Aggregate Shocks or Aggregate Information? Costly Information and Business Cycle Comovement Technical Appendix

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A Appendix

A.1 Proof of proposition 1

Using equation (4), the covariance of productivity is $\beta_i \beta_j \sigma_z^2$. For a given information choice, $Var[z_i|\mathcal{F}_i]$ is not random. The only random variable in labor choice is $E[z_i|\mathcal{F}_i]$. Since correlations are invariant to linear transformations, $corr(n_i, n_j) = corr(E[z_i|\mathcal{F}_i], E[z_j|\mathcal{F}_i])$.

For firms with aggregate information, the conditional expectation is given by equation (9); the only random variable is s_0, the common signal both agents observe. The aggregate signal s_0 enters in both conditional expectations linearly. Thus, $corr(E[z_i|s_0], E[z_j|s_0]) = corr(s_0, s_0) = 1$, and therefore $corr(n_i^a, n_j^a) = 1$. Since the correlation of the informed firms labor input cannot exceed one, the correlation of aggregate-information labor input must be weakly greater.

To establish strict inequality, we must compute the correlation of informed firms' labor, using (3) and (7): $corr(n_i^{FI}, n_j^{FI}) = \beta_i \beta_j \sigma_z^2 [(\beta_i^2 \sigma_z^2 + \sigma_\eta^2)(\beta_j^2 \sigma_z^2 + \sigma_\eta^2)]^{-1/2}$. Note that the denominator is strictly larger than the numerator, and thus the correlation is strictly less than one whenever $\sigma_\eta^2 > 0$. Therefore $corr(n_i^a, n_j^a) > corr(n_i^{FI}, n_j^{FI})$ whenever $\sigma_\eta^2 > 0$.

A.2 Output Covariance in the Island Model

Corollary 1 *When any two industries observe the aggregate signal only (AG), the covariance of their output is*

$$cov(y_i^{AG}, y_j^{AG}) = \alpha_i \alpha_j \left\{ \beta_i \beta_j \sigma_z^2 (3\sigma_z^2 + \phi_0 + \gamma_i \gamma_j) + \mu_z \sigma_z^2 (\mu_z - \gamma_i \beta_i - \gamma_j \beta_j) + \phi_0^2 \mu_z^2 \right\}. \quad (1)$$

For two industries that observe their industry-specific signal, the industry-information (II) output covariance is

$$cov(y_i^{II}, y_j^{II}) = \frac{\alpha_i^{II} \alpha_j^{II}}{\rho^2 Var[z_i|s_i] Var[z_j|s_j]} \left\{ \sigma_\eta^4 + \sigma_\eta^2 \sigma_z^2 (\beta_i + \beta_j) + \beta_i \beta_j \mu_z^2 (\sigma_z^2 + \phi_0^2) \right. \quad (2)$$

$$\left. + (\beta_i \beta_j)^2 \sigma_z^2 (3\sigma_z^2 + \phi_0^2) + \frac{\beta_i \beta_j \sigma_z^2}{\alpha_i^{II} \alpha_j^{II}} (\mu_z - \psi)(\mu_z (1 + \alpha_i^{II} + \alpha_j^{II}) - \psi) \right\}$$

With Aggregate Signal For firms that observe the aggregate signal, their labor input is given by (11). Combining with the expression for z_i from (4) and substituting in the definition of s_0:

$$z_i n_i = \alpha_i(\beta_i \bar{z} + \eta_i + e_i)(\bar{z} + e_0 + \gamma_i) \tag{3}$$

After removing additive constant terms, the covariance is

$$cov(y_i, y_j) = \alpha_i \alpha_j \beta_i \beta_j \{E[(\tilde{z} + \mu_z)^2)(\tilde{z} + e_0 + \gamma_i)(\tilde{z} + e_0 + \gamma_j)] - E[\tilde{z}^2 + \mu_z \gamma_i] E[\tilde{z}^2 + \mu_z \gamma_j]\} \tag{4}$$

where $\tilde{z}$ is the mean-zero variable $\bar{z} - \mu_z$. Taking expectations, using the fact that $E[\tilde{z}^4] = 3\sigma_z^2$, $E[\tilde{z}^3] = 0$, $E[e_0] = \phi_0^2$ and rearranging delivers the expression in the corollary.

With Full Information The full-information optimal labor supply is $n_i = (\beta_i \bar{z} + \eta_i - \psi)/(\rho \phi_i^2)$. Combining this with the expression for z_i yields $z_i n_i = (\beta_i \bar{z} + \eta_i + e_i)(\beta_i \bar{z} + \eta_i - \psi)/(\rho \phi_i^2)$. Expected output is $E[z_i n_i] = (\beta_i^2(\sigma_z^2 + \mu_z^2) - \psi \beta_i \mu_z + \sigma_\eta^2)/(\rho \phi_i^2)$.

To compute output covariance, we first take $E[y_i y_j] - E[y_i]E[y_j]$ and cancel out the cross-terms equal to zero, in expectation. This leaves us with

$$cov(y_i, y_j) = \frac{1}{\rho^2 \phi_i^2 \phi_j^2} \left\{ E[(\beta_i^2 \bar{z}^2 + +\eta_i + \psi \beta_i \bar{z})(\beta_j^2 \bar{z}^2 + +\eta_j + \psi \beta_j \bar{z})] \right. \tag{5}$$

$$\left. (\beta_i^2(\sigma_z^2 + \mu_z^2) - \psi \beta_i \mu_z + \sigma_\eta^2)(\beta_j^2(\sigma_z^2 + \mu_z^2) - \psi \beta_j \mu_z + \sigma_\eta^2) \right\}$$

Simplifying this expression and using the formulas for the higher moments detailed above, we get the expression in the corollary.

A.3 Proof of proposition 2 (Derivation of information value)

Substituting the optimal labor choice in the utility function and applying the law of iterated expectations yields

$$U = E[E[-exp\left(-\rho(z_i - \psi)\frac{1}{\rho Var[z_i|\mathcal{I}_i]}(E[z_i|\mathcal{I}_i] - \psi)\right)|E[z_i|\mathcal{I}_i]]] \cdot K \tag{6}$$

where $K = exp(\rho \sum_j(-\pi_j + L_{ij}p_j))$ is the utility benefit from information sales or cost of purchases. That part of utility is deterministic. Inside the inner expectation, the only random variable is z_i, which is normally distributed about $E[z_i|\mathcal{I}_i]$ with variance $Var[z_i|\mathcal{I}_i]$. Applying the formula for the expectation of a log normal variable, and combining terms yields

$$U = E[-exp\left(-\frac{1}{2}\frac{(E[z_i|\mathcal{I}_i] - \psi)^2}{Var[z_i|\mathcal{I}_i]}\right)] \cdot K. \tag{7}$$

The one random variable left in the expectation is $E[z_i|\mathcal{I}_i]$. Because beliefs are a martingale, its expectation must be equal to the prior mean μ_i. The variance of beliefs after observing the signal is $\sigma_i^2 - Var[z_i|\mathcal{I}_i]$. Using the moment-generating formula for a non-central chi-square, the expectation can be re-written as

$$U = -\left(\frac{Var[z_i|\mathcal{I}_i]}{\sigma_i^2}\right)^{1/2} \exp\left(\frac{-1}{2\sigma_i^2}(\mu_i - \psi)^2\right) \cdot K. \tag{8}$$

The exponential term contains only parameters and prior beliefs. Information only affects utility multiplicatively. The lower the standard deviation of posterior beliefs, the less negative utility is.

To derive the willingness to pay for information, substitute back in the constant K. For an agent the purchases a signal s_j at cost p_j

$$U(s_j) = -\left(\frac{Var[z_i|s_j]}{\sigma_i^2}\right)^{1/2} \exp\left(\frac{-1}{2\sigma_i^2}(\mu_i - \psi)^2\right) \cdot \exp\left(-\rho\sum_k \pi_k + \rho p_j\right). \tag{9}$$

For the agent that does not purchase a signal, the posterior and prior variances are equal:

$$U_{no\ info} = -\exp\left(\frac{-1}{2\sigma_i^2}(\mu_i - \psi)^2\right) \cdot \exp\left(\rho\sum_k \pi_k\right). \tag{10}$$

Information increases expected utility when $U(s_j) > U_{no\ info}$, which is true when

$$-\left(\frac{Var[z_i|s_j]}{\sigma_i^2}\right)^{1/2} \exp(\rho p_j) > -1. \tag{11}$$

Rearranging and taking logs on both sides yields the condition in the text. □

A.4 Proof of proposition 3

Part I: If only one industry l chooses to observe its industry-specific productivity, but industry i and industry j both choose not to, then $corr(n_i, n_j) = 1$ or -1.

If l learns, then $(z_l + \eta_l)$ is the public signal about aggregate productivity. Posterior beliefs are $\hat{\hat{z}} = (z_l + \eta_l)\phi_l^{-2}/(\phi_l^{-2} + \sigma_z^{-2})$. Note that this posterior is comprised of known constants and $(z_l + \eta_l)$, and is linear in $(z_l + \eta_l)$.

Substituting these posteriors into equation (16), tells us that the wage is

$$w = 1/K_1[\sum_{i \neq l} \beta_i \hat{\hat{z}}/V_i + (z_l + \eta_l)/V_l] + \mu_z$$

where K_1 is a known constant, as are the posterior variances V_i and V_l. Since $\hat{\hat{z}}$ is linear in $(z_l + \eta_l)$, we can rewrite $(z_l + \eta_l) = K_2\hat{\hat{z}}$. Thus,

$$w = 1/K_1[\sum_{i \neq l} \beta_i/V_i + K_2/V_l]\hat{\hat{z}} + \mu_z.$$

Substituting the posterior and the wage into equation (3) tells us that labor inputs in an uninformed sector i are

$$n_i = 1/(\rho V_i)((\beta_i - 1/K_1(\sum_{i \neq l} \beta_i/V_i + K_2/V_l))\hat{\hat{z}} + \mu_z)$$

as long as the non-negativity constraints on n_i don't bind. The labor input of sector j is defined analogously. Since both are linear functions of one random variable $\hat{\hat{z}}$, their correlation is 1 if $(\beta_i - 1/K_1(\sum_{i \neq l} \beta_i/V_i + K_2/V_l)$ and $(\beta_j - 1/K_1(\sum_{i \neq l} \beta_i/V_i + K_2/V_l)$ have the same sign and -1 otherwise.

There is a knife-edge case where $(\beta_i - 1/K_1(\sum_{i\neq l}\beta_i/V_i + K_2/V_l) = 0$ for either industry, in which case the correlation will be zero. Since with any random draw of parameters, this is a measure-zero event, the proposition focuses on the other two cases.

Part II: If more than one industry chooses to observe its industry-specific productivity, but industry i and industry j both choose not to, then $corr(n_i, n_j) = 1$ iff $\beta_i = \beta_j$.

Let $\hat{\hat{z}}$ be the posterior belief about aggregate technology, derived from the public signals. Equation (16), tells us that the wage is

$$w = 1/K_1[\sum_{i\in Un}\beta_i\hat{\hat{z}}/V_i + \sum_{l\in In}(z_l + \eta_l)/V_l] + \mu_z$$

where K_1 is a known constant, as are the posterior variances V_i and V_l, Un represents the set of firms who are uninformed and In is the set of informed firms. The two sum terms can be rewritten as $K_2\hat{\hat{z}} + e_z$, where $K_2 = 1/K_1(\sum_i \beta_i/V_i)$ and $e_z = 1/K_1\sum_{l\in In}(z_l + \eta_l - \hat{\hat{z}})/V_l$, which is independent of $\hat{\hat{z}}$.

Substituting the posterior and the wage into equation (3) tells us that labor inputs in an uninformed sector i are

$$n_i = 1/(\rho V_i)((\beta_i - K_2)\hat{\hat{z}} + e_z + \mu_z)$$

as long as the non-negativity constraints on n_i don't bind. The labor input of sector j is defined analogously.

Labor covariance is

$$cov(n_i, n_j) = (\beta_i - K_2)(\beta_j - K_2)Var(\hat{\hat{z}}) + Var(e_z).$$

The product of standard deviations of labor input is

$$std(n_i)std(n_j) = ((\beta_i - K_2)Var(\hat{\hat{z}}) + Var(e_z))^{1/2}((\beta_j - K_2)Var(\hat{\hat{z}}) + Var(e_z))^{1/2}.$$

The necessary condition for a correlation of 1 is that $cov(n_i, n_j) = std(n_i)std(n_j)$. This is the case, if an only if $\beta_i = \beta_j$. □

B Data Description

We detrend the annual data from (Basu, Fernald and Kimball 2006) using a Hodrick-Prescott filter. We set the smoothing parameter to 6, as suggested by Ravn and Uhlig (2002). Given the similarity of our approaches, it is reassuring that our description of industry comovement is largely similar to that in Christiano and Fitzgerald (1999). But there are differences in our data sources, industry categorizations, sample periods and detrending procedures, although none that lead us to expect important differences. One of the advantages of the data provided by Basu et al. (2006) is that they have constructed a "purified" measure of sectoral total factor productivity (TFP)—a measure of the Solow residual, constructed to take account of non-constant returns to scale in industry production functions, imperfect competition, and varying utilization of labor and capital inputs.

Table 1 provides greater detail about the cyclical behavior of these industries. Column one shows the correlation of sectoral value-added with aggregate value-added, while column two shows the correlation of sectoral input use with aggregate input use.

Industry	**C**orrelation of industry data with aggregates.		
	Value-added	**I**ndex of inputs	**T**FP
Construction	0.70	0.79	0.72
Food	0.47	0.09	0.29
Tobacco	0.30	-0.12	-0.02
Textiles	0.18	0.68	-0.20
Apparel	0.52	0.40	0.08
Lumber	-0.02	0.76	0.40
Furniture	0.86	0.84	0.12
Paper	0.60	0.70	0.27
Printing	0.68	0.61	0.30
Chemicals	0.73	0.55	0.52
Petroleum	0.34	0.30	0.29
Rubber	0.67	0.83	-0.08
Leather	-0.37	0.53	-0.31
Stone	0.90	0.85	0.28
Primary metal	0.83	0.81	0.34
Fab. metal	0.87	0.86	0.37
Machinery	0.74	0.82	0.35
Electrical machinery	0.86	0.80	0.15
Autos	0.72	0.56	-0.06
Transport equip	0.25	0.35	0.26
Instruments	0.78	0.65	0.08
Misc. Manufacturing	0.39	0.56	0.14
Transportation	0.75	0.91	0.18
Communications	0.17	0.37	-0.10
Elec. Utilities	0.32	0.29	-0.17
Gas Utilities	-0.01	0.17	-0.35
Trade	0.68	0.84	0.61
FIRE	0.30	0.12	0.33
Services	0.58	0.61	0.16
Simple average	0.51	0.57	0.17
Share-weighted average	0.58	0.61	0.32

Table 1: Coherence of Output, Inputs and TFP across industries.
Each cell shows the correlation of industry output, inputs or TFP with the corresponding aggregate.

	Summary stats		Regression Results					Single factor: TFP growth in the rest of the economy		Dependent variable: Industry TFP growth		Residuals: Industry-specific shock		Fitted values	
Industry	Observations	Average output share	Industry Beta	(se)	Industry Fixed Effects	(se)	R-sq.	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Construction	48	7.3%	1.10	0.20	-0.5%	0.2%	30.7%	0.4%	1.2%	-0.1%	2.3%	0.0%	1.9%	-0.1%	1.3%
Food	48	2.8%	0.16	0.27	1.1%	0.4%	1.5%	0.3%	1.4%	1.1%	1.9%	0.0%	1.9%	1.1%	0.2%
Tobacco	48	0.3%	-0.05	0.88	0.1%	1.3%	0.0%	0.3%	1.4%	0.1%	6.9%	0.0%	6.9%	0.1%	0.1%
Textiles	48	0.7%	0.23	0.53	1.8%	0.8%	0.9%	0.4%	1.5%	1.9%	3.8%	0.0%	3.8%	1.9%	0.4%
Apparel	48	1.4%	0.55	0.38	1.4%	0.6%	9.6%	0.4%	1.5%	1.6%	2.6%	0.0%	2.5%	1.6%	0.8%
Lumber	48	0.9%	0.71	0.47	1.0%	0.7%	13.3%	0.4%	1.5%	1.2%	2.9%	0.0%	2.7%	1.2%	1.1%
Furniture	48	0.5%	0.18	0.61	0.8%	0.9%	1.9%	0.4%	1.5%	0.8%	1.9%	0.0%	1.9%	0.8%	0.3%
Paper	48	1.3%	0.59	0.39	0.2%	0.6%	12.9%	0.4%	1.4%	0.4%	2.3%	0.0%	2.2%	0.4%	0.8%
Printing	48	1.8%	0.38	0.33	0.3%	0.5%	10.9%	0.3%	1.4%	0.5%	1.7%	0.0%	1.6%	0.5%	0.5%
Chemicals	48	2.8%	0.82	0.28	-2.0%	0.4%	8.5%	0.4%	1.3%	-1.7%	3.7%	0.0%	3.5%	-1.7%	1.1%
Petroleum	48	0.9%	0.43	0.48	0.2%	0.7%	2.1%	0.3%	1.4%	0.3%	4.3%	0.0%	4.2%	0.3%	0.6%
Rubber	48	0.9%	0.15	0.46	1.4%	0.7%	0.3%	0.3%	1.4%	1.4%	3.9%	0.0%	3.9%	1.4%	0.2%
Leather	48	0.3%	0.20	0.80	-0.9%	1.3%	0.6%	0.6%	1.5%	-0.8%	3.9%	0.0%	3.9%	-0.8%	0.3%
Stone, clay, glass	48	1.1%	0.40	0.42	0.1%	0.6%	15.4%	0.4%	1.5%	0.3%	1.5%	0.0%	1.4%	0.3%	0.6%
Primary metal	48	2.3%	0.71	0.32	-0.2%	0.5%	11.7%	0.5%	1.4%	0.1%	2.9%	0.0%	2.7%	0.1%	1.0%
Fabricated metal	48	2.4%	0.58	0.29	0.0%	0.4%	14.1%	0.3%	1.4%	0.2%	2.2%	0.0%	2.0%	0.2%	0.8%
Machinery	48	3.3%	0.36	0.25	0.6%	0.4%	2.3%	0.3%	1.4%	0.7%	3.4%	0.0%	3.3%	0.7%	0.5%
Electrical Machinery	48	2.4%	0.21	0.29	1.4%	0.4%	0.9%	0.3%	1.4%	1.4%	3.2%	0.0%	3.2%	1.4%	0.3%
Motor Vehicles	48	2.0%	-0.17	0.31	0.1%	0.5%	0.6%	0.4%	1.5%	0.0%	3.2%	0.0%	3.2%	0.0%	0.3%
Transport equip	48	1.9%	0.80	0.33	0.4%	0.5%	13.0%	0.3%	1.4%	0.6%	3.1%	0.0%	2.9%	0.6%	1.1%
Instruments	48	1.4%	-0.05	0.39	1.8%	0.6%	0.1%	0.2%	1.4%	1.8%	2.3%	0.0%	2.3%	1.8%	0.1%
Misc Manufacturing	48	0.6%	0.36	0.57	0.5%	0.9%	1.9%	0.4%	1.5%	0.6%	3.9%	0.0%	3.9%	0.6%	0.5%
Transportation	48	5.7%	-0.05	0.18	0.9%	0.3%	0.1%	0.3%	1.5%	0.9%	2.2%	0.0%	2.2%	0.9%	0.1%
Communications	48	2.9%	-0.17	0.26	-0.7%	0.4%	1.1%	0.4%	1.4%	-0.8%	2.3%	0.0%	2.3%	-0.8%	0.2%
Electric Utilities	48	2.3%	-0.31	0.29	-2.0%	0.4%	2.2%	0.4%	1.4%	-2.1%	3.0%	0.0%	3.0%	-2.1%	0.4%
Gas Utilities	48	0.7%	-0.68	0.51	0.1%	0.8%	7.9%	0.4%	1.5%	-0.2%	3.5%	0.0%	3.4%	-0.2%	1.0%
Trade	48	19.5%	-0.30	0.11	0.9%	0.1%	3.3%	0.1%	1.4%	0.9%	2.2%	0.0%	2.2%	0.9%	0.4%
FIRE	48	11.7%	0.10	0.14	1.2%	0.2%	0.8%	0.1%	1.4%	1.2%	1.5%	0.0%	1.5%	1.2%	0.1%
Services	48	18.2%	-0.39	0.11	-1.5%	0.2%	9.4%	0.9%	1.4%	-1.9%	1.8%	0.0%	1.7%	-1.9%	0.6%

Figure 1: Descriptive statistics for industry TFP 1-factor model.

References

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