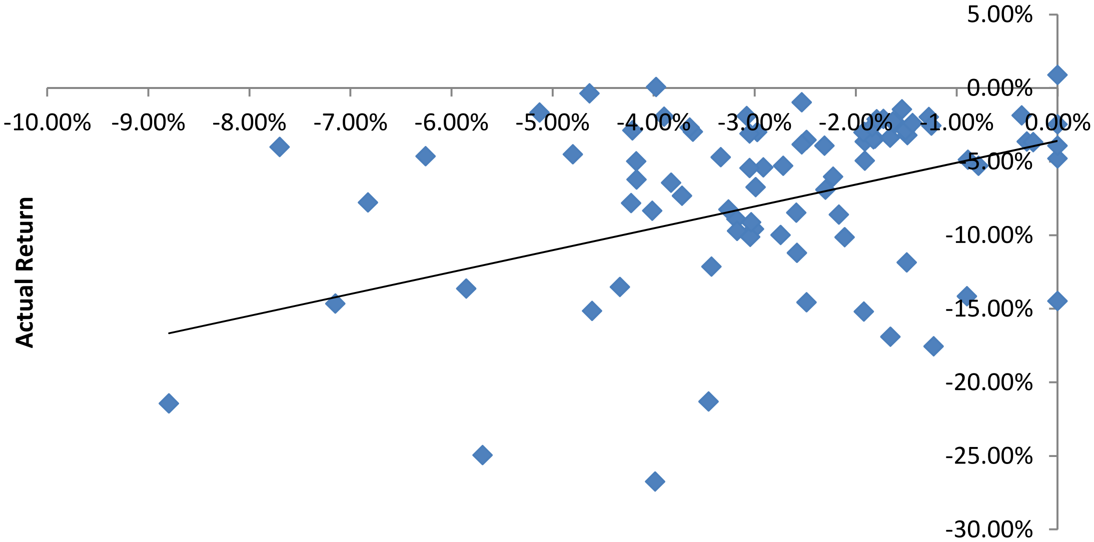


Figure 7. Vulnerability and Maximum Crisis Drawdown. We plot the maximum drawdown during the crisis against the ranking of the bank's vulnerability in January 2008. Maximum drawdown is the minimum cumulative rolling return from July 2007 through March 2011. We also show the corresponding regression, above the picture.

Maximum Drawdown(i) = -0.65 – 2.88 Vulnerability(i) [t=-3.68]

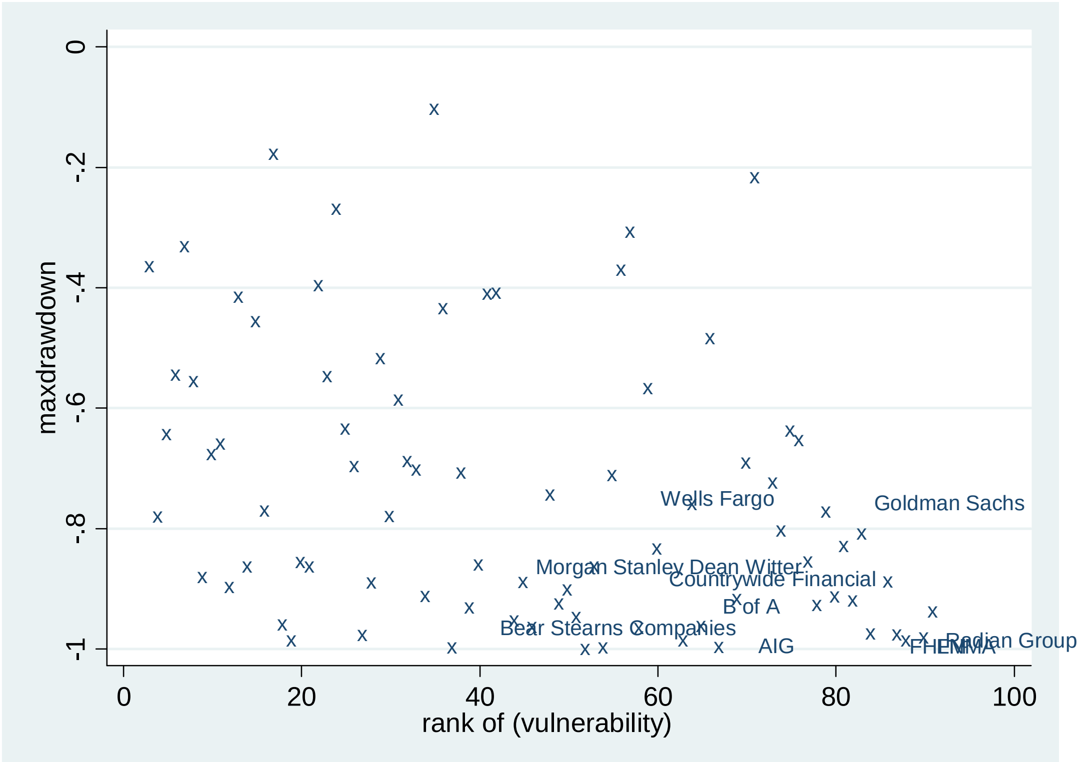


Figure 8. Vulnerability and Direct Exposure. Vulnerability $V(i)$ is a bank's exposure to deleveraging following an initial shock S. Direct Exposure (called "Round-0 exposure on the picture) is the simple levered exposure to the initial shock. The plot is drawn based on data as of January 2008.

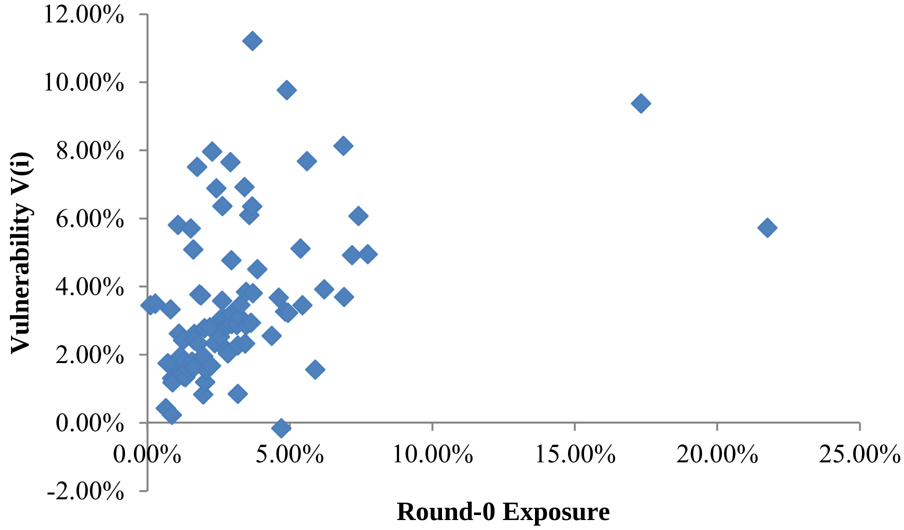


Table 1. Vulnerability to a 50% write-off on all GIIPS Debt. We compute the vulnerability of the major European banks to a 50% write-down on all sovereign debt of Greece, Italy, Ireland, Portugal, and Spain. In column 1, *IV(n)* denotes the indirect vulnerability via sector-wide deleveraging as we define it in Equation (10), adjusted for the fact that total fire sales are capped by total assets (see Section II.A.). In column 3, *DV(n)* denotes the direct vulnerability to the write-down on balance-sheets, as defined in Equation (9), adjusted for maximal fire sales. Both measures are normalized by bank equity. In column 5, the table also reports the leverage, capped at 30. We only report bank-by-bank values for the 10 largest banks in terms of deleveraging vulnerability. In the last line of the table, we also report sample averages: Hence, a 50% write-down on all GIIPS debt would wipe out 111% of the equity of the average bank through the direct impact, while the indirect impact via deleveraging would create an additional loss of 302% of equity.

Bank_Name	**Indirect Vulnerability as a Fraction of Equity**		**Direct Vulnerability as a Fraction of Equity**		**Leverage Ratio**
	IV(n)	Rank	DV(n)	Rank	Leverage Ratio b_{nn}
ALLIED IRISH BANKS PLC	35.24	1	11.9	2	30
AGRICULTURAL BANK OF GREECE	12.98	2	33.5	1	30
WESTLB AG, DÜSSELDORF	8.80	3	0.9	25	30
BANCA MONTE DEI PASCHI DI SIENA	5.08	4	3.7	3	30
OESTERREICHISCHE VOLKSBANK AG	4.83	5	0.2	56	30
SNS BANK NV	4.71	6	0.3	55	30
CAIXA DE AFORROS DE GALICIA, VIGO	4.70	7	1.4	11	30
NORDDEUTSCHE LANDESBANK	4.61	8	0.4	51	30
COMMERZBANK AG	4.54	9	1.0	21	30
CAIXA D'ESTALVIS DE CATALUNYA	4.36	10	0.8	31	30
Full sample average	3.02		1.11		22.1

Table 2. Vulnerability to GIIPS and Cumulative Stock Returns. For each publicly listed bank in our sample, we calculate the cumulative return between Dec 31, 1999 and Sep 16, 2011. We then regress this return on our measure of indirect vulnerability, controlling for direct exposure to a 50% write-off on GIIPS debt, bank size and leverage. Columns 1-3 report plain OLS estimates. Columns 4-6 report median regressions to account for outliers.

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	Cumulative return: 2009/12 - 2011/9					
Indirect Vulnerability	-0.017***	-0.008**	-0.010***	-0.013***	-0.010**	-0.010**
	[-4.34]	[-2.58]	[-2.92]	[-2.70]	[-2.59]	[-2.52]
Direct Vulnerability		-0.016***	-0.010*		-0.010***	-0.003
		[-2.93]	[-1.96]		[-2.74]	[-0.51]
log(assets)			0.069***			0.081
			[2.70]			[1.46]
Debt to Equity			-0.001			-0.004
			[-0.08]			[-0.33]
Constant	-0.435***	-0.441***	-0.099	-0.472***	-0.467***	-0.037
	[-9.24]	[-9.60]	[-0.47]	[-6.42]	[-6.53]	[-0.08]
Observations	49	49	49	49	49	49
R-squared	0.088	0.136	0.213			

Robust t-statistics in brackets
*** p<0.01, ** p<0.05, * p<0.1
Columns 1-3 report OLS estimates; columns 4-6 report median regression results. Debt-to-equity ratio is capped to 30

Table 3. Systemicness ranking in a response to a GIIPS shock. We calculate the systemicness *S(n)* of each individual bank, assuming a 50% write-off on GIIPS sovereign debt. Column 1 reports systemicness as computed in equation (7). We only report detailed information for the top 10 banks in terms of systemicness. Columns 2-4 report the element of the decomposition of systemicness as in equation (8), except that we take into account the fact that fire sales induced by the write-off are capped by total assets (see Section II.A.). Column 2 reports total exposure of each bank, normalised by aggregate equity. Column 3 reports the fraction of assets that would be fire-sold as a fraction of total exposure. Because of our cap, it is always smaller than 1. Column 4 focuses on the linkage effect. By virtue of equation (8), systemicness is the product of the elements in columns 2,3 and 4. Banks are sorted by systemicness. Through Santander, a GIIPS write-off would lead, through deleveraging, to a 21% reduction in aggregate bank equity. The last line present the aggregate sum (over the 90 banks) of systemicness, which is equal to Aggregate Vulnerability (equation (5)). A 50% write-down on GIIPS debt would wipe out, through deleveraging 245% of total bank equity.

Bank Name	Systemicness *S(n)*	Assets / Aggregate Equity (a_{nn}/E)	Fire sales $min(-b_{nn.}\square'_n MF_1, 1+\square'_n MF_1)$	Linkage effect $(1'AMLM'\square_n)$
BANCO SANTANDER S.A.	0.21	1.06	0.58	0.34
UNICREDIT S.p.A	0.19	0.88	0.69	0.31
INTESA SANPAOLO S.p.A	0.19	0.62	0.95	0.33
BBVA	0.18	0.57	0.94	0.33
BNP PARIBAS	0.15	1.37	0.36	0.30
BFA-BANKIA	0.12	0.29	0.95	0.42
CAJA DE AHORROS Y PENSIONES DE BARCELONA	0.10	0.27	0.93	0.38
SOCIETE GENERALE	0.07	0.75	0.32	0.32
COMMERZBANK AG	0.07	0.66	0.48	0.23
BANCA MONTE DEI PASCHI DI SIENA	0.06	0.22	0.92	0.32
Full Sample Average	0.03	0.27	0.44	0.30
Full Sample Total *AV*	2.45			

Table 4. Impact of Various Policies on Aggregate Vulnerability of European Banking Sector. The first line reports the aggregate vulnerability of the European banks to a 50% GIIPS write-down: induced deleveraging would destroy 245% of aggregate bank equity. The remaining rows of the table show this calculation under different hypothetical policy interventions.

Policy intervention	Detail	Summary Statistics	Aggregate Vulnerability (deviation / benchmark)	Contribution of change in distribution of Asset	Connectedness	Exposure
Baseline			0			
		Number of banks capped				
Size cap (bn euros)	500	17	0.06	0.16	0.00	-0,09
	900	8	0.04	0.07	0.00	-0,03
	1300	2	0.00	0.01	0.00	0,00
GIIPS debt re-nationalization (bn euros)		*Fraction of total renationalized*				
	96	0,2	-0.08	0.01	-0.01	-0,08
Eurobonds (swap individual sov. holdings		*Total amount of sovereign reshuffled (in bn €)*				
for the same basket of sovereigns)		1672	0.08	0.00	0.00	0,09
		Number of banks merged				
Merge banks on which a GIIPS shock	50%	47	0.13			
is at least xx% of equity	100%	20	0.01			
	150%	9	0.00			
		Number of Banks Merged				
Merge banks on which a GIIPS shock	Merge exposed only	20	0.01			
is at least 100% of equity	Merge unexposed only	6	0.00			
with banks totally unexposed	Merge all	26	0.08			
		Equity Injection (in bn €)				
Leverage cap; max D/E =	15	480	-0.28			
	20	173	-0.11			
	25	45	-0.03			
Optimized equity injection (in €bn)		*Countries*				
	200	All Europe	-0.26			
	200	German banks	-0.05			
	200	German + French	-0.09			
	200	GIIPS	-0.24			

Table 5. Optimal Equity Allocation to Reduce Aggregate Vulnerability to a GIIPS shock. We assume the social planner has 200bn euros to inject, and seeks the allocation of capital increases that maximizes the reduction in Aggregate Vulnerability. We only report here the top 20 receivers. Column 1 reports optimal equity injection in bn euros. Column 2 reports systemicness as in equation (8). Columns 3-6 provide the four components of systemicness as in equation (9): their product equals systemicness: debt to common equity ratio (col 4), total assets relative to aggregate bank equity (col. 5), bank exposure w.r.t. to the GIP shock (col. 6), and the linkage term (col. 7).

Bank	Equity Injection (bn euros)	Systemic ness	Target leverage	Size (ai / Agg. E)	Exposure to GIP shock (ei'MS)	Linkage effect (1'AML M'ei)
Banca Monte Dei ...Siena	18.20	0.17	30.00	0.22	0.08	0.32
Intesa Sanpaolo S.P.A	18.20	0.23	21.43	0.62	0.05	0.33
Caja De Ahorros Y Pensiones De Barcelona	17.90	0.16	22.38	0.27	0.07	0.38
Banco Bilbao Vizcaya Argentaria	17.77	0.22	20.87	0.57	0.06	0.33
Bfa-Bankia	17.40	0.16	28.63	0.29	0.05	0.42
Banco Santander S.A.	12.04	0.21	22.99	1.06	0.03	0.34
Unicredit S.P.A	12.00	0.19	22.39	0.88	0.03	0.31
Banco Popolare	8.11	0.07	30.00	0.13	0.05	0.36
Bnp Paribas	6.04	0.15	22.62	1.37	0.02	0.30
Banco De Sabadell	4.68	0.04	25.26	0.10	0.04	0.40
Banco Comercial Português	4.34	0.04	27.16	0.10	0.04	0.34
Ubi Banca	4.13	0.04	20.37	0.15	0.04	0.33
Banco Popular Español	3.53	0.03	18.50	0.14	0.04	0.35
National Bank Of Greece	3.52	0.03	12.64	0.11	0.09	0.28
Efg Eurobank Ergasias	3.26	0.03	22.88	0.08	0.06	0.26
Commerzbank Ag	3.14	0.07	30.00	0.66	0.02	0.23
Bank Of Ireland	2.98	0.03	29.36	0.17	0.02	0.32
Caja De Ahorros Del Mediterráneo	2.96	0.03	30.00	0.07	0.04	0.34
Piraeus Bank Group	2.69	0.02	16.69	0.05	0.09	0.34
Caixa De Aforros De Galicia	2.66	0.03	30.00	0.07	0.04	0.36

Table 6: Robustness to Liquidation Rules. We calculate the aggregate vulnerability AV to a 50% writedown of GIIPS debt. In line 1, we report the baseline. In line 2, we assume only sovereigns can be sold. In line 3, we assume sovereigns and commercial real estate only can be sold. In line 4, we add mortgages to the list of assets that can be sold. In line 7, we include all known assets (typically about 80 % of total exposure). Implicitly, the difference here with the first line is that we assume banks have no cash to adjust.

	GIIPS	**Liquid assets / total**
Benchmark	-2.85	1.00
Sovereigns only	-0.23	0.12
+ Commercial real estate	-0.47	0.18
+ Mortgages	-2.40	0.41
+ Corporate loans	-4.11	0.68
+ Consumer loans	-4.02	0.70
+ SME loans	-3.84	0.75

Table 7. Risk factors used to proxy for bank holdings. The factors consist of the weekly returns on S&P non-financial firms, returns on US Mortgage REITs, returns on the US10yr Treasury, the return on the GSCI Commodities index, and the return on high yield bonds. The data span 2001 through March 2011.

Panel A. Summary Statistics

	Full sample		Crisis period (March 2007-May 2009)	
	Mean Return (%)	Volatility (%)	Mean Return (%)	Volatility (%)
SP Returns	0.19	3.21	-0.28	4.55
Mortgage REITs	-0.01	3.64	-0.74	5.82
US 10 yr Return	-0.02	0.55	-0.05	0.69
Commodities	0.12	3.59	-0.16	4.62
High Yield Returns	0.15	1.26	-0.05	2.13

Panel B. Correlations

	SP Returns	Mortgage REITs	US 10 yr Return	Commodities	High Yield Returns
SP Returns	1.00				
Mortgage REITs	0.57	1.00			
US 10 yr Return	0.28	0.07	1.00		
Commodities	0.24	0.06	0.14	1.00	
High Yield Returns	0.54	0.37	0.21	0.25	1.00

Table 8. The impact of the Lehman Brothers failure on other banks. We regress stock returns on September 15, 2008 on V(I,Lehman) which is the impact of Lehman induced fire sales on each bank. T-statistics are shown in brackets.

	Dep. Var = Return on September 15, 2008	
Predicted Return from deleveraging *V*(*i*, Lehman)	1.48	1.31
	[3.04]	[2.44]
Log(Size)		-0.01
		[-1.86]
Log(Leverage)		-0.09
		[-0.11]
R^2	0.10	0.16

Table 9. Top 10 Systemic Banks, selected dates. We show $S(i)$ as well as $S(i)/AV$. $S(i)$ is systemicness, and is the impact of each bank on aggregate vulnerability AV. It is defined in Equation (9).

Jan-07			Jan-08			Jan-09		
Name	**S(i)**	**S(i)/AV % of total**	**Name**	**S(i)**	**S(i)/AV % of total**	**Name**	**S(i)**	**S(i)/AV % of total**
AIG	0.07%	19.6%	Citigroup Inc	0.66%	17.4%	Wells Fargo	1.60%	20.4%
Jpmorgan Chase	0.05%	13.6%	Goldman Sachs	0.49%	12.9%	Jpmorgan Chase	1.26%	16.0%
Morgan Stanley	0.03%	7.0%	Jpmorgan Chase	0.36%	9.4%	Bank Of America	0.88%	11.3%
Goldman Sachs	0.02%	5.7%	FNMA	0.33%	8.6%	Citigroup	0.74%	9.4%
Lehman Brothers	0.02%	4.4%	Bank Of America	0.19%	5.0%	Intercontinentalexchange	0.23%	3.0%
Metlife Inc	0.02%	4.2%	AIG	0.17%	4.5%	BONY Mellon	0.18%	2.2%
Wachovia Corp	0.01%	3.3%	American Express	0.13%	3.5%	Merrill Lynch & Co Inc	0.18%	2.2%
FNMA	0.01%	3.1%	FHLM	0.13%	3.4%	Goldman Sachs	0.15%	1.9%
Merrill Lynch	0.01%	2.7%	Lehman Brothers	0.10%	2.5%	Regions Financial	0.15%	1.9%
State Street Corp	0.01%	2.6%	Metlife Inc	0.09%	2.4%	Capital One Financial	0.14%	1.8%

Table 10. Top 10 Vulnerable Financial Institutions, selected dates. We show vulnerability expressed as a percentage of equity value. Vulnerability is the impact of an aggregate shock to all factors on each single bank. We a;sp sjpw the direct exposure of each bank to the shocks considered. For each date and in each panel, we show the 10 most vulnerable banks in the sample. Banks are ranked by Vulnerability V(i)

2007			2008			2009		
Name	**Round 0 Exposure**	**V(i) %**	**Name**	**Round 0 Exposure**	**V(i) %**	**Name**	**Round 0 Exposure**	**V(i) %**
Radian Group	2.31%	1.19%	Radian Group	20.33%	19.43%	M G I C Investment Wis	38.09%	30.49%
AIG	1.06%	1.18%	Federal National Mortgage	3.27%	11.68%	Intercontinentalexchange	19.00%	24.18%
M G I C Investment Wis	1.75%	1.15%	C B Richard Ellis Group	7.57%	9.09%	American Capital Ltd	21.27%	23.94%
Sovereign Ban	0.86%	1.10%	Citigroup	2.87%	8.23%	C B Richard Ellis Group	11.46%	23.18%
M B I A	1.88%	0.95%	Federal Home Loan Mortgage	2.07%	7.95%	C M E Group	6.20%	16.47%
Ambac Financial Group	1.12%	0.84%	American Capital Ltd	3.01%	7.24%	Fifth Third Ban	10.18%	15.78%
Metlife	1.26%	0.79%	E Trade Financial	11.38%	6.96%	Legg Mason	10.80%	14.14%
State Street	1.80%	0.76%	Synovus Financial	1.90%	6.88%	Regions Financial New	14.06%	13.94%
C B Richard Ellis Group	4.32%	0.75%	Goldman Sachs Group	4.72%	6.65%	Wells Fargo New	9.43%	13.87%
Jpmorgan Chase	1.35%	0.74%	Fifth Third Ban	2.11%	6.57%	M B I A	8.57%	13.66%

APPENDIX

We prove below that when some banks have similar leverage and similar asset mix, merging them has no impact on aggregate *AV*. Equivalently, dividing a bank into several banks having the same levels of leverage and the same asset mix has no impact on *AV*.

The proof is the following: Assume there are *N+s-1* banks and that the last *s* banks all have same leverage b_N and same portfolio weights $m_{n,k}$. Since they have the same mix of assets, they also have same asset returns r_N

Developing formula (6) yields:

$$AV \times E_1 = \sum_{n\in[1,N+s]} \sum_{k\in[1,K]} \sum_{m\in[1,N+s]} a_m m_{m,k} l_k m_{n,k} b_n a_n r_n$$

$$AV \times E_1 = \sum_{n\in[1,N-1]} \sum_{k\in[1,K]} \sum_{m\in[1,N-1]} a_m m_{m,k} l_k m_{n,k} b_n a_n r_n$$

$$+ \sum_{n\in[N,N+s]} \sum_{k\in[1,K]} \sum_{m\in[N,N+s]} a_N m_{N,k} l_k m_{N,k} b_N a_n r_N$$

$$AV \times E_1 = \sum_{n\in[1,N-1]} \sum_{k\in[1,K]} \sum_{m\in[1,N-1]} a_m m_{m,k} l_k m_{n,k} b_n a_n r_n$$

$$+ \sum_{k\in[1,K]} \left(\sum_{m\in[N,N+s]} a_m \right) m_{N,k} l_k m_{N,k} b_N \left(\sum_{n\in[N,N+s]} a_n \right) r_N$$

This expression is strictly identical to the *AV* of a system where the first *N-1* banks are similar to the previous system ($\tilde{a}_m = a_m; m \in [1, N-1]$) and the last one, bank *N*, is the combination of the previous last *s* banks: $\left(\tilde{a}_N = \sum_{n\in[N,N+s]} a_n\, ;\ \tilde{b}_N = b_N;\quad \widetilde{m}_{N,k} = m_{N,k}\right)$.