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Technical Appendix:
Unmeasured Investment and the Puzzling U.S. Boom in the 1990s*

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* The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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Chapter 1.

Introduction

In this Appendix, we provide supporting evidence for claims made in our paper "Unmeasured Investment and the Puzzling U.S. Boom in the 1990s" and address issues raised in seminars and by referees. Here, we summarize our findings as responses to four common myths (which arise in most discussions of the paper).

The first common myth is that intangible investment is simply a free parameter that "makes up for whatever is missing to make standard theory work." (Here, we are quoting one of our referees.) To dispel this myth, we apply our methodology to three theories of the U.S. boom in the 1990s. *All three theories generate paths of GDP, consumption, investment, and hours that match U.S. data perfectly.* Despite the perfect fit for all theories, only one of the three theories satisfies our criteria for a successful theory.[1] One of the unsuccessful theories does include intangible capital and does generate a boom in the 1990s, but does not satisfy our criteria.

We also demonstrate that we would get a very different result if the data-generating mechanism were in fact inconsistent with our theory of intangible investment and non-neutral technological change. This is a slightly different way of making the point that intangible investment is not a free parameter that makes up for whatever is missing. We set up the following experiment. First, we generate artificial data from a model with *no* intangible investment that has hours fluctuating only because there have been changes in

[1] Our *input justification criterion* requires the exogenous inputs to be consistent with micro and macro evidence. Our *prediction criterion* requires conformity of theory and observed time series that were not used to set parameters or exogenous inputs.

labor market distortions (which are not due to changes in government labor tax rates). Second, with these data, we assess the theory *with* intangible investment and non-neutral technology. In this case, we find that our theory would satisfy neither our input justification criterion nor our prediction criterion.

The second common myth is that the neoclassical growth model does poorly over the *entire postwar period*—especially with regard to movements in hours of work—and not just in the 1990s as we claimed. For example, Stephanie Schmitt-Grohe and Martin Uribe (2005) motivated their work in an interview with the *NBER Reporter* by noting that "by the late 1990s empirical research using macroeconomic data from industrialized countries had cast compelling doubts on the ability of the neoclassical growth model to provide a satisfactory account of aggregate fluctuations" (p. 19). In dispelling the myth that neoclassical theory is doomed, we draw heavily on the work of Uhlig (2003) and Chen, İmrohoroğlu, and İmrohoroğlu (2007).[2]

A third common myth is that the class of new Keynesian models that Schmitt-Grohe, Uribe and others have adopted provides a better understanding of business cycles. With a new Keynesian model developed and used by Smets and Wouters (2007) to study U.S. business cycles, we show that this current-generation model, *which is designed to perfectly fit seven U.S. time series—GDP, consumption, investment, business hours, business wages, the federal funds rate, and inflation—fails to satisfy our criteria for a successful theory.*

The fourth and final myth is that conclusions based on perfect-foresight analyses are not robust. We use models with stochastic variation in the key exogenous variables to

[2] We thank Harold Uhlig, Kaiji Chen, and Rafael Wouters for providing us with data and codes used in their papers that we discuss in this Appendix. This made it relatively easy to reproduce their research and to make direct comparisons between our work and theirs.

show that the specific realizations of these exogenous stochastic variables, not the choice of household expectations, are what is important. We also do sensitivity analysis with respect to choices of parameters of preferences and technologies to show that our findings and conclusions are robust.

Chapter 2.

Assessing Three Theories

In this chapter we consider three theories. All three generate equilibrium paths for GDP, consumption, investment, and hours *that exactly match the U.S. time series during the 1990s.* The fact that all three theories can generate the boom of the 1990s—the phenomenon that is central to our paper—does not mean, however, that we view all of them as "successful." In fact, we will demonstrate later on that two of these theories are actually unsuccessful *in our sense of the word* because they do not satisfy the input justification criterion or the prediction criterion described in the paper. We deem them unsuccessful *even though they can generate the U.S. boom of the 1990s.* We finish the chapter by addressing the question, Could our preferred theory with intangible investment and non-neutral technological change ever fail to satisfy the criteria for a successful theory that we propose, or are we simply setting the bar too low for ourselves?

To satisfy the input justification criterion, the exogenous inputs of the theoretical model must be consistent with micro and macro empirical evidence. This criterion requires a theory for the exogenous inputs. To satisfy the prediction criterion, the model must not make counterfactual predictions. This is a minimum requirement. A stronger requirement is that the theory must predict time series that were not used to set parameters or exogenous inputs. For example, we can use the theories to make predictions for incomes and capital gains—data that were not used in setting any of the exogenous variables.

For each theory that we investigate, we generate an exact match between predicted and actual paths for GDP, consumption, investment, and hours by introducing either a *labor wedge* or an *investment wedge* or both. The labor wedge is an exogenous input that results in an exact fit for the household's intratemporal first-order condition relating the marginal rate of substitution between leisure and consumption and the marginal product of labor.[3] The investment wedge is an exogenous input that results in an exact fit for the household's intertemporal first-order condition relating the intertemporal marginal rate of substitution and the intertemporal marginal rate of transformation.

For a theory to satisfy the input justification criterion, we require either (*i*) the variation in U.S. time series attributed to the wedges is small or (*ii*) some empirical justification for these wedges. If no empirical support is available, then theory is, for all practical purposes, vacuous. For a theory to successfully resolve the puzzling U.S. boom of the 1990s, it must then satisfy the more demanding prediction criterion.

The first theory we analyze in this section is the standard *theory without intangible capital*, commonly referred to as neoclassical growth theory. We consider a specific model with fluctuations driven by TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. We consider a version of the model with one sector that combines business and non-business activity and another version that distinguishes between them. We do both because the behavior of economy-wide TFP and of business-sector TFP was quite different during the 1990s. The economy-wide TFP was a little below trend throughout the 1990s, and the business-sector TFP started below trend and rose rapidly in the late 1990s. To give the simple theory the best chance of success, we want to allow for a rapid increase in business TFP.

Neither model for the standard theory satisfies our criteria for a successful theory.

[3] Unlike Chari, Kehoe, and McGrattan (2007), we separately include tax rates on labor. The labor wedge would have to be a proxy for labor distortions other than taxes.

The labor wedge has to be huge and is inconsistent with all measures of effective tax rates and all measures of worker benefits available (e.g., tax credits and welfare). Furthermore, the model's predictions of factor incomes and capital gains are not consistent with U.S. observations.

The second and third theories include intangible capital and also distinguish between business and non-business activity. The second theory assumes that technological change is neutral with respect to two activities: production of final goods and services and production of new intangible investment goods. We refer to this as the *theory with intangible capital and neutral technology.* The specific model we analyze has fluctuations driven by the same exogenous variables used for the standard model: TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. Like the standard model without intangible capital, this extension with intangible capital *does not* satisfy our criteria for a successful theory. The labor wedge and the implied intangible investment are wildly oscillatory and inconsistent with all micro evidence on labor distortions and all direct measures of intangible investments. Furthermore, the model's predictions are also grossly inconsistent with the U.S. data. This theory shows that intangible capital is not "making up" for whatever is missing in standard theory. In fact, the theory of intangible capital with neutral technology does considerably worse than the standard theory.

The third theory assumes that technological change is non-neutral. The non-neutrality is at the heart of the theoretical contribution. We consider two different activities within the business sector and refer to the theory as the *theory with intangible capital and non-neutral technology.* The specific model that we analyze has fluctuations driven by TFP in the final goods and services sector, TFP in the intangible-investment sector, tax rates on hours and consumption, and an investment wedge. The effect of the investment wedge—

which is simply an addition to get a perfect fit—is small and well within the range of estimates of capital tax rate changes. The other inputs are consistent with micro evidence of this period in which a technology boom occurred. Specifically, they are consistent with micro evidence on R&D, which is an important component of intangible investment, and they are consistent with the shift in employment to occupations in which sweat equity is important. We compare the model's predictions for factor incomes and capital gains, which were not used in the determination of sectoral TFPs. We show that the model does well here too.

These findings lead us to conclude that there is now one theory of the 1990s boom, whereas before there was none.

2.1. Theory without Intangible Capital

In this section we describe a particular growth model without intangible capital. Fluctuations in the model are driven by changes in TFP, tax rates on hours and consumption, a labor wedge, and an investment wedge. We use the model to demonstrate that this theory fails to satisfy the criteria we propose for a successful theory.

2.1.1. A Specific Model

Given an initial capital stock k_0, the stand-in household chooses sequences of consumption $\{c_t\}$, investment $\{x_t\}$, and hours $\{h_t\}$ to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$c_t + x_t = r_t k_t + w_t h_t - \tau_{ct} c_t - \tau_{ht} w_t h_t - \tau_{kt} k_t - \tau_{pt}(r_t - \delta - \tau_{kt}) k_t - \tau_{xt} x_t$$
$$- \tau_{dt}\{r_t k_t - x_t - \tau_{kt} k_t - \tau_{pt}(r_t - \delta - \tau_{kt}) k_t - \tau_{xt} x_t\} + Tr_t \qquad (2.1.1)$$

$$k_{t+1} = [(1-\delta)k_t + x_t]/(1+\eta), \qquad (2.1.2)$$

where variables are written in per capita terms, N_t is population at t which grows at rate η, and r_t and w_t are rental and wage rates. Taxes are assessed on consumption (τ_c), investment (τ_x), property (τ_k), profits (τ_p), dividends (τ_d), and labor income (τ_h). Transfers are given by Tr_t.

In equilibrium factors are paid their marginal products. Per capita output is given by

$$y_t = k_t^\theta (Z_t h_t)^{1-\theta},$$

and, therefore, $r_t = \theta y_t / k_t$ and $w_t = (1-\theta) y_t / h_t$ are the rental rate and wage rate, respectively, in equilibrium. The parameter Z_t is labor-augmenting technology change which grows at rate γ, that is, $Z_t = z_t(1+\gamma)^t$ with z_t stationary.

Suppose $U(c,h) = \log c + \psi \log(1-h)$. In this case, the household's first-order conditions—after substituting for r_t and w_t—are given by

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta)\hat{y}_t}{h_t} \qquad (2.1.3)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R_{t,t+1} + (1-\delta)\xi_{t,t+1}\right], \qquad (2.1.4)$$

where $\hat{\beta} = \beta/(1+\gamma)$, $\mu_t = 1/[(1+\tau_{ct})\hat{c}_t]$, and

$$R_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})}\left[(1-\tau_{p,t+1})\left(\theta\frac{\hat{y}_{t+1}}{\hat{k}_{t+1}} - \tau_{k,t+1}\right) + \delta\tau_{p,t+1}\right] \qquad (2.1.5)$$

$$\xi_{t,t+1} = \frac{1-\tau_{d,t+1}}{1-\tau_{dt}} \cdot \frac{1-\tau_{p,t+1}}{1-\tau_{pt}}. \qquad (2.1.6)$$

The hat on a variable indicates that it has been detrended by $(1+\gamma)^t$, e.g., $\hat{c}_t = c_t/(1+\gamma)^t$. To close the model, we add the resource constraint,

$$\hat{c}_t + \hat{x}_t + \hat{g}_t = \hat{y}_t.$$

When setting parameters for our numerical experiments, we use 1990 estimates from U.S. data for $\hat{y}$, $\hat{c}$, $\hat{g}$, h, and $\hat{k}$ along with estimates for the growth rates γ, η, the tax rate on labor τ_h, the tax rate on consumption τ_c, tax rates on capital τ_p, τ_d, τ_x, τ_k, and an interest rate i. We can use these estimates to evaluate the following expressions for β, δ, θ, ψ, and z:

$$\beta = \frac{1+\gamma}{1+i} \tag{2.1.7}$$

$$\delta = \hat{x}/\hat{k} + 1 - (1+\eta)(1+\gamma) \tag{2.1.8}$$

$$\theta = \frac{(1-\hat{\beta}(1-\delta))(1+\tau_x) - \hat{\beta}\delta\tau_p + \hat{\beta}(1-\tau_p)\tau_k}{\hat{\beta}(1-\tau_p)} \frac{\hat{k}}{\hat{y}} \tag{2.1.9}$$

$$\psi = \frac{(1-\tau_h)(1-\theta)(1-h)\hat{y}}{(1+\tau_c)\hat{c}h} \tag{2.1.10}$$

$$z = \left(\hat{k}/y\right)^{\theta/(\theta-1)} \frac{\hat{y}}{h} \tag{2.1.11}$$

with $\hat{x} = \hat{y} - \hat{c} - \hat{g}$.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are as follows: $\hat{y} = 1$ (which is a normalization), $\hat{c} = .7626$, $\hat{x} = .2377$, $\hat{g} = 0$, $h = .2751$, and $\hat{k} = 3.91$.[4] The growth rates are set equal to $\eta = 1\%$ and $\gamma = 2\%$ and the interest rate to $i = 4.1\%$. Tax rates on labor and consumption are $\tau_h = .3109$ and $\tau_c = .0657$, respectively. When we compute equilibrium paths for the 1990s, we assume

[4] All estimates are based on the data described in the paper. Note that we included public consumption in $\hat{c}$ and public investment in $\hat{x}$. See the data appendix for further details.

that these tax rates affecting the intratemporal margin (2.1.3) are varying. We assumed that capital tax rates were roughly constant throughout the 1990s, since there was little change in corporate tax policy and capital tax rates have little effect on hours. The constant rates we use are as follows: $\tau_k = .0073$, $\tau_x = 0$, $\tau_p = 0.1487$, and $\tau_d = 0.0637$. The tax on profits τ_p and distributions τ_d are computed by multiplying effective corporate tax rates times the ratio of business capital to total capital. Substituting these values in the expressions (2.1.7)–(2.1.11) implies $\beta = .98$, $\delta = 0.0306$, $\theta = .3358$, $\psi = 1.4841$, and $z = 1.8243$.

2.1.2. Business Cycle Accounting in the 1990s

We observe sequences for $\hat{y}_t$, $\hat{c}_t$, $\hat{x}_t$, $\hat{g}_t$, h_t, τ_{ct}, and τ_{ht}, and an initial capital stock for capital $\hat{k}_0$. The initial capital stock plus sequence of investments imply a sequence of capital stocks if we apply the capital accumulation equation in (2.1.2). Given inputs for capital and labor, we have a measure of (detrended) total factor productivity, $A_t = \hat{y}_t/(\hat{k}_t^{\theta} h_t^{1-\theta})$, which is also equal to $z_t^{1-\theta}$.

If we compute a perfect-foresight equilibrium path for this model, assuming households take as given time paths for TFP and tax rates on hours and consumption, we cannot get a perfect match between the model predictions and the data.[5] For example, if we substitute U.S. data for $\hat{c}_t$, h_t, $\hat{y}_t$, τ_{ht}, and τ_{ct} into (2.1.3), the relation does not hold exactly.

We could get a perfect match if we introduce a labor wedge that forces (2.1.3) to hold and an investment wedge that forces (2.1.4) to hold. Specifically, we define the labor wedge

[5] Later, we relax that assumption to determine if our results are sensitive to this specification of expectations.

L_{wt} and investment wedge X_{wt} as follows:

$$L_{wt} = \frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} \cdot \frac{h_t}{(1-\theta)\hat{y}_t} \cdot \frac{1}{1-\tau_{ht}} \tag{2.1.12}$$

$$X_{w,t+1} = \frac{\hat{\beta}(1-\delta)\mu_{t+1}X_{wt}}{\mu_t - \hat{\beta}R_{t,t+1}\mu_{t+1}X_{wt}}\xi_{t,t+1} \tag{2.1.13}$$

with equation (2.1.13) solved recursively starting with $X_{w0} = 1$. Then we replace $1 - \tau_{ht}$ in (2.1.3) with $(1 - \tau_{ht})L_{wt}$ and $1/(1 + \tau_{xt})$ in (2.1.4) with $X_{wt}/(1 + \tau_{xt})$. If there is some mismeasurement in the effective rates on labor and capital, these wedges will pick it up. Ideally, they should be quantitatively insignificant.

Increases in the wedges have a positive effect on output and hours. The labor wedge has the same effect as a tax on labor (in the form $1-\tau_{ht}$), and the investment wedge has the same effect as a tax on investment (in the form $1/(1+\tau_{xt})$). We distinguish movements in the wedges from movements in these tax rates because we want to set τ_{ht} and τ_{xt} equal to effective rates set by the government. Without further interpretation, these time-varying inputs are just wedges that force first-order conditions to hold. Thus, it is desirable, unless we have some theory of these wedges, that their effect be quantitatively insignificant. They should be interpreted as small measurement errors in constructing national accounts and tax data.

In Table 1, we report the values of the implied exogenous variables; when all are fed into the model, the model exactly reproduces the U.S. sequences for detrended output $\hat{y}_t$, detrended consumption $\hat{c}_t$, detrended investment $\hat{x}_t$, and hours of work h_t. This is true by construction.

Figure 1 is a comparison of U.S. per capita hours and the model's prediction of per capita hours in the case that only TFP and tax rates on labor and consumption are varying

Year (t)	A_t	τ_{ht}	τ_{ct}	L_{wt}	X_{wt}
1990	1.4909	0.3109	0.0657	1.0000	1.0000
1991	1.4651	0.3070	0.0675	0.9838	1.0132
1992	1.4760	0.3028	0.0678	0.9716	1.0101
1993	1.4609	0.3034	0.0678	0.9859	1.0164
1994	1.4544	0.3068	0.0702	1.0076	1.0196
1995	1.4435	0.3116	0.0686	1.0228	1.0255
1996	1.4441	0.3190	0.0674	1.0403	1.0274
1997	1.4448	0.3254	0.0674	1.0561	1.0316
1998	1.4530	0.3327	0.0670	1.0730	1.0252
1999	1.4612	0.3335	0.0662	1.0870	1.0119
2000	1.4568	0.3424	0.0649	1.0960	1.0112
2001	1.4469	0.3472	0.0625	1.0874	1.0156
2002	1.4459	0.3076	0.0617	1.0119	1.0159
2003	1.4328	0.2885	0.0621	0.9990	1.0153

Table 1. Exogenous Variables for Model without Intangible Capital

(i.e., $L_{wt} = X_{wt} = 1$ for all t). By construction, if the wedges were varying, then the model would fit exactly and the predicted and actual series would lie on top of each other. The difference in the actual and predicted series is therefore attributed to the wedges. Clearly, this difference is large.

Figure 2 compares U.S. per capita real GDP and the model's prediction for per capita real GDP. We divide both series by 1.02^t, since our technological growth rate is chosen to be $\gamma = .02$. The model predicts a depressed economy (relative to a 2 percent trend), but the U.S. economy boomed. Figure 3 shows GDP per hour relative to the 2 percent trend for both the data and the model. This figure shows that the deviations in Figures 1 and 2 are not offsetting and, therefore, the prediction for labor productivity is also inconsistent with observations.

Figures 4 and 5 show the model predictions for per capita real investment and consumption along with U.S. data. Neither match up well.

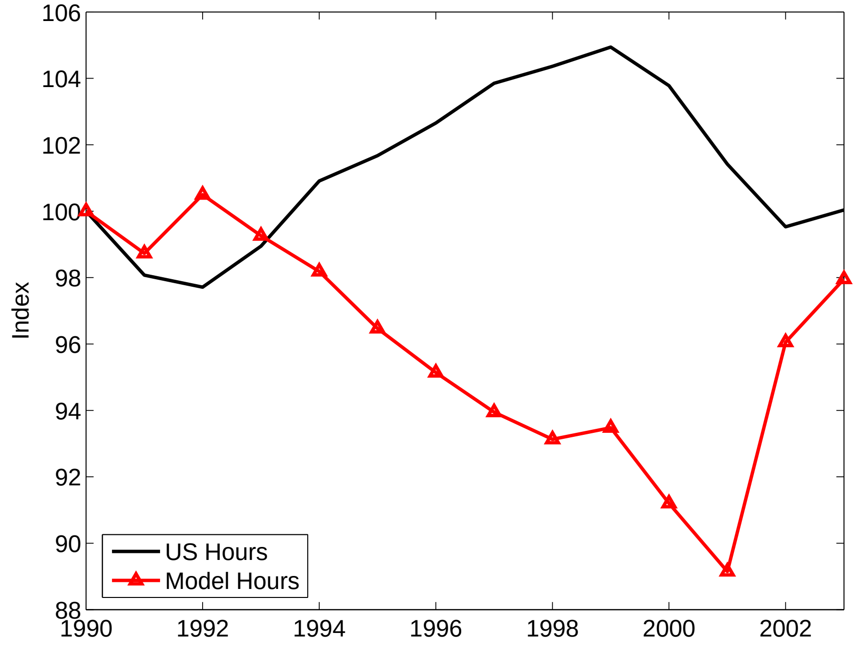

Figure 1. U.S. Per Capita Hours and Prediction of Model without Intangible Capital

(Labor and investment wedges constant)

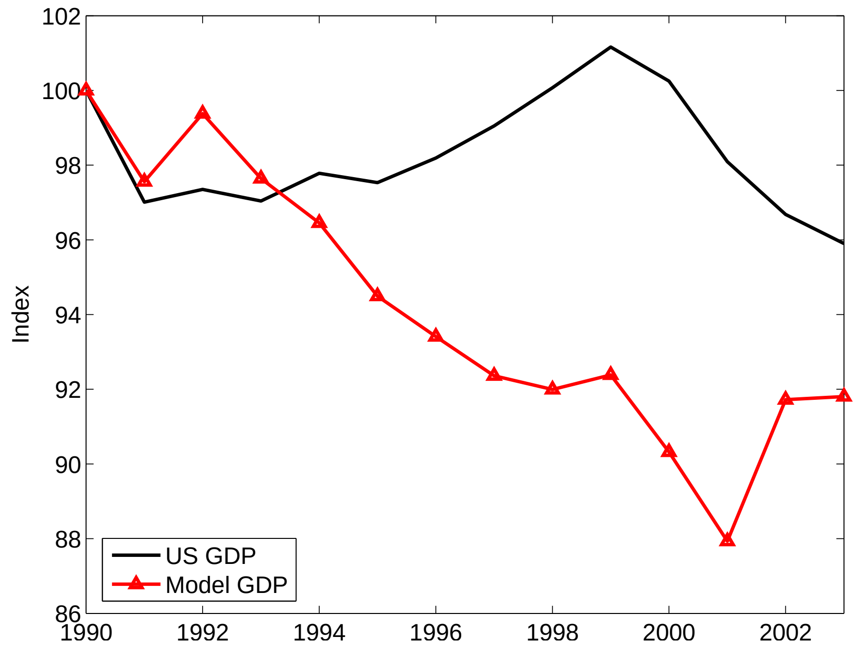

Figure 2. U.S. Per Capita Real GDP and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

In Figures 6 and 7, we examine the model's predictions for per capita hours without TFP or tax rates varying. In Figure 6, we plot U.S. per capita hours along with the model's prediction for hours in the case that only L_{wt} is varying. Figure 7 is the prediction when only X_{wt} is varying.[6] These figures show that the labor wedge is key to getting the hours boom. To generate an hours boom of 7 percent between 1992 and 1999, the labor wedge has to rise nearly 10 percent.

Summary

The main problem with the standard theory driven by the labor wedge is the interpretation of the wedge. It certainly cannot be interpreted as mismeasurement of effective labor tax rates, which were rising—not falling—for all estimates we have seen. Could it be that other policies such as the Earned Income Tax Credit (EITC) or welfare benefits were affecting how much people work?[7] The answer, given the aggregate spending and coverage of these programs, is most surely no. For example, in 1990, the EITC total amount of credit was 7.5 billion, or roughly 0.13 times GDP. (See the U.S. House of Representatives Green Book, Table 13-14.) That figure rose over the 1990s to 0.34 times GDP and then flattened. It is not clear whether it had a positive or negative effect on hours, but the upper bound of the effect on tax rates is tiny. Furthermore, the EITC credits did not decline after 1999, but hours did. There are other tax credits and income-tested benefit programs that affect hours but are much smaller than the EITC.[8]

Without some other empirical motivation for this wedge, the theory does not sat-

[6] When summed, the predicted series in Figures 1, 6, and 7 are approximately but not exactly equal to the U.S. series. It is not exact because there are endogenous movements in the capital stock.

[7] These policies appear in transfers to persons and do not come into the calculation of τ_{ht}.

[8] Examples include the Work Opportunity Tax Credit, the Welfare-to-Work Tax Credit, the Welfare-to-Work Grant Program, and work-related Temporary Assistance for Needy Families. For further details, see Appendix K of the the U.S. House of Representatives Green Book.

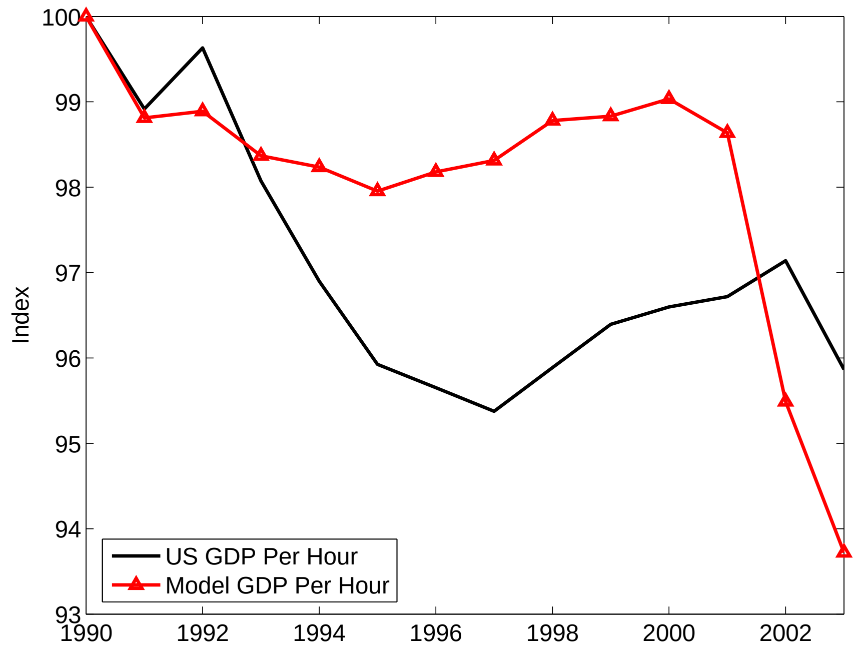

Figure 3. U.S. Real GDP Per Hour and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

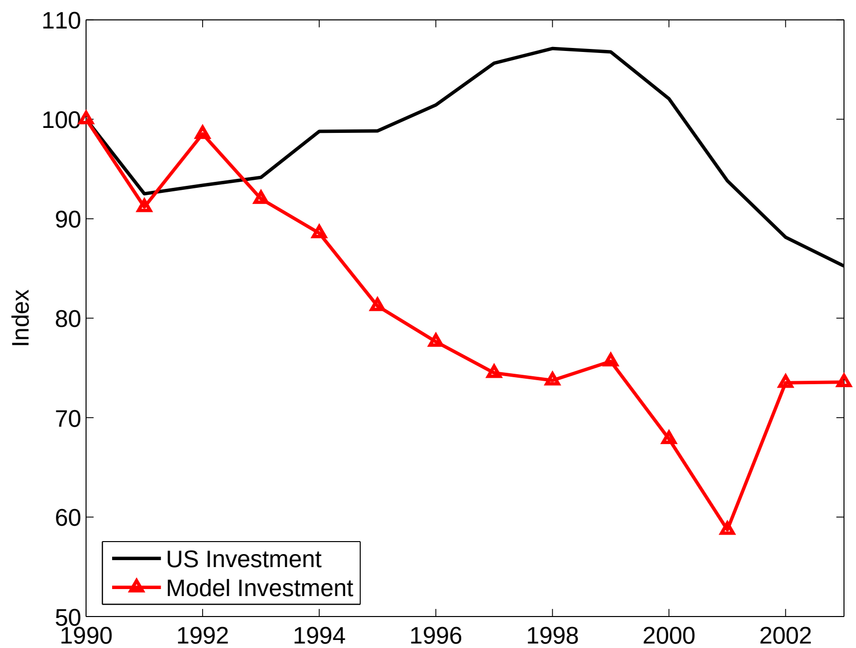

Figure 4. U.S. Per Capita Real Investment and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

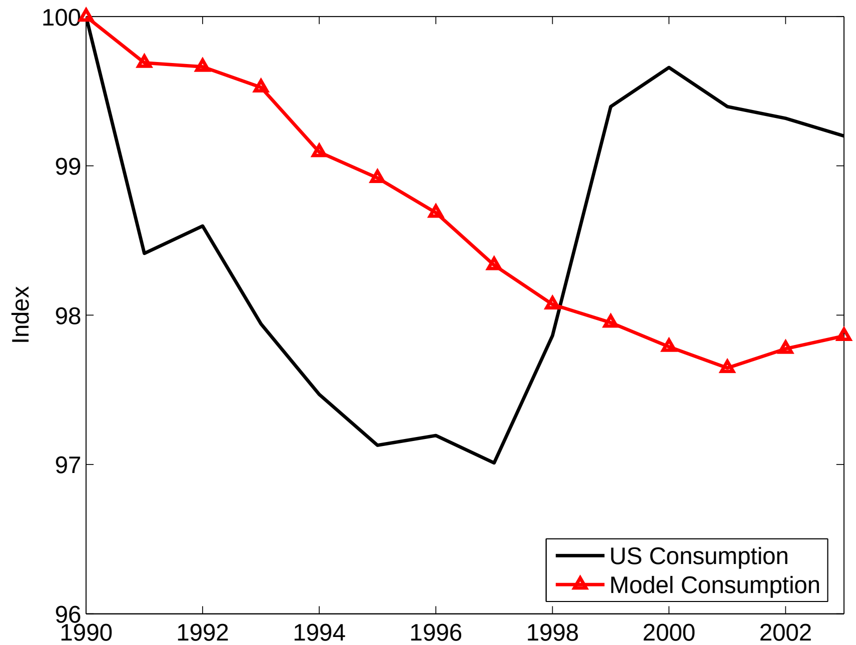

Figure 5. U.S. Per Capita Real Consumption and Prediction of Model without Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

isfy the input justification criterion and does not provide a plausible answer the question, Why did hours boom in the 1990s? It also does not satisfy the prediction criterion. The model's predictions for factor incomes and capital gains are inconsistent with U.S. observations. The model's estimate of compensation is $(1-\theta)y_t$. U.S. wages rose by more than U.S. output. The model's estimate for the market value of capital is $(1-\tau_d)k_t$. Changes in the reproducible stock of tangible capital are much too small to rationalize the large U.S. capital gains in the late 1990s.

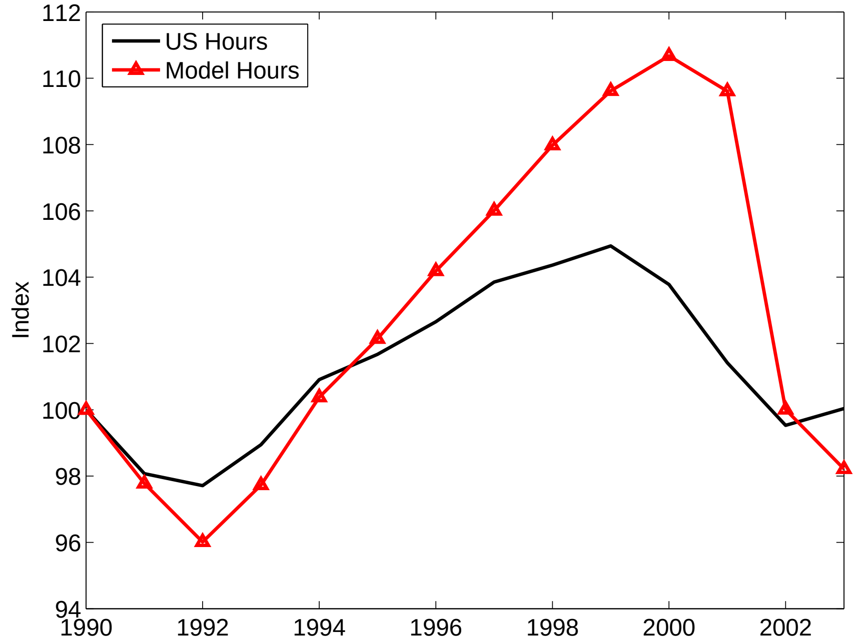

Figure 6. U.S. Per Capita Hours and Prediction of Model without Intangible Capital

(Labor wedge only)

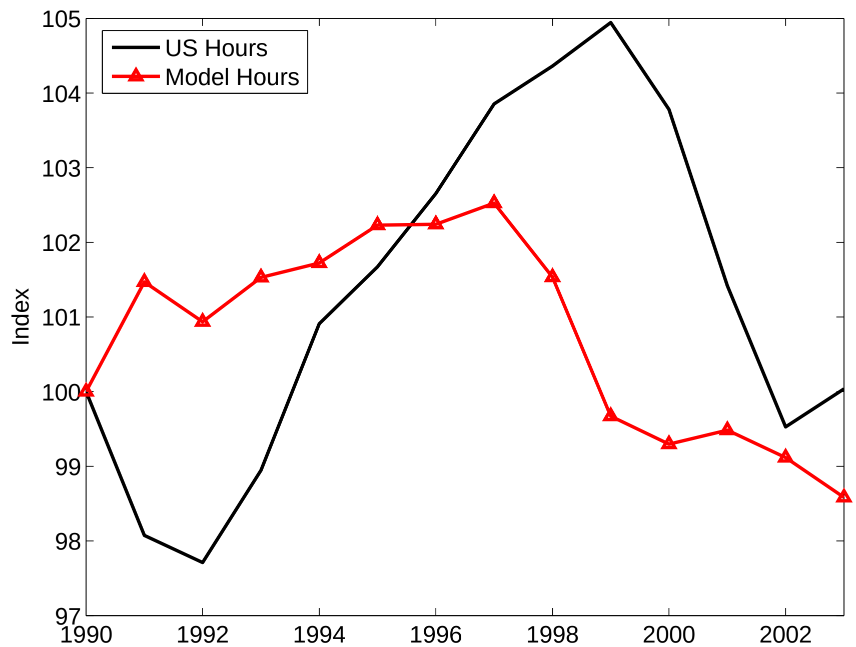

Figure 7. U.S. Per Capita Hours and Prediction of Model without Intangible Capital

(Investment wedge only)

2.1.3. A Version with a Business Sector

We extend the standard model slightly to include both a business and non-business sector, where the latter includes households, government, and nonprofits. As we saw from Table 1, the TFP of the aggregate economy is falling relative to trend. Business TFP, on the other hand, rose rapidly at the end of the 1990s. Here, we investigate whether focusing on the business sector helps the standard theory satisfy our criteria for a successful theory.

Assume now that measured investment includes business sector investment x_{bt} plus non-business investment $\bar{x}_{nt}$. Throughout, we will use b as a subscript for business and n for non-business. The problem for the household given the initial capital stock k_{b0} is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned} c_t + x_{bt} = {} & r_t k_{bt} + w_t h_{bt} - \tau_{ct} c_t - \tau_{ht} w_t h_{bt} - \tau_{kt} k_{bt} \\ & - \tau_{pt}(r_t - \delta - \tau_{kt}) k_{bt} - \tau_{xt} x_{bt} \\ & - \tau_{dt}\{r_t k_{bt} - x_{bt} - \tau_{kt} k_{bt} - \tau_{pt}(r_t - \delta - \tau_{kt}) k_{bt} - \tau_{xt} x_{bt}\} \\ & + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t \end{aligned} \tag{2.1.14}$$

$$k_{b,t+1} = [(1-\delta) k_{bt} + x_{bt}]/(1+\eta) \tag{2.1.15}$$

$$h_t = h_{bt} + \bar{h}_{nt} \tag{2.1.16}$$

where $\bar{y}_{nt}$, $\bar{x}_{nt}$, $\bar{h}_{nt}$, and $\bar{\tau}_{nt}$ are value added, investment, hours, and taxes paid, respectively, in the non-business sector.

Here and later, we assume that sequences for non-business investment, hours, and output are taken as given by the households. Essentially we are assuming that prices in the non-business sector are such that households optimally chose the U.S. levels. Treating the non-business sector this way simplifies the modeling and allows us to directly compare the model national accounts and U.S. national accounts. Furthermore, our interest is U.S. boom in the 1990s, which occurred in the business sector.

The resource constraint is now

$$c_t + x_{bt} + \bar{x}_{nt} + g_t = y_{bt} + \bar{y}_{nt} = y_{mt},$$

where model GDP is the measured output y_{mt}. Value added in the business sector is

$$y_{bt} = k_{bt}^{\theta}(Z_{bt}h_{bt})^{1-\theta},$$

where $Z_{bt} = z_{bt}(1+\gamma)^t$. A NIPA accountant in this economy would measure the following product and income:

NIPA product $= c + x_b + \bar{x}_n + g$

Private consumption $= c$

Public consumption $= g$

Investment $= x_b + \bar{x}_n$

NIPA income $= y_b + \bar{y}_n$

Business profits $= (r - \tau_k - \delta)k_b$

Business wages $= wh_b$

Business depreciation $= \delta k_b$

Business production tax $= \tau_k k_b$

Nonbusiness income $= \bar{y}_n$.

The first-order conditions for the household's problem (assuming factors are paid their marginal product) in the case of log utility are as follows:

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta)\hat{y}_{bt}}{h_{bt}} \tag{2.1.17}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^b_{t,t+1} + (1-\delta)\xi_{t,t+1} \right], \tag{2.1.18}$$

where $\hat{\beta} = \beta/(1+\gamma)$ and $\mu_t = 1/[(1+\tau_{ct})\hat{c}_t]$ as before,

$$R^b_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta \frac{\hat{y}_{b,t+1}}{\hat{k}_{b,t+1}} - \tau_{k,t+1} \right) + \delta \tau_{p,t+1} \right] \tag{2.1.19}$$

and $\xi_{t,t+1}$ is defined in (2.1.6). Notice that the only difference between the first-order conditions (2.1.3)–(2.1.4) and (2.1.17)–(2.1.18) are the marginal products of labor and capital, which in the first case is economy-wide and in the second case is for the business sector.

We'll again assume that we have values for some of the endogenous variables in 1990 and use them to set some of the parameters. Suppose we observe $\hat{y}_b$, $\hat{y}_n$, $\hat{c}$, $\hat{g}$, h, h_b, and $\hat{k}_b$ along with estimates for the growth rates γ, η, the tax on labor τ_h, the tax on consumption τ_c, tax rates on capital, τ_p, τ_d, τ_x, τ_k, and an interest rate i. We can use these estimates to evaluate the following expressions for δ, θ, ψ, and z_b:

$$\delta = \hat{x}_b/\hat{k}_b + 1 - (1+\eta)(1+\gamma) \tag{2.1.20}$$

$$\theta = \frac{(1-\hat{\beta}(1-\delta))(1+\tau_x) - \hat{\beta}\delta\tau_p + \hat{\beta}(1-\tau_p)\tau_k}{\hat{\beta}(1-\tau_p)} \frac{\hat{k}_b}{\hat{y}_b} \tag{2.1.21}$$

$$\psi = \frac{(1-\tau_h)(1-\theta)(1-h)\hat{y}_b}{(1+\tau_c)\hat{c}h_b} \tag{2.1.22}$$

$$z_b = \left(\hat{k}_b/y_b \right)^{\theta/(\theta-1)} \frac{\hat{y}_b}{h_b}, \tag{2.1.23}$$

where $\hat{x}_b = \hat{y}_b + \bar{y}_n - \bar{x}_n - \hat{g} - \hat{c}$ and $\beta = (1+\gamma)/(1+i)$ as before.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are as follows: $\hat{y}_m = 1$ (which is a normalization), $\hat{y}_b = .6621$, $\bar{y}_n = .3379$, $\hat{c} = .7626$, $\hat{x} = .2377$, $\hat{g} = 0$, $h = .2751$, and $\hat{k}_b = 1.66$. Growth rates, the interest rate, the discount

Year (t)	A_t	τ_{ht}	τ_{ct}	L_{wt}	X_{wt}
1990	1.7544	0.3109	0.0657	1.0000	1.0000
1991	1.7136	0.3070	0.0675	0.9882	1.0122
1992	1.7267	0.3028	0.0678	0.9761	1.0081
1993	1.7127	0.3034	0.0678	0.9866	1.0135
1994	1.7066	0.3068	0.0702	1.0088	1.0162
1995	1.7069	0.3116	0.0686	1.0163	1.0224
1996	1.7118	0.3190	0.0674	1.0327	1.0253
1997	1.7269	0.3254	0.0674	1.0403	1.0316
1998	1.7639	0.3327	0.0670	1.0394	1.0284
1999	1.7837	0.3335	0.0662	1.0449	1.0185
2000	1.7985	0.3424	0.0649	1.0402	1.0218
2001	1.7559	0.3472	0.0625	1.0488	1.0285
2002	1.7330	0.3076	0.0617	0.9864	1.0298
2003	1.7118	0.2885	0.0621	0.9746	1.0297

Table 2. Exogenous Variables for Model with Business but No Intangible Capital

factor, and taxes on labor and consumption are as before. The capital tax rates are now $\tau_k = .0144$, $\tau_x = 0$, $\tau_p = 0.35$, and $\tau_d = 0.15$, which are the effective rates for the business sector. Substituting these values in the expressions (2.1.20)–(2.1.23) implies $\delta = 0.0331$, $\theta = .277$, $\psi = 1.375$, and $z_b = 2.176$.

2.1.4. Business Cycle Accounting in the 1990s

We observe sequences for $\hat{y}_{bt}$, $\bar{y}_{nt}$, $\hat{c}_t$, $\bar{x}_{bt}$, $\bar{x}_{nt}$, $\hat{g}_t$, h_t, h_{bt}, $\bar{h}_{nt}$, τ_{ht}, and τ_{ct}, and an initial business capital stock $\hat{k}_{b0}$. Given $\hat{k}_{b0}$ and the sequence for $\hat{x}_{bt}$, we can use the law of motion for business capital (2.1.15) to derive the sequence of stocks $\{k_{bt}\}$. Then, we have business TFP as follows: $A_t = y_{bt}/[k_{bt}^{\theta} h_{bt}^{1-\theta}]$.

As in the one-sector version of the model, we can define the labor wedge as follows:

$$L_{wt} = \frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} \cdot \frac{\hat{h}_{bt}}{(1-\theta)\hat{y}_{bt}} \cdot \frac{1}{1-\tau_{ht}} \tag{2.1.24}$$

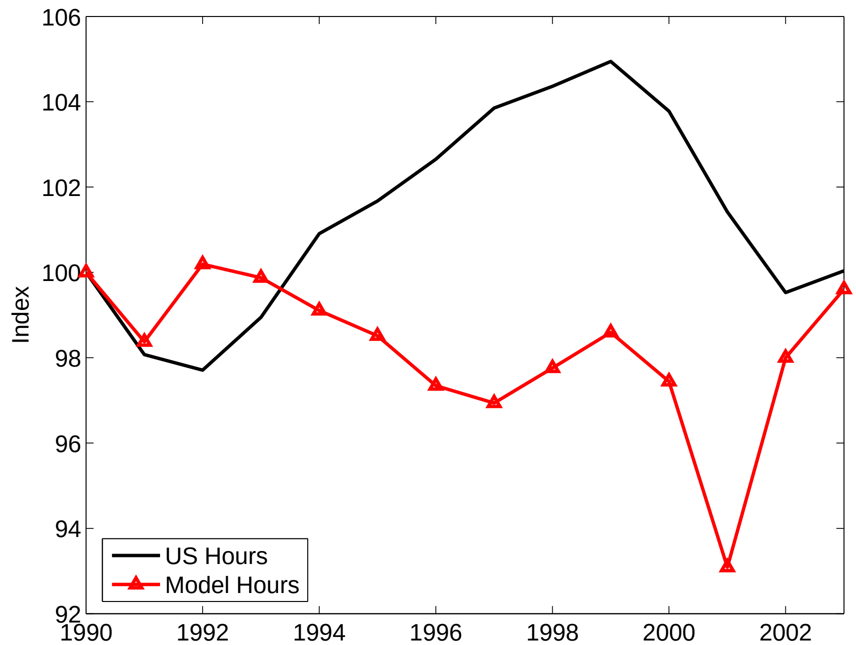

Figure 8. U.S. Per Capita Hours and Prediction of Model with Business but No Intangible Capital

(Labor and investment wedges constant)

and the investment wedge is as in (2.1.13), except that the capital return appearing in this expression is now the return on business capital $R^b_{t,t+1}$ instead of $R_{t,t+1}$.

We conduct the same experiment as before of computing equilibrium paths for per capita hours. In Table 2, we report the values of the implied exogenous variables.

Figures 8–14 show the results in the case that only TFP (A_t) and tax rates (τ_{ht}, τ_{ct}) vary. Figure 8 is a comparison of per capita hours for the United States and for the model. As in the one-sector version of the model, there is a large deviation. Figures 9 and 10 show per capita real GDP and value added in the business sector. As before, the model predicts that the U.S. economy should have been depressed (relative to trend). Because TFP in the business sector rises at the end of the 1990s, there is a rise in business value added. However, the growth is too modest relative to what we observed in the actual economy.

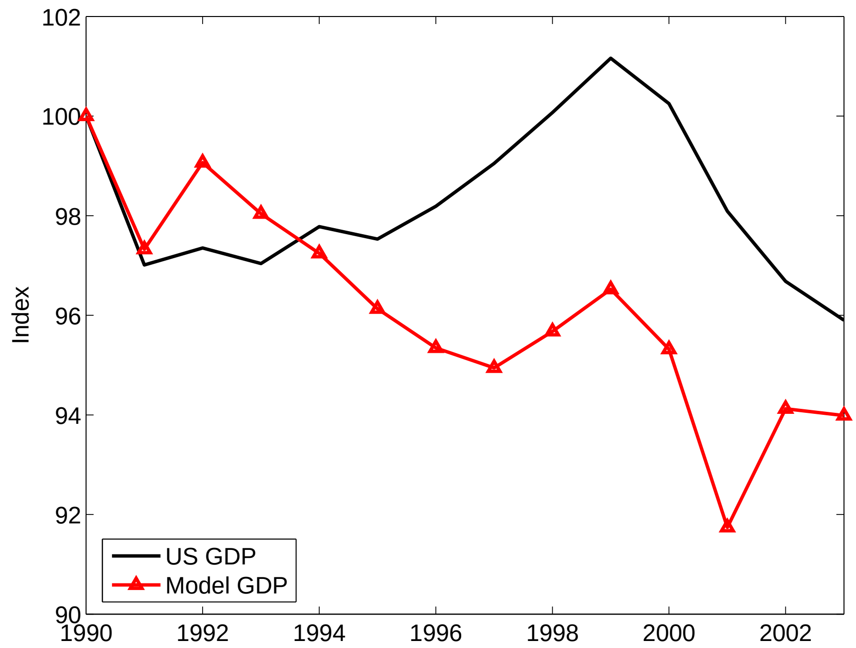

Figure 9. U.S. Per Capita Real GDP and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

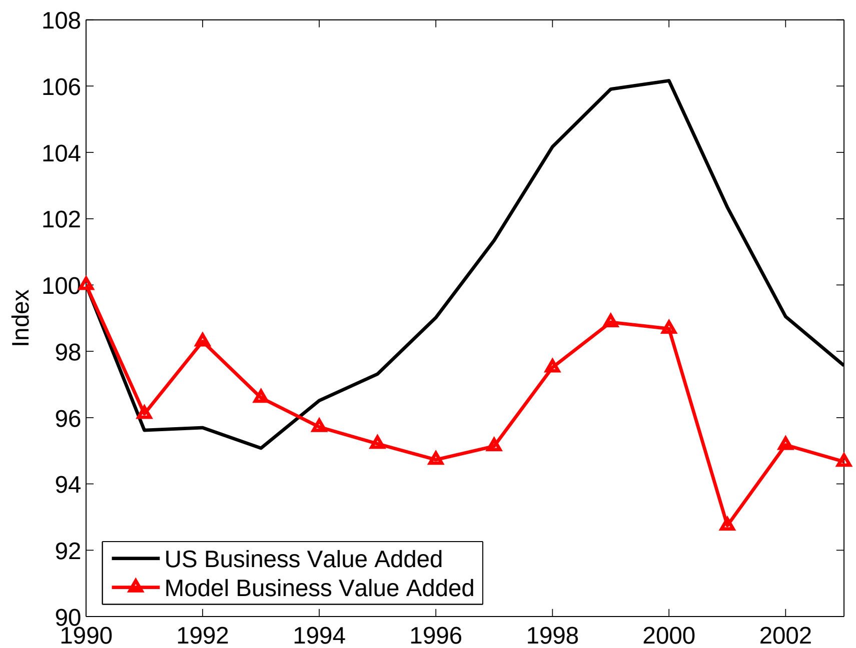

Figure 10. U.S. Per Capita Real Business Value Added and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

Figures 11–12 show labor productivity for the aggregate economy and for the business sector. The prediction of business labor productivity is reasonable, but the prediction of overall labor productivity is no better than in the one-sector version of the growth model.

For completeness, we also include figures for the components of GDP. These are shown in Figures 13 and 14. In this case, there is little improvement in the model's predictions relative to the one-sector model.

In Figures 15 and 16, we show the predictions for per capita hours when we allow only the labor wedge or only the investment wedge to vary. These results are comparable to those in Figures 6 and 7 for the one-sector version of the model. They can also be compared to Figure 8, which assumes no variation in either the labor wedge or the investment wedge. These results make it clear that to generate the hours boom, we again need an implausible boom in the labor wedge. However, as before, we have no empirical evidence supporting any theory of this labor wedge boom.

Summary

To account for the boom in the U.S. economy using the standard growth model, whether we use the one-sector model or the two-sector model, we must rely on implausible movements in the labor and investment wedges.

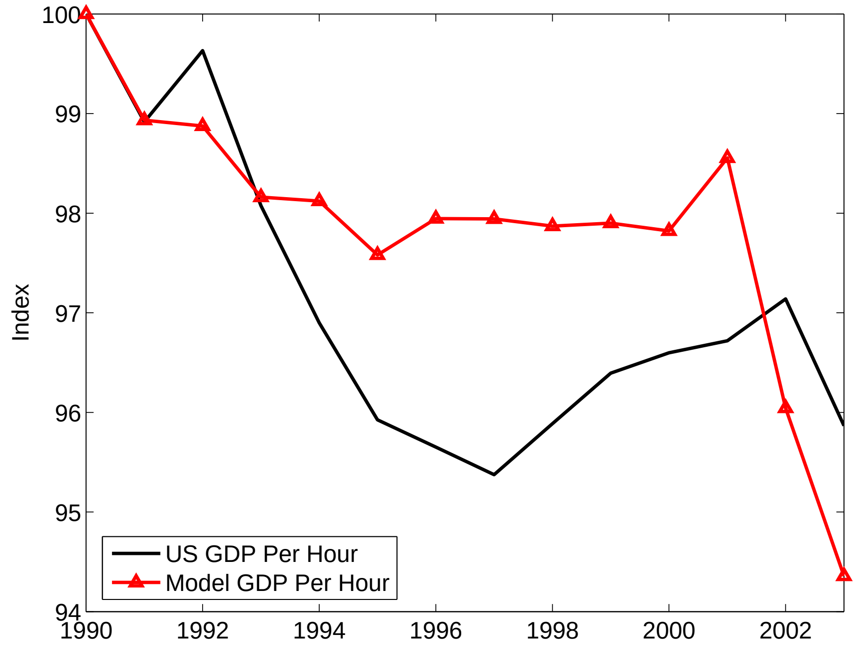

Figure 11. U.S. Real GDP Per Hour and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

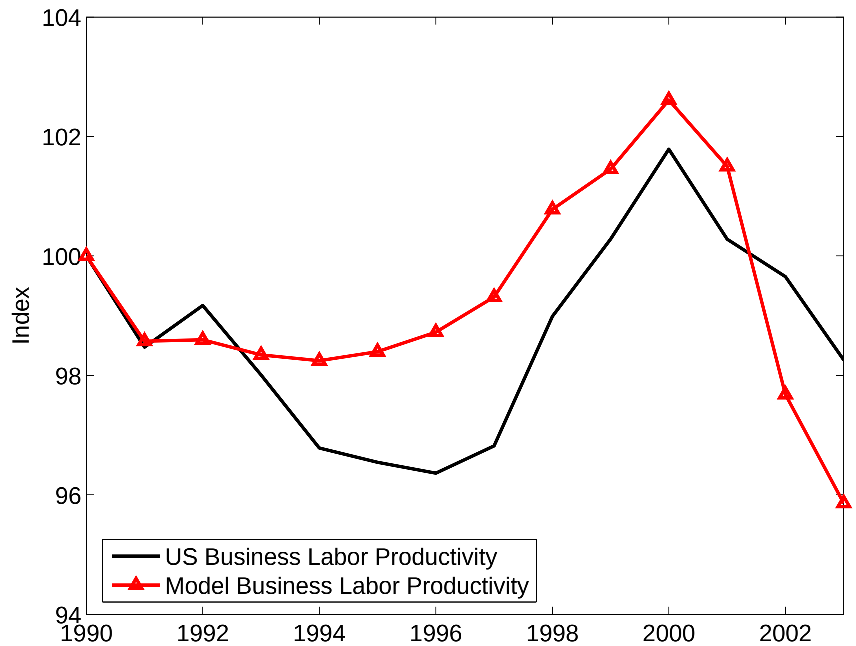

Figure 12. U.S. Real Business Value Added Per Hour and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

2.2. Theory with Intangible Capital and Neutral Technology

We now extend the basic theory described above by incorporating intangible capital. We have used this theory before to study the U.S. and U.K. stock markets. We show, however, that to generate a boom like that observed in the United States during the 1990s, we require wildly implausible exogenous wedges as before, thus demonstrating that intangible capital per se cannot make up for whatever is missing in standard theory.

2.2.1. A Specific Model

The problem for the household given initial stocks of tangible capital k_b and intangible capital k_u (which is *unmeasured* in NIPA) is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned} c_t + x_{Tt} + x_{It} = {} & r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_{bt} \\ & - \tau_{ct} c_t - \tau_{ht} w_t h_{bt} - \tau_{kt} k_{Tt} \\ & - \tau_{pt} \{ r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - x_{It} \} - \tau_{xt} x_{Tt} \\ & - \tau_{dt} \{ r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \tau_{kt} k_{Tt} - x_{It} \\ & \qquad - \tau_{pt} (r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - x_{It}) - \tau_{xt} x_{Tt} \} \\ & + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t \end{aligned} \tag{2.2.1}$$

$$k_{T,t+1} = [(1 - \delta_T) k_{Tt} + x_{Tt}]/(1 + \eta) \tag{2.2.2}$$

$$k_{I,t+1} = [(1 - \delta_I) k_{It} + x_{It}]/(1 + \eta) \tag{2.2.3}$$

$$h_t = h_{bt} + \bar{h}_{nt}. \tag{2.2.4}$$

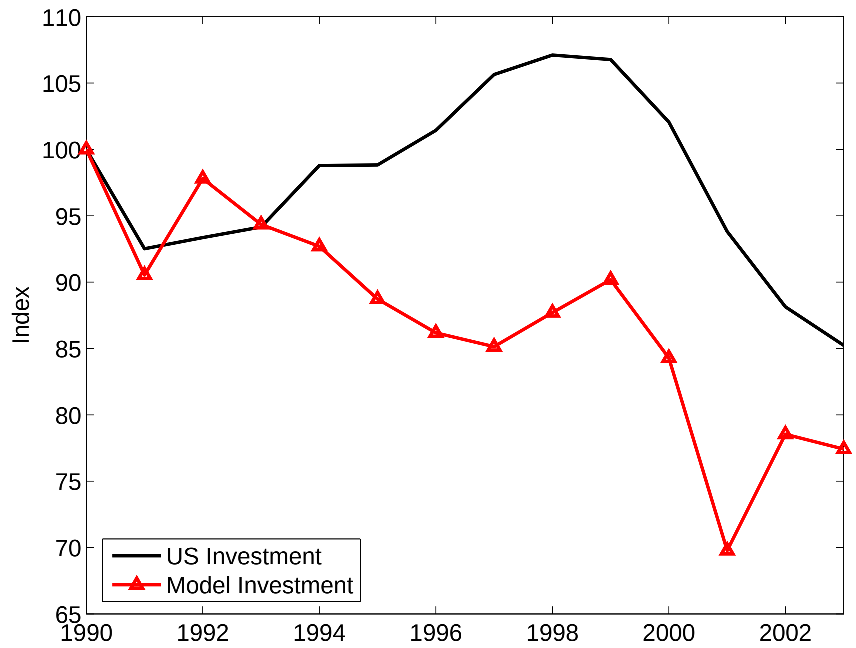

Figure 13. U.S. Per Capita Real Investment and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

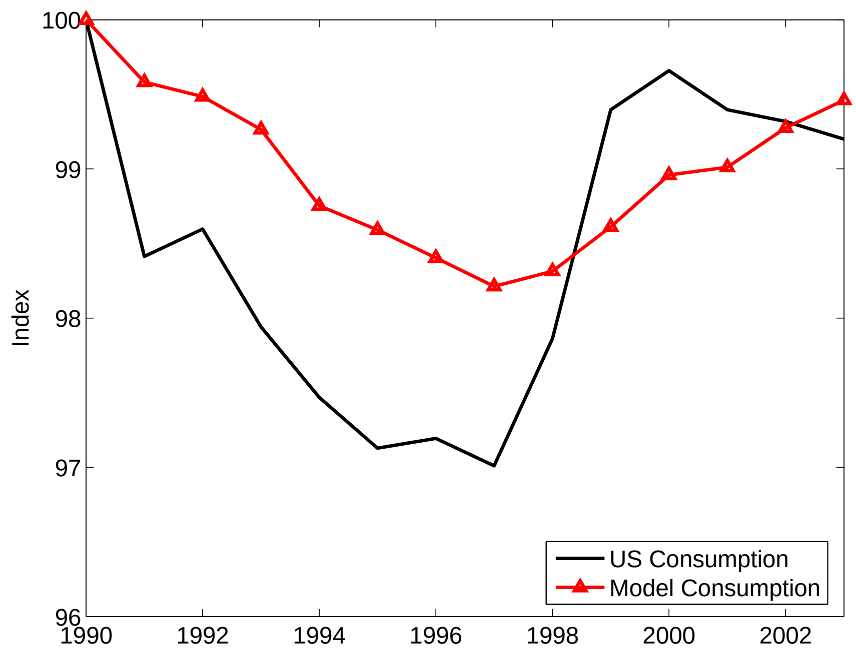

Figure 14. U.S. Per Capita Real Consumption and Prediction of Model with Business but No Intangible Capital, Series Divided by 1.02^t

(Labor and investment wedges constant)

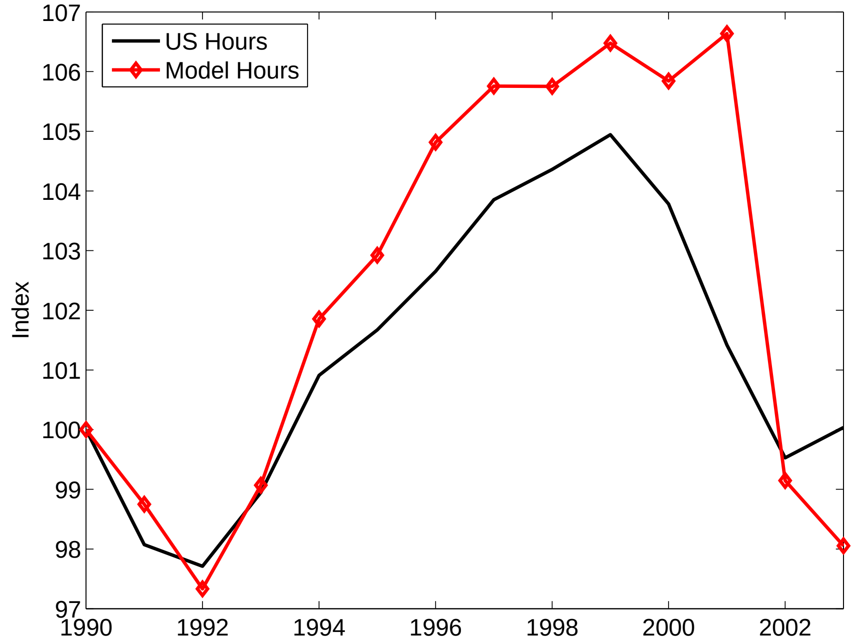

Figure 15. U.S. Per Capita Hours and Prediction of Model with Business but No Intangible Capital

(Labor wedge only)

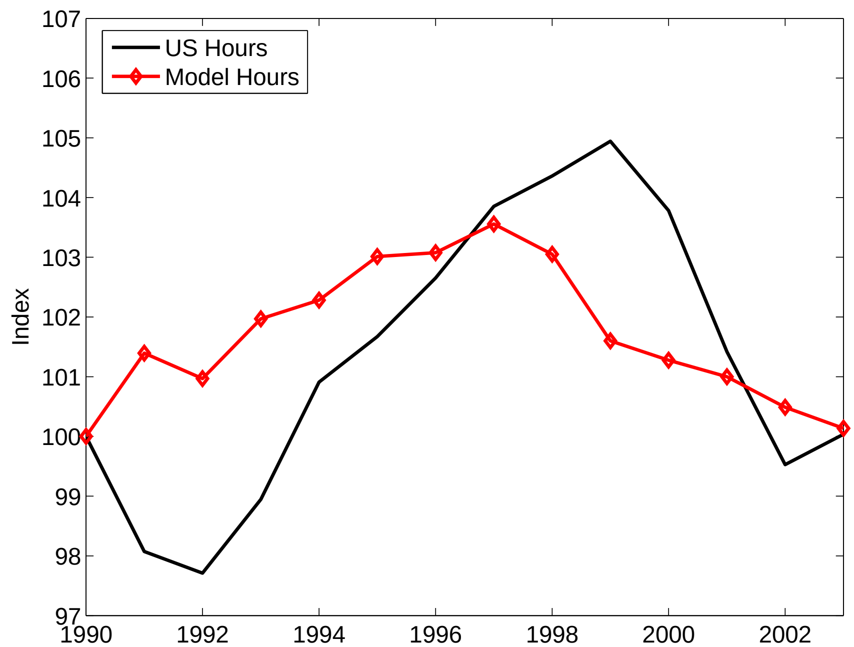

Figure 16. U.S. Per Capita Hours and Prediction of Model with Business but No Intangible Capital

(Investment wedge only)

All variables are as defined in Section 2.1.3 except that we have added the stock k_{It} and investment x_{It} of intangibles. There are no additional exogenous variables. Notice that x_{It} is expensed and thus subtracted from taxable profits in (2.2.1).

Total output in the business sector is equal to

$$y_t = k_{Tt}^{\theta} k_{It}^{\phi} (Z_{bt} h_{bt})^{1-\theta-\phi},$$

where $Z_{bt} = z_{bt}(1+\gamma)^t$, and the economy's resource constraint is

$$c_t + x_{Tt} + x_{It} + \bar{x}_{nt} + g_t = y_t + \bar{y}_{nt}.$$

Measured value added in the business sector is $y_{bt} = y_t - x_{It}$ and aggregate GDP is $y_{bt} + \bar{y}_{nt}$. A NIPA accountant in this economy would measure the following product and income:

NIPA product $= c + x_T + \bar{x}_n + g$

- Private consumption $= c$
- Public consumption $= g$
- Investment $= x_T + \bar{x}_n$

NIPA income $= y_b + \bar{y}_n$

- Business profits $= (r_T - \tau_k - \delta_T)k_T + r_I k_I - x_I$
- Business wages $= w h_b$
- Business depreciation $= \delta_T k_T$
- Business production tax $= \tau_k k_T$
- Nonbusiness income $= \bar{y}_n$,

which differs from the earlier model only in the category of business profits. Business profits now include a dividend to intangible, $r_I k_I - x_I$.

Assuming log utility, we can derive the first-order conditions, which are given by

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta-\phi)\hat{y}_t}{h_{bt}} \qquad (2.2.5)$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^T_{t,t+1} + (1-\delta_T)\xi_{t,t+1} \right] \tag{2.2.6}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^I_{t,t+1} + (1-\delta_I)\zeta_{t,t+1} \right], \tag{2.2.7}$$

where

$$R^T_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta \frac{\hat{y}_{t+1}}{\hat{k}_{T,t+1}} - \tau_{k,t+1} \right) - \delta_T \tau_{p,t+1} \right] \tag{2.2.8}$$

$$R^I_{t,t+1} = \phi \frac{\hat{y}_{t+1}}{\hat{k}_{I,t+1}} \tag{2.2.9}$$

$$\zeta_{t,t+1} = \frac{1-\tau_{d,t+1}}{1-\tau_{dt}} \cdot \frac{1-\tau_{p,t+1}}{1-\tau_{pt}} \tag{2.2.10}$$

and $\xi_{t,t+1}$ is given by (2.1.6). Relative to the standard theory without intangible capital, we are adding one dynamic equation (2.2.7).

Assigning parameters for the model is done as above except we need to also assign δ_I and θ_I. We assume the intangible capital is long-lived (e.g., organizations) and set $\delta_I = 0$. This choice will not matter for our results. To set θ_I we use one piece of additional information from the national accounts for 1990, namely business compensation. Then, given estimates for the rental rate r_T, the intangible capital stock $\hat{k}_I$, and the intangible investment $\hat{x}_I$, we have θ and ϕ:

$$r_T = \frac{(1-\hat{\beta}(1-\delta_T))(1+\tau_x) - \delta_T \hat{\beta}\tau_p + \hat{\beta}(1-\tau_p)\tau_k}{\hat{\beta}(1-\tau_p)} \tag{2.2.11}$$

$$\hat{k}_I = \frac{\hat{y}_b - r_b \hat{k}_b - \text{1990 NIPA business compensation}}{1+i-(1+\gamma)(1+\eta)} \tag{2.2.12}$$

$$\hat{x}_I = ((1+\gamma)(1+\eta) - 1 + \delta_I)\hat{k}_I \tag{2.2.13}$$

$$\phi = \frac{(i+\delta_u)\hat{k}_u}{\hat{y}_b + \hat{x}_u} \tag{2.2.14}$$

$$\theta = \frac{r_T \hat{k}_T}{\hat{y}_T + \hat{x}_I} \tag{2.2.15}$$

where the values for $\hat{y}_b$, $\hat{k}_T$, i, γ, and η are as before.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are exactly the same as those used above. The only additional information is NIPA business compensation, which is equal to 0.443 times GDP in 1990. Growth rates, the interest rate, the discount factor, tax rates, and the depreciation rate on tangible capital are also exactly the same as in Section 2.1.3. Because total output includes intangible investment, some parameters are changed slightly. The new parameters are $\theta = 0.240$, $\phi = 0.180$, $\psi = 1.273$, and $z = 1.635$.

2.2.2. Business Cycle Accounting for the 1990s

Suppose that we have observations on $\hat{y}_{bt}$, $\bar{y}_{nt}$, $\hat{c}_t$, $\hat{x}_{Tt}$, $\bar{x}_{nt}$, $\hat{g}_t$, h_t, $\bar{h}_{nt}$, τ_{ht}, and τ_{ct}. Then, given $\hat{x}_{Tt}$, we can use the law of motion for capital to get

$$\hat{k}_{T,t+1} = [(1-\delta_T)\hat{k}_{Tt} + \hat{x}_{Tt}]/[(1+\gamma)(1+\eta)]$$

given an initial condition $\hat{k}_{T0}$.

We can infer the magnitude of intangible capital and investment using (2.2.3) and (2.2.7) along with data on consumption, business output, and tax rates.[9] Given sequences for intangible investments and stocks, we can compute TFP,

$$A_t = \frac{\hat{y}_{bt} + \hat{x}_{It}}{\hat{k}_{Tt}^{\theta}\hat{k}_{It}^{\phi}h_{bt}^{1-\theta-\phi}}.$$

To get a perfect match to U.S. data, we again rely on a labor wedge and an investment

[9] We use the steady-state level of $\hat{k}_I$ (given 1990 values of other variables) to initialize the stock. We assume the growth in the per capita stock in the last period is γ.

Year (t)	A_t	τ_{ht}	τ_{ct}	L_{wt}	X_{wt}
1990	1.6586	0.3109	0.0657	0.8020	1.0000
1991	0.9014	0.3070	0.0675	1.4051	0.9911
1992	1.4223	0.3028	0.0678	0.8955	0.9919
1993	1.1352	0.3034	0.0678	1.1216	0.9875
1994	1.2306	0.3068	0.0702	1.0634	0.9846
1995	1.1315	0.3116	0.0686	1.1695	0.9788
1996	1.2365	0.3190	0.0674	1.0995	0.9740
1997	1.1647	0.3254	0.0674	1.1924	0.9668
1998	1.4442	0.3327	0.0670	0.9874	0.9627
1999	1.6581	0.3335	0.0662	0.8722	0.9625
2000	1.3061	0.3424	0.0649	1.0993	0.9575
2001	1.1055	0.3472	0.0625	1.2776	0.9471
2002	1.1913	0.3076	0.0617	1.1043	0.9358
2003	1.1942	0.2885	0.0621	1.0793	0.9235

Table 3. Exogenous Variables for Model with Intangible Capital and Neutral Technology

wedge,

$$L_{wt} = \frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} \cdot \frac{h_{bt}}{(1-\theta)\hat{y}_t} \cdot \frac{1}{1-\tau_{ht}} \tag{2.2.16}$$

$$X_{w,t+1} = \frac{\hat{\beta}(1-\delta)\mu_{t+1}X_{wt}}{\mu_t - \hat{\beta}R^T_{t,t+1}\mu_{t+1}X_{wt}}\xi_{t,t+1} \tag{2.2.17}$$

with equation (2.1.13) solved recursively starting with $X_{w0} = 1$. As before, we would replace $1-\tau_{ht}$ with $(1-\tau_{ht})L_{wt}$ and $1/(1+\tau_{xt})$ with $X_{wt}/(1+\tau_{xt})$.

In Figures 17–19, we repeat the exercise of plotting actual hours and predicted hours for versions of the model with wedges off and wedges on. Table 3 has the values of the implied exogenous variables. Figure 17 shows the predicted hours for the case that $L_{wt} = X_{wt} = 1$ in all periods. Clearly, this is not an improvement on standard theory because the hours prediction is wildly oscillatory.[10] The same strange behavior is evident

[10] An alternative strategy is to use (2.2.5) to infer the sequence of intangible investments. In this case,

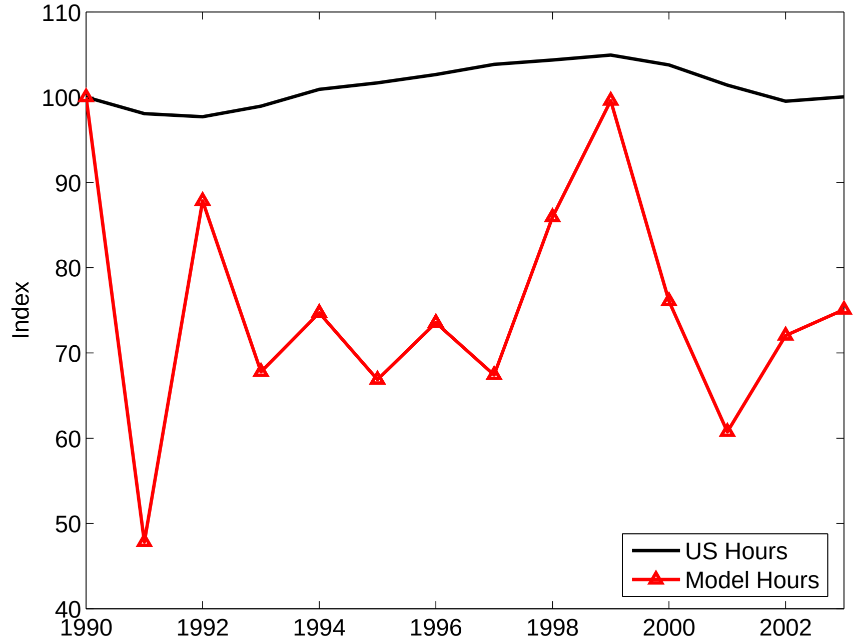

Figure 17. U.S. Per Capita Hours and Prediction for Model with Intangible Capital and Neutral Technology

(Labor and investment wedges constant)

in the labor-wedge-only case shown in Figure 18. Essentially, the labor wedge is canceling out the other exogenous variables in such a way that the prediction of Figure 19, with the investment-wedge-only case, is relatively smooth. Interestingly, if we input all of these exogenous variables, *we get a perfect fit.*

Why do the results look so strange? The logic that intangible capital makes up for whatever is missing is faulty. When we added a labor wedge to the theory without intangible capital, we simply added an exogenous term to one of the existing first-order conditions and let the wedge be whatever it had to be. *In the model with intangible capital, we are adding two more endogenous variables, and, therefore, we are adding two more equations and more restrictions in the dynamical system.* The new larger system

a "tangible investment wedge" and an "intangible investment wedge" are needed for (2.2.6) and (2.2.7) to hold, given U.S. observations. We applied this strategy and found again that the model's predictions were grossly at odds with the data when the wedges were off.

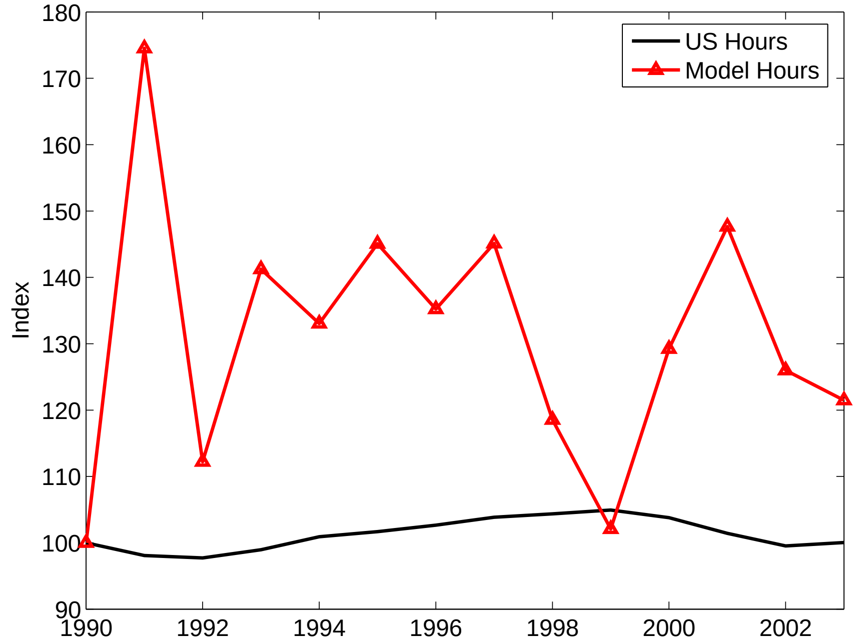

FIGURE 18. U.S. PER CAPITA HOURS AND PREDICTION FOR MODEL WITH INTANGIBLE CAPITAL AND NEUTRAL TECHNOLOGY

(Labor wedge only)

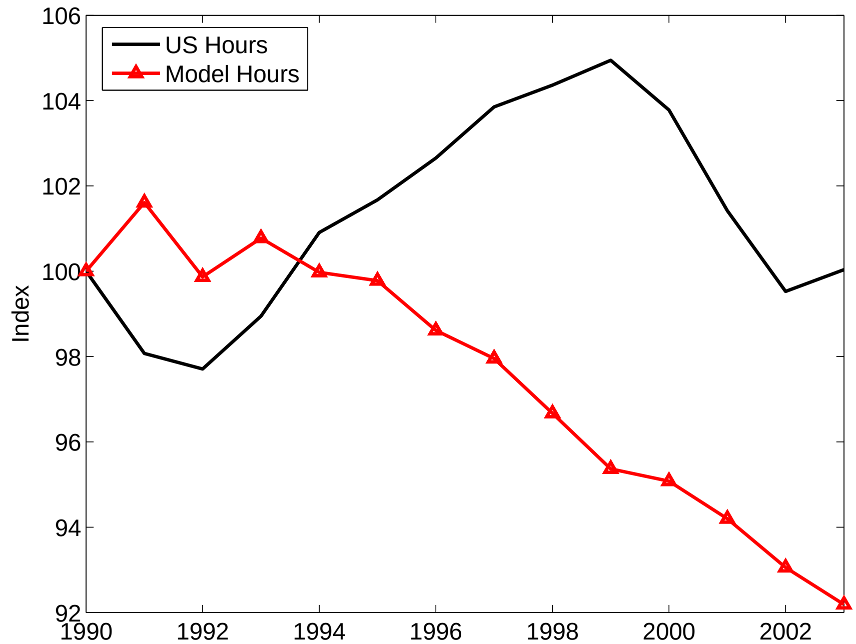

FIGURE 19. U.S. PER CAPITA HOURS AND PREDICTION FOR MODEL WITH INTANGIBLE CAPITAL AND NEUTRAL TECHNOLOGY

(Investment wedge only)

of first-order conditions are not in block form. Thus, the intangible variables enter the original first-order conditions, and the observed variables enter the additional conditions. There is no mathematical basis for thinking that adding the intangible capital will improve the predictive power of the model with the wedges turned off. Indeed, it is worse in this case.

Summary

We showed that intangible capital is not a free parameter that makes up for whatever is missing to make standard theory work. Next, we show that central to understanding the boom of the 1990s is non-neutral technological change.

2.3. Theory with Intangible Capital and Non-neutral Technology

We turn now to the "extended model" developed in our paper that has *both* intangible capital and non-neutral technological change. This theory satisfies our input justification criterion, and we show here that it also satisfies our prediction criterion. We also demonstrate that we would get a very different result if the data-generating mechanism were inconsistent with the theory proposed, implying that the bar we set is not too low.

2.3.1. A Specific Model

There are two additions relative to the model of Section 2.2. First, we allow for *non-neutral technology*. The idea is that we are modeling a technology boom that is concentrated in intangible activity. Second, we allow for *sweat equity*. Some intangible investment is financed by shareholders and some is financed by worker-owners of businesses. We were motivated to include this, since the pattern of incomes suggests that both types of financing are done.

The problem for the household given initial stocks of business tangible capital k_{T0} and business intangible capital k_{I0} is to maximize

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + q_t x_{It} = {} & r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_{bt} \\
& - \tau_{ct} c_t - \tau_{ht}(w_t h_{bt} - (1-\chi) q_t x_{It}) - \tau_{kt} k_{Tt} \\
& - \tau_{pt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - \chi q_t x_{It}\} - \tau_{xt} x_{Tt} \\
& - \tau_{dt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \tau_{kt} k_{Tt} - \chi q_t x_{It} \\
& \qquad - \tau_{pt}(r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \tau_{kt} k_{Tt} - \chi q_t x_{It}) \\
& \qquad - \tau_{xt} x_{Tt}\} - \tau_{gt} q_t x_{It} \\
& + [\bar{y}_{nt} - \bar{x}_{nt} - \bar{\tau}_{nt}] + Tr_t
\end{aligned} \tag{2.3.1}$$

$$k_{T,t+1} = [(1-\delta_T) k_{Tt} + x_{Tt}]/(1+\eta) \tag{2.3.2}$$

$$k_{I,t+1} = [(1-\delta_I) k_{It} + x_{It}]/(1+\eta) \tag{2.3.3}$$

$$h_t = h_{bt}^1 + h_{bt}^2 + \bar{h}_{nt} \tag{2.3.4}$$

$$k_{Tt} = k_{Tt}^1 + k_{Tt}^2, \tag{2.3.5}$$

where q_t is the relative price of intangible investment goods and final output, and χ is the fraction of intangible investment financed by shareholders. The remaining $1-\chi$ of intangible investment is financed by workers who own their businesses and put in sweat equity, which is uncompensated labor. The effective compensation is through capital gains when the business is sold.

The total produced in the business sector is $y_t = y_{bt} + q_t x_{It}$, where

$$y_{bt} = (k_{Tt}^1)^{\theta_1} k_{It}^{\phi_1} (Z_t^1 h_{bt}^1)^{1-\theta_1-\phi_1}$$

$$x_{It} = (k_{Tt}^2)^{\theta_2} k_{It}^{\phi_2} (Z_t^2 h_{bt}^2)^{1-\theta_2-\phi_2}$$

and $Z_t^1 = z_t^1(1+\gamma)^t$, $Z_t^2 = z_t^2(1+\gamma)^t$. If $\chi = 1$ and technologies are neutral (so that $q_t = 1$ in equilibrium), then we are back to the model of Section 2.2.1.

The economy's resource constraint is

$$c_t + x_{Tt} + q_t x_{It} + \bar{x}_{nt} + g_t = y_t + \bar{y}_{nt}$$

and aggregate GDP is $y_{bt} + \bar{y}_{nt}$. A NIPA accountant in this economy would measure the following product and income:

NIPA product $= c + x_T + \bar{x}_n + g$

Private consumption $= c$

Public consumption $= g$

Investment $= x_T + \bar{x}_n$

NIPA income $= y_b + \bar{y}_n$

Business profits $= (r_T - \tau_k - \delta_T)k_T + r_I k_I - \chi q x_I$

Business wages $= w h_b - (1-\chi) q x_I$

Business depreciation $= \delta_T k_T$

Business production tax $= \tau_k k_T$

Nonbusiness income $= \bar{y}_n$

Simplifying the first-order conditions for the log utility case, we get

$$\frac{\psi(1+\tau_{ct})\hat{c}_t}{1-h_t} = (1-\tau_{ht})\frac{(1-\theta_1-\phi_1)\hat{y}_{bt}}{h_{bt}^1} \tag{2.3.6}$$

$$(1-\theta_1-\phi_1)\frac{\hat{y}_{bt}}{h_{bt}^1} = (1-\theta_2-\phi_2)\frac{q_t \hat{x}_{It}}{h_{bt}^2} \tag{2.3.7}$$

$$\theta_1 \frac{\hat{y}_{bt}}{\hat{k}^1_{Tt}} = \theta_2 \frac{q_t \hat{x}_{It}}{\hat{k}^2_{Tt}} \tag{2.3.8}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^T_{t,t+1} + (1-\delta_T)\xi_{t,t+1} \right] \tag{2.3.9}$$

$$\mu_t = \hat{\beta} E_t \mu_{t+1} \left[R^I_{t,t+1} + (1-\delta_I) q_{t+1} \zeta_{t,t+1}/q_t \right], \tag{2.3.10}$$

where

$$R^T_{t,t+1} = \frac{1-\tau_{d,t+1}}{(1-\tau_{dt})(1+\tau_{xt})} \left[(1-\tau_{p,t+1}) \left(\theta_1 \frac{\hat{y}_{b,t+1}}{\hat{k}^1_{b,t+1}} - \tau_{k,t+1} \right) - \delta_b \tau_{p,t+1} \right]$$

$$R^I_{t,t+1} = \frac{\phi_1 \hat{y}_{b,t+1} + \phi_2 q_{t+1} \hat{x}_{I,t+1}}{q_t \hat{k}_{I,t+1}} \left(\frac{(1-\tau_{d,t+1})(1-\tau_{p,t+1})}{\chi(1-\tau_{dt})(1-\tau_{pt}) + (1-\chi)(1-\tau_{ht})} \right)$$

$$\zeta_{t,t+1} = \frac{\chi(1-\tau_{d,t+1})(1-\tau_{p,t+1}) + (1-\chi)(1-\tau_{h,t+1})}{\chi(1-\tau_{dt})(1-\tau_{pt}) + (1-\chi)(1-\tau_{ht})} \tag{2.3.11}$$

and $\xi_{t,t+1}$ is given by (2.1.6). Assigning parameters for the model is done as above except that we have three additional parameters, χ, θ_2, and ϕ_2 (and we normalize q to 1). In our benchmark experiments, we set $\chi = 1/2$ and equated capital shares, $\theta_2 = \theta_1$ and $\phi_2 = \phi_1$. In this case, the shares θ_1 and ϕ_1 are constructed as follows:

$$r_I = \frac{q(1-\hat{\beta}(1-\delta_I))[(1-\chi)(1-\tau_h) + \chi(1-\tau_d)(1-\tau_p)]}{\hat{\beta}(1-\tau_d)(1-\tau_p)}$$

$$\hat{k}_I = \frac{\hat{y}_b - r_T \hat{k}_T - \text{1990 NIPA business compensation}}{r_I - \chi q[(1+\gamma)(1+\eta) - 1 + \delta_I]}$$

$$\hat{x}_I = ((1+\gamma)(1+\eta) - 1 + \delta_I)\hat{k}_I$$

$$\phi_1 = \frac{r_I \hat{k}_I}{\hat{y}_b + q\hat{x}_I}$$

$$\theta_1 = \frac{r_T \hat{k}_T}{\hat{y}_b + q\hat{x}_I},$$

where r_T is given by (2.2.11). In Chapter 5, we will check to see if our results are sensitive to our choices of χ, α_b, and α_u.

The U.S. levels of (detrended) variables in 1990 that we use when parameterizing the model are exactly the same as those used in Section 2.2.1. Because $\chi < 1$ and technology is non-neutral, some parameters are changed. These are changed to $\theta_1 = 0.263$, $\phi_1 = 0.076$, $\psi = 1.323$, $z^1 = 2.161$, and $z^2 = 1.539$.

2.3.2. Business Cycle Accounting for the 1990s

We have observations on $\hat{y}_{bt}$, $\bar{y}_{nt}$, $\hat{c}_t$, $\hat{x}_{Tt}$, $\bar{x}_{nt}$, $\hat{g}_t$, h_t, h_{bt}, $\bar{h}_{nt}$, τ_{ht}, and τ_{ct}. Given $\hat{x}_{Tt}$, we can use the law of motion for capital as before to get the sequence of capital stocks $\{\hat{k}_{Tt}\}$ given an initial condition $\hat{k}_{T0}$.

We can infer how many hours are used in final production in the business sector by rearranging the intratemporal condition as follows:

$$\hat{h}^1_{bt} = \frac{(1-\theta_1-\phi_1)(1-\tau_{ht})\hat{y}_{bt}(1-h_t)}{\psi(1+\tau_{ct})\hat{c}_t}$$

in terms of observables. Since we observe total business hours, we know the hours spent accumulating intangible, $h^2_{bt} = h_{bt} - h^1_{bt}$. The fact that we had to put in a large L_{wt} in the standard theory works in favor of the technology boom hypothesis. If a rise in h^2_{bt} is the source of the hours boom, then we will get a boom in $\hat{y}_{bt}/\hat{h}^1_{bt}$. This is our story for the earlier labor wedge.

Total output is $\hat{y}_{bt} + \bar{y}_{nt} + q_t\hat{x}_{It}$ where

$$q_t\hat{x}_{It} = \left(\frac{1-\theta_1-\phi_1}{1-\theta_2-\phi_2}\right)\frac{\hat{y}_t}{h^1_t}h^2_t. \tag{2.3.12}$$

The relation follows from the fact that households equate wages in the two business-sector activities. Similarly, households equate returns to tangible business capital in the two business-sector activities and, therefore, it must be that case that

$$\hat{k}^1_{Tt} = \frac{\theta_1 \hat{y}_{bt}}{\theta_1 \hat{y}_{bt} + \theta_2 q_t \hat{x}_{It}} \hat{k}_{Tt}$$

and $\hat{k}^2_{Tt} = \hat{k}_{Tt} - \hat{k}^1_{Tt}$.

To infer sequences for intangible capital $\hat{k}_{It}$, we guess a path for the price q_t and use (2.3.10) and the capital accumulation equation (2.3.3) to derive sequences for intangible flows and stocks. It must be the case that x_{It} multiplied by the guess q_t is equal to the left-hand side of (2.3.12).

Technology parameters are given as follows:

$$A^1_t = \hat{y}_{bt}/[(\hat{k}^1_{Tt})^{\theta_1}(\hat{k}_{It})^{\phi_1}(h^1_t)^{1-\theta_1-\phi_1}]$$

$$A^2_t = \hat{x}_{It}/[(k^2_{Tt})^{\theta_2}(k_{It})^{\phi_2}(h^2_t)^{1-\theta_2-\phi_2}]$$

We also include an investment wedge in (2.3.9) to account for any mismeasurement in capital tax rates (which we assumed to be constant over this period). The expression is exactly as in the three earlier examples with $R^T_{t,t+1}$ given by (2.2.8). We need the effect of the investment wedge to be quantitatively small if our theory is to satisfy the input justification criterion. Otherwise, we need to rethink our assumption of constant tax policy.[11]

We turn now to the numerical experiments. Table 4 has the values of the implied exogenous variables. In Figure 20, we plot U.S. per capita hours along with the model's

[11] In our sensitivity analysis, we consider feeding in some estimates of capital tax rates for the 1990s.

Year (t)	A_t^1	A_t^2	τ_{ht}	τ_{ct}	X_{wt}
1990	1.6534	1.3111	0.3109	0.0657	1.0000
1991	1.5956	1.2357	0.3070	0.0675	1.0116
1992	1.5910	1.1608	0.3028	0.0678	1.0064
1993	1.5940	1.2445	0.3034	0.0678	1.0110
1994	1.6213	1.3818	0.3068	0.0702	1.0140
1995	1.6312	1.4321	0.3116	0.0686	1.0208
1996	1.6579	1.5176	0.3190	0.0674	1.0252
1997	1.6814	1.5740	0.3254	0.0674	1.0336
1998	1.7125	1.5948	0.3327	0.0670	1.0327
1999	1.7358	1.6245	0.3335	0.0662	1.0254
2000	1.7380	1.6132	0.3424	0.0649	1.0314
2001	1.7027	1.6180	0.3472	0.0625	1.0416
2002	1.5901	1.3214	0.3076	0.0617	1.0431
2003	1.5556	1.2302	0.2885	0.0621	1.0428

Table 4. Exogenous Variables for Model with Intangible Capital and Non-neutral Technology

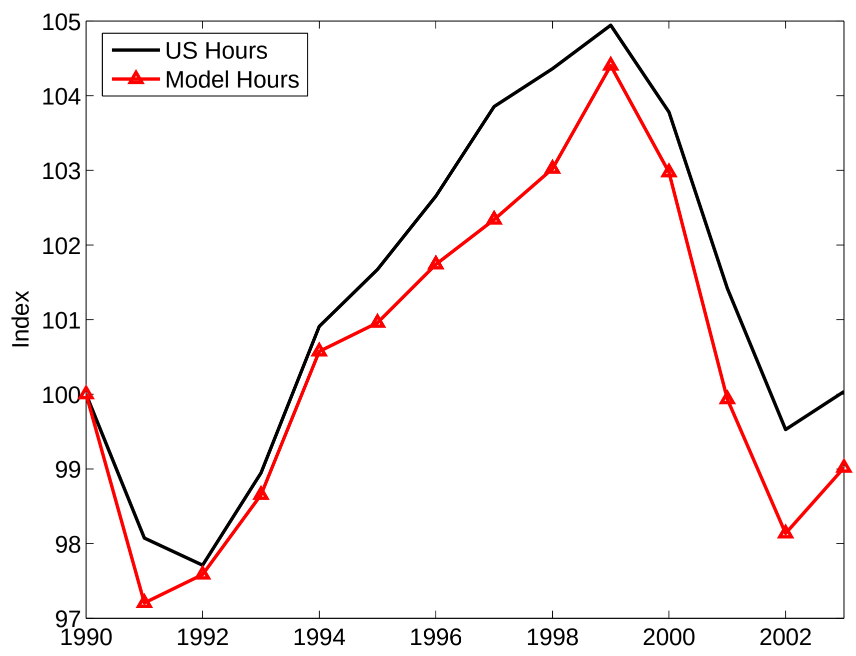

Figure 20. U.S. Per Capita Hours and Prediction of Model with Intangible Capital and Non-neutral Technology

(Investment wedge constant)

prediction for the case with only the TFPs and tax rates on labor and consumption varying

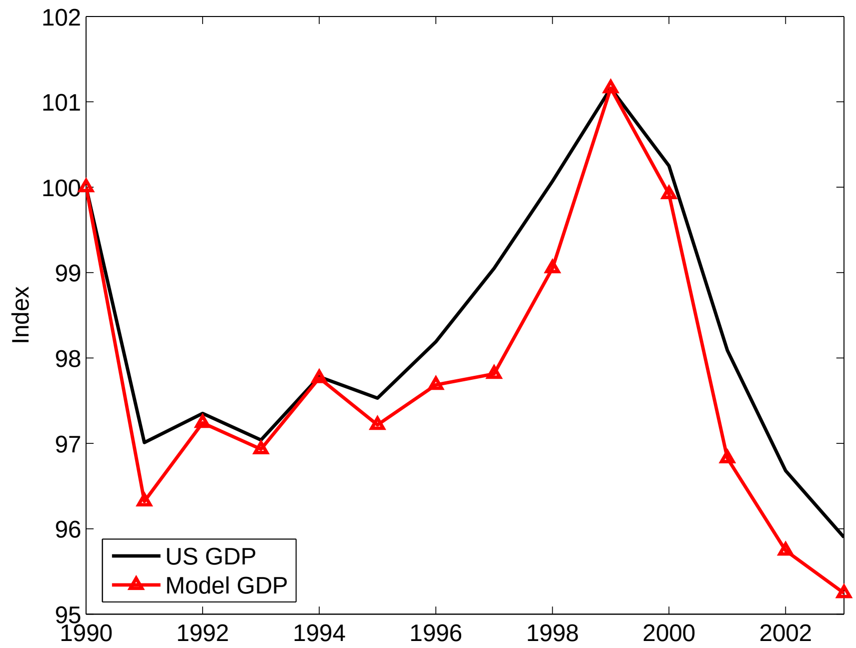

Figure 21. U.S. Per Capita Real GDP and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

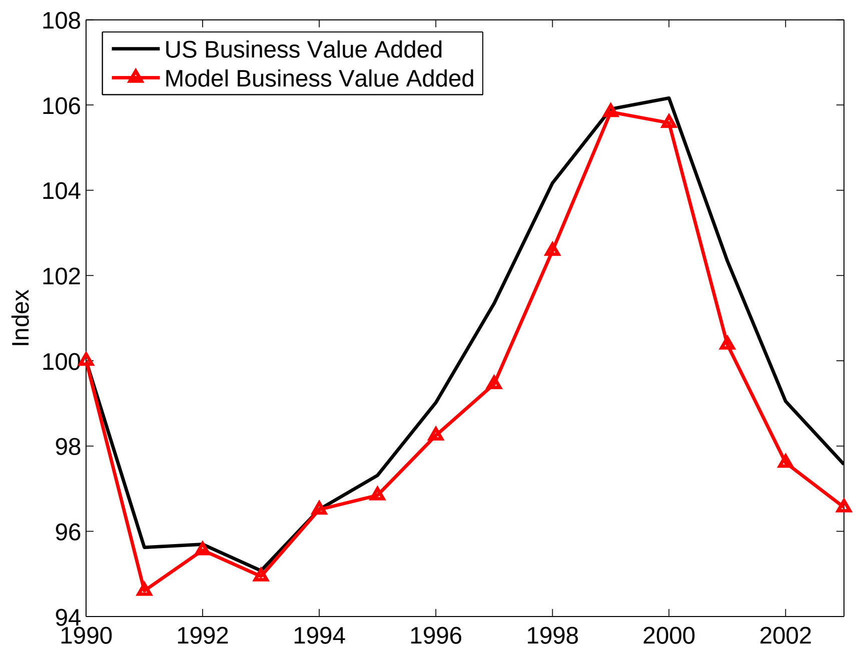

Figure 22. U.S. Per Capita Real Business VA and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

(i.e., $X_{wt} = 1$ for all t). By construction, if the investment wedge were varying, then the model would fit exactly and the predicted and actual series would lie on top of each other. The difference in the actual and predicted series is therefore attributed to the wedges. Clearly, this difference is small.

Figures 21 and 22 show output for the aggregate economy and the business sector. In this case, the match is extremely close. As a contrast, compare these figures to Figures 9 and 10.

Figures 23 and 24 show labor productivity for the aggregate economy and the business sector. Given the good agreement between theory and data for both outputs and hours, it is not a surprise that predicted and actual series are close. The significant improvement in fit can be seen by comparing these figures to Figures 11 and 12.

Figure 25 displays tangible investment for both the model and the data. The deviation from theory seems larger than it really is. For example, if we set 1991=100, the series line up nicely. The same is true for consumption displayed in Figure 26.

As a check on the model's predictions, we compared the model's prediction for factor incomes and capital gains to U.S. measures in, respectively, the NIPA and the U.S. Flow of Funds accounts. This is shown in Figures 27 and 28. In Figure 28, the data have been averaged. The line at the beginning of the 1990s is the average for the period 1953–1994, and the line at the end of the 1990s is the average for the period 1995–2003. Neither the income data nor the capital gains data are used to infer our measures of sectoral TFPs. We find that the model's predictions of both incomes and capital gains are in conformity with U.S. observations.

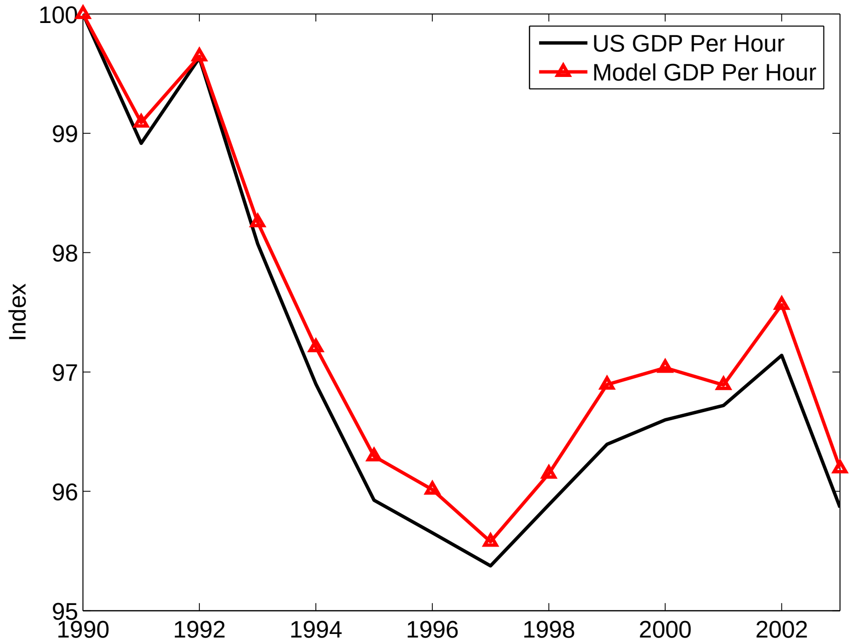

FIGURE 23. U.S. REAL GDP PER HOUR AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

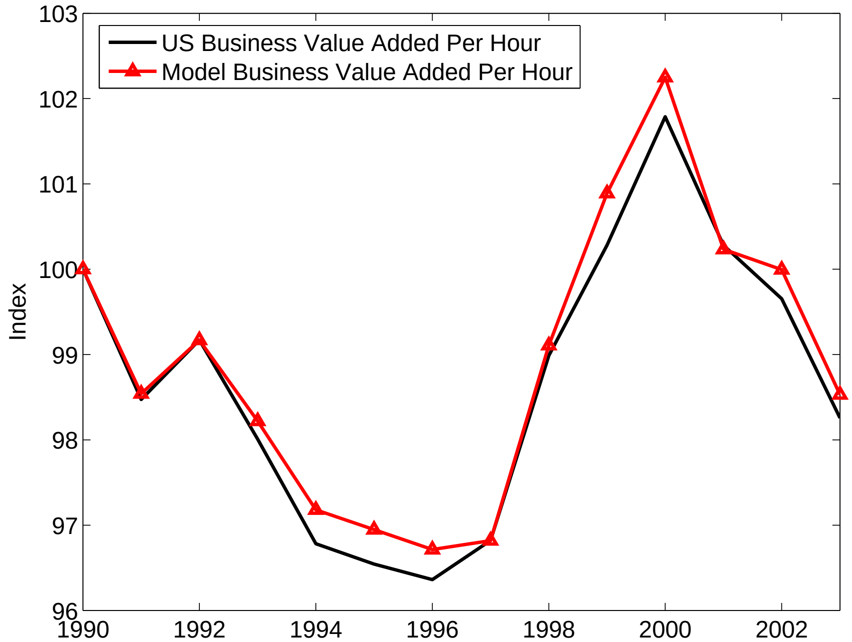

FIGURE 24. U.S. REAL BUSINESS VALUE ADDED PER HOUR AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

2.3.3. Can Our Theory Be Unsuccessful?

An issue is whether our theory can ever be unsuccessful given that the path of intangible capital is inferred from first-order conditions of the theory. To address this issue, we carry out the following experiment. We simulate data for the model with no intangible capital, assuming that TFP is on trend, no change in capital tax rates, and a large decline in labor market distortions other than τ_{ht} or τ_{ct}. In other words, we simulate a large rise in the labor wedge that proxies for labor market distortions other than government taxes. (See Section 2.1.1.) We treat these simulated data as the true economic data.

With these data, we ask, If we analyzed these data using our preferred theory with intangible capital and non-neutral technology, would we say that the theory satisfies our two criteria for a successful theory?

The short answer is no: the model with intangible capital and non-neutral technology would not satisfy either criterion. The theory would not satisfy the input justification criterion because it predicts a huge boom in R&D and other expensed investments ($q_t x_{It}$) when there was not a shred of micro evidence for it—in fact, the true economy being analyzed has *no intangible capital at all*. Even if there was a sizable intangible capital stock, we would face the same problem if it was not in fact abnormally high during the period of interest. With our model, we would infer an abnormally high intangible stock, but would find no micro evidence for the rise. We would also predict a big temporary shift in employment in certain occupations (h_{2t} high relative to h_{1t}) but would find no evidence of this given that the changes in the true economy are neutral with respect to employment sectors. The model would not satisfy the prediction criterion because the capital gains in the true economy barely changed, whereas we would be predicting a huge boom.

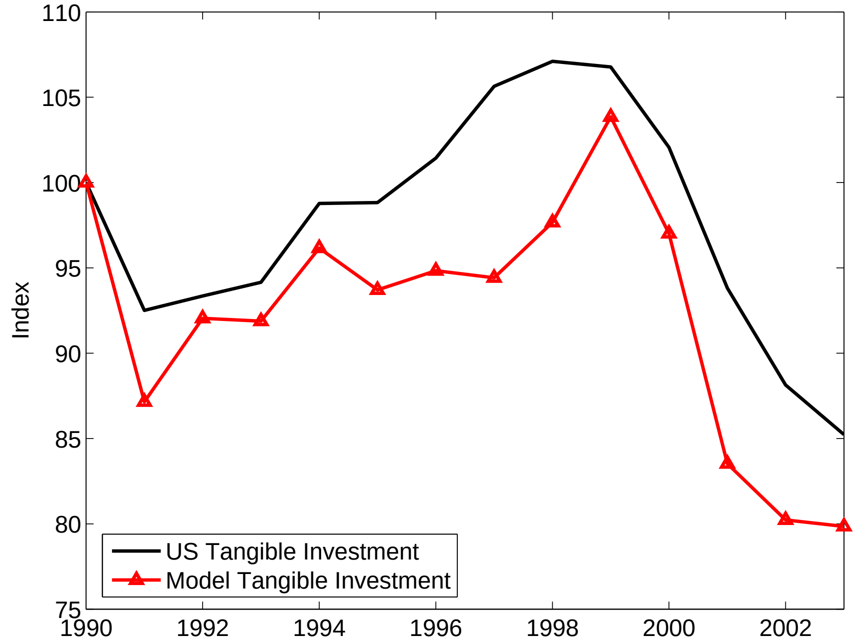

FIGURE 25. U.S. PER CAPITA REAL INVESTMENT AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

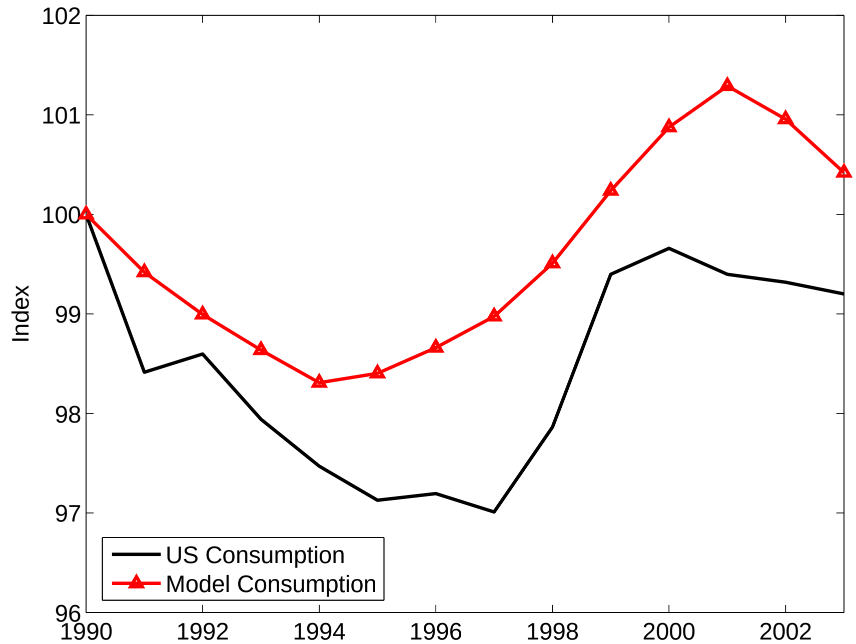

FIGURE 26. U.S. PER CAPITA REAL CONSUMPTION AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

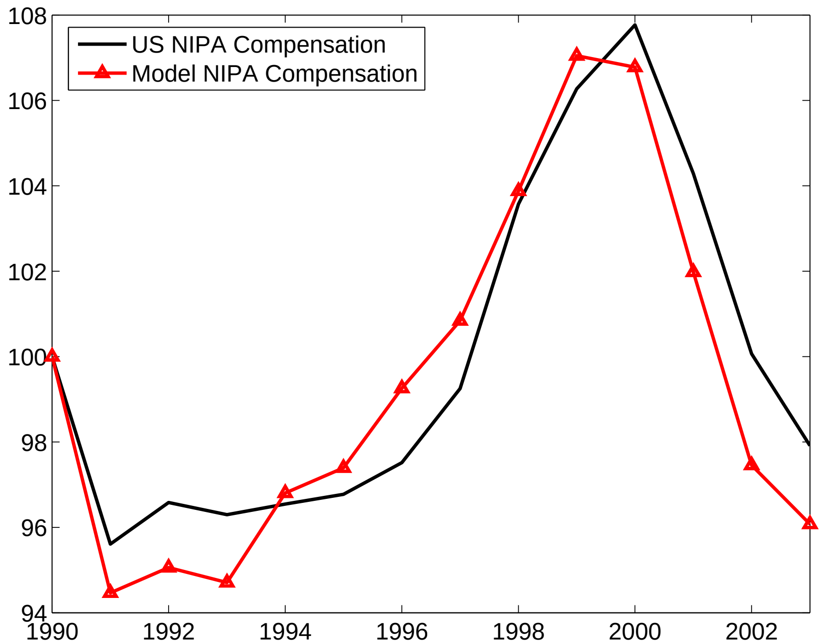

Figure 27. U.S. Per Capita Real Business Compensation and Prediction of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

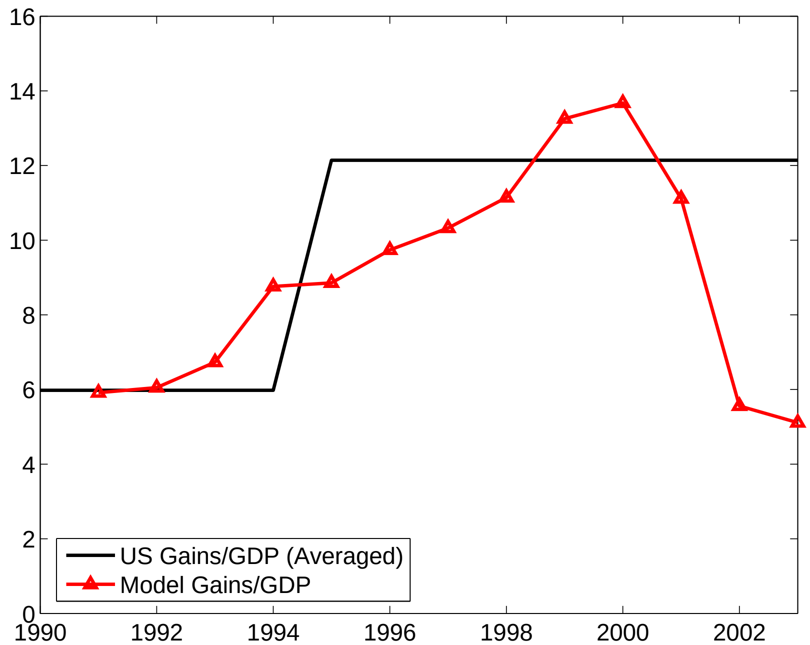

Figure 28. U.S. Real Holding Gains as % of GDP and Prediction of Model with Intangible Capital and Non-neutral Technology

(Investment wedge constant)

Summary

In this chapter, we assessed three theories designed to generate equilibrium paths for GDP, consumption, investment, and hours that matched the U.S. time series exactly. Despite the perfect fit, only one theory satisfied our criteria for a successful theory. We also showed that this was not a foregone conclusion just because intangible investment was included in the model.

Chapter 3.

Accounting for Business Cycles During 1960–1989

In our paper we focus on the 1990s as a period during which a huge unexplained deviation from theory occurred. Many people have noted "that the baseline model from which you [McGrattan and Prescott] start has difficulties in replicating the unfiltered and TFP-detrended macroeconomic time series not only in the 90s, but generally." (Here, we are quoting the editor handling the paper.) In this chapter, we demonstrate that the basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated.

The view that the "basic" neoclassical growth model does poorly in general is largely due to key missing factors in the basic model. For example, to account for movements in hours and labor productivity, one must account for key distortions to the labor market, in particular tax rates on labor. If we compare the predictions of Uhlig's (2003) real business cycle model and Chen et al.'s (2007) real business cycle model, we reach a different conclusion about the performance of the "basic" neoclassical growth model driven by real factors.

Figure 29 shows Uhlig's (2003) prediction for the log deviation in business along with the U.S. measure. In Uhlig's model, fluctuations are driven by changes in TFP, government spending, and population growth. Periods in his model are quarters. The figure is reproduced from Uhlig's Table 6, which has the caption "uncomfortably big gaps are visible." In terms of hours, the most uncomfortable gaps are during the recessions of

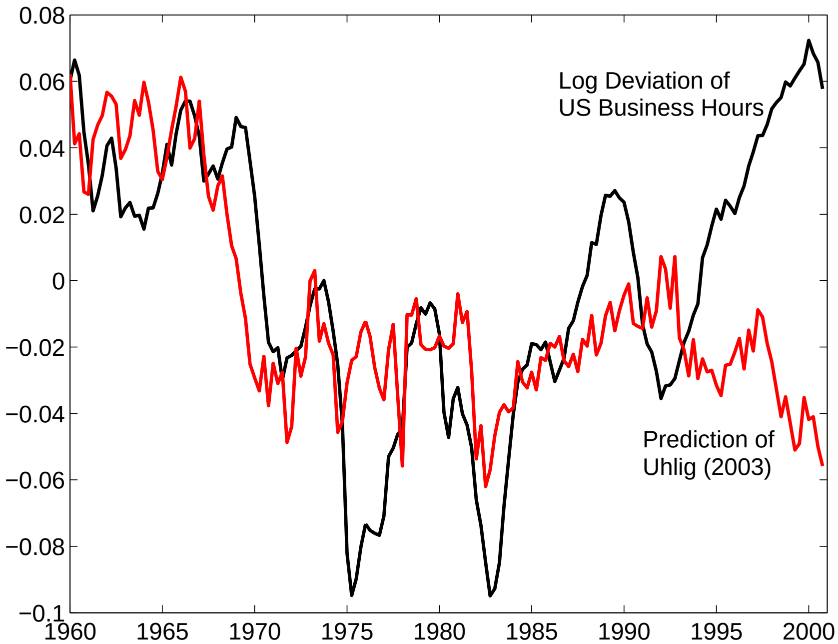

FIGURE 29. DEVIATION OF LOGARITHM OF U.S. PER CAPITA BUSINESS HOURS AND PREDICTION OF UHLIG (2003, TABLE 6) MODEL, 1960:1–2000:4

the 1970s and 1980s and especially during the boom of the 1990s. The model gets 1/3 of the 1970s decline in hours, 1/2 of the 1980s decline, and predicts a depression in the 1990s when in fact there was a boom.

Figure 30 shows Chen et al.'s prediction of total hours along with the U.S. measure (based on the establishment survey). This figure is reproduced from their paper (page 15). In Chen et al.'s model, fluctuations are driven by changes in TFP, labor tax rates, capital tax rates, depreciation rates, government spending, and population growth. Periods in their model are years. Interestingly, if we focus on the cyclical movements of hours prior to the 1990s, the model does quite well. To see this better, we apply a Hodrick-Prescott (1997) filter (with the smoothing parameter set equal to 100). Figure 31 shows the filtered series for 1960–1989. The cyclical predictions are extremely good until the late 1980s. In contrast to Uhlig's model, this model accounts for most of the decline in the 1970s and all

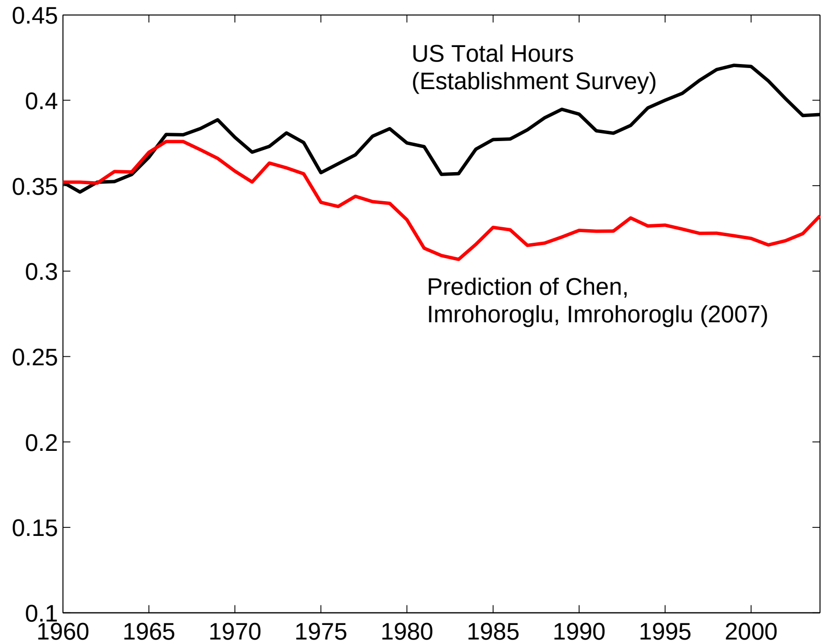

Figure 30. U.S. Per Capita Total Hours and Prediction of Chen et al. (2007, p.15) Model, 1960–2004

of the decline in the 1980s. An important difference between Uhlig (2003) and Chen et al. (2007)—especially in the case of hours worked—is that Chen et al. include variations in taxes, whereas Uhlig does not.

However, during the 1990s, both the Uhlig model and the Chen et al. model do poorly. Both predict depressions when in fact there was a boom. Figure 32—which displays the filtered series of Chen et al. for 1990–2004—is a dramatic contrast to Figure 31 for the earlier period. Clearly, there is something missing. This was our starting point. This is what puzzled us for many years.

An open question remains about the basic model's (of Chen et al.) predictive ability for secular trends. Some of the deviation between theory and data in Figure 30 may be due to measurement. For example, the aggregate CPS-based hours series that we use does not rise as much as the aggregate establishment-based hours series that Chen et al. use.

Some of the deviation between theory and data may be due to treatment of households as uni-sex. Work is beginning on this important topic. In our opinion, the deviation in the 1990s was a big and important puzzle, so we started there.

One final note concerns the role of intangible capital in the pre-1990 period. If technological change is neutral, the predictions of our model *with intangible capital* is the same as Chen et al.'s model *without intangible capital* for the relevant set of (measured) variables. In particular, we too would generate results like that in Figure 32 as long as we allow for variations in TFP, tax rates, government spending, population, and depreciation as in Chen et al. (2007).

Summary

In this chapter, we displayed the results of Chen et al. (2007) to motivate our claim that the basic neoclassical growth model accounts well for the postwar cyclical behavior of the U.S. economy prior to the 1990s, provided that variations in population growth, depreciation rates, total factor productivity, and taxes are incorporated. We contrasted these results with the model in Uhlig (2003), which does not include variation in tax rates and, therefore, does poorly in accounting for movements in hours and productivity.

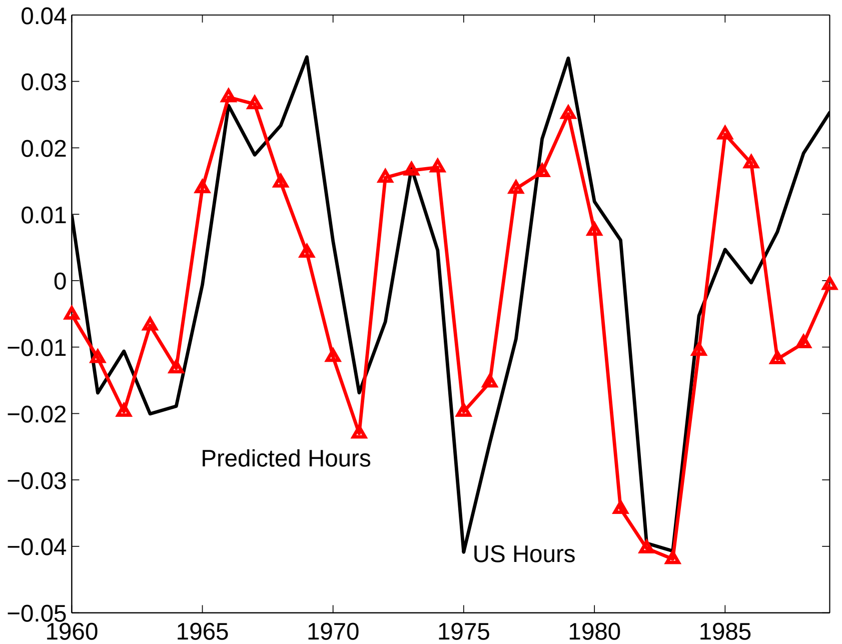

Figure 31. Filtered U.S. Per Capita Hours and Prediction of Chen et al. (2007) Model, 1960–1989

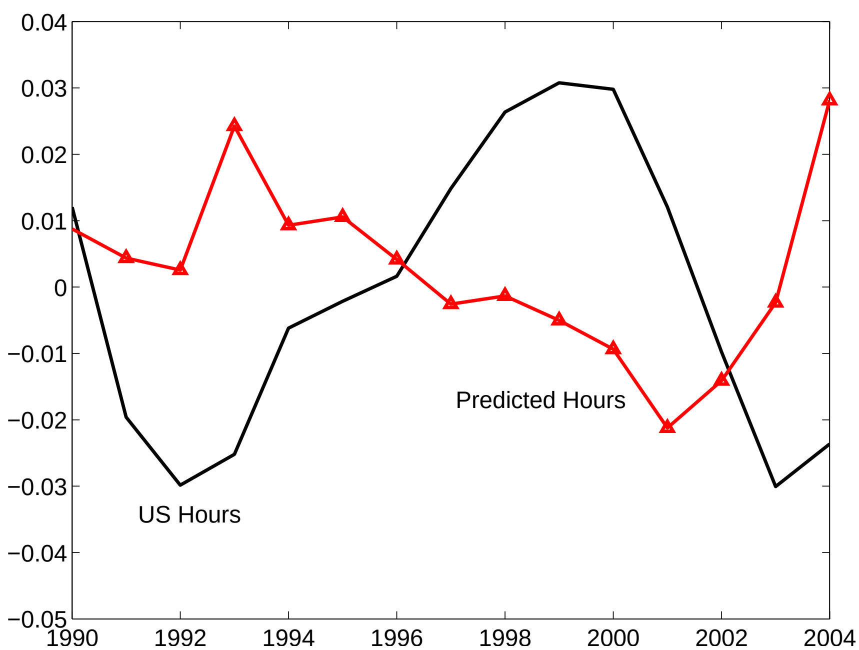

Figure 32. Filtered U.S. Per Capita Hours and Prediction of Chen et al. (2007) Model, 1990–2004

Chapter 4.

Assessing New Keynesian Business Cycle Theory

One reasonable reaction to predictions such as those shown in Figures 1–16 is to abandon the standard neoclassical theory and replace it with something else. That has happened to some extent in the macro literature as many researchers have switched to analyzing and using new Keynesian theories. In this chapter, we briefly describe a typical model in this class that Smets and Wouters (2007) analyzed. We then ask, in the context of their model, Why did hours boom in the 1990s? The answer is the same one given in Section 2.1: because a labor wedge boom occurred. The Smets-Wouters model says that there was a shock to wage markups and point out that "alternatively, we could interpret this disturbance as a labour supply disturbance coming from changes in preferences for leisure" (p. 15). Here, we report on some key predictions of the Smets-Wouters model and compare them to the same predictions of the model of Section 2.1.

The new Keynesian models are more complicated than the models worked out in Chapter 2 because they typically have many nominal and real "rigidities" intended to help propagate shocks. Here, we will report results based on the Smets-Wouters (2007) model, which has sticky wages and prices, habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. Fluctuations are driven by seven exogenous variables (to account perfectly for seven observables). Five of the exogenous stochastic variables are modeled as AR(1) processes, and two are modeled as ARMA(1,1) processes. None of the exogenous variables are taken from outside data sources or estimated outside the model (e.g., tax rates or government spending). (See Smets and Wouters 2007 for details.)

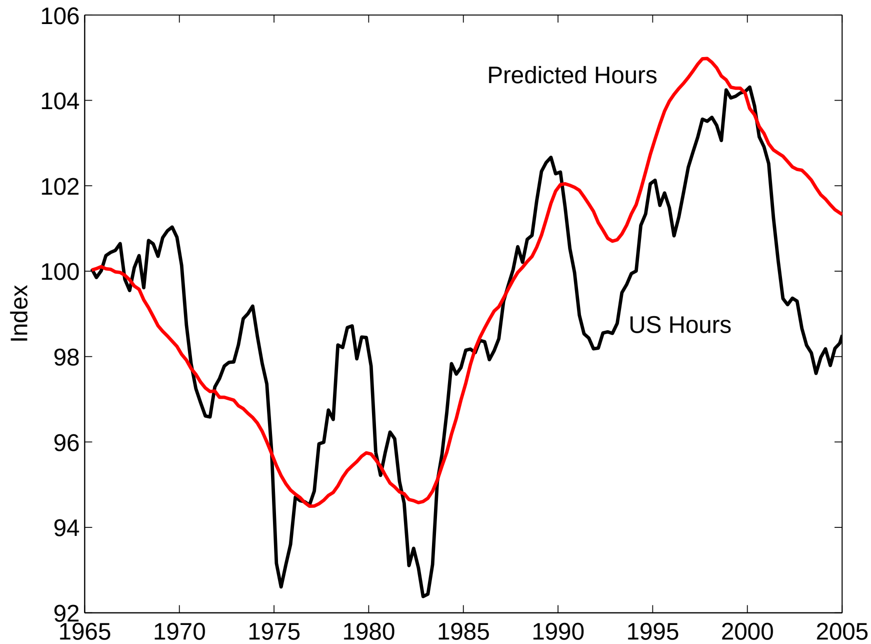

FIGURE 33. U.S. PER CAPITA BUSINESS HOURS AND SMOOTHED PREDICTION FOR SMETS-WOUTERS (2007) MODEL

(Preference shock to leisure only, 1965:1–2004:4)

Smets and Wouters (2007) estimate the parameters of their model and use the estimates to compute a variance decomposition for the observed variables. For business hours, they find that 67 percent of variation in the 1965–2005 period is due to shocks to preferences for leisure—what we call the labor wedge. In Figure 33, we plot the series for business hours that they use (which is based on establishment data) along with the model's prediction of hours. It is not a perfect fit because we shut off all shocks except the shock to preferences for leisure. (Note that if we turn on all shocks, the model's prediction will be the same as the data by construction.) The figure shows why most of the variation in hours is being attributed to the labor wedge.

If we zoom in on the figure in the 1990s, we get Figure 34. We can compare this Smets-Wouters prediction for the path of business hours due to the labor wedge with the prediction of standard theory (of Section 2.1) for business hours that is shown in Figure

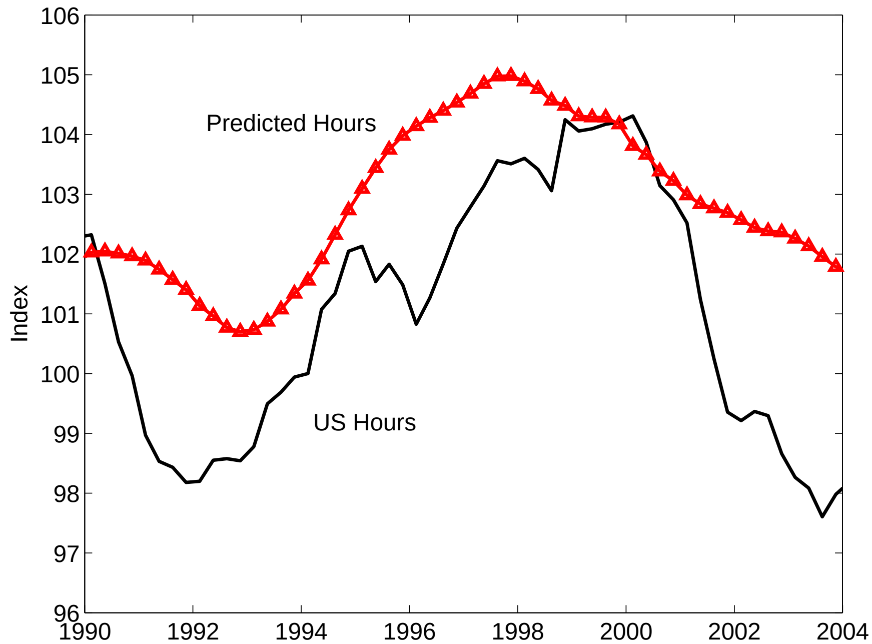

FIGURE 34. U.S. PER CAPITA BUSINESS HOURS AND SMOOTHED PREDICTION FOR SMETS-WOUTERS (2007) MODEL

(Preference shock to leisure only, 1990:1–2003:4)

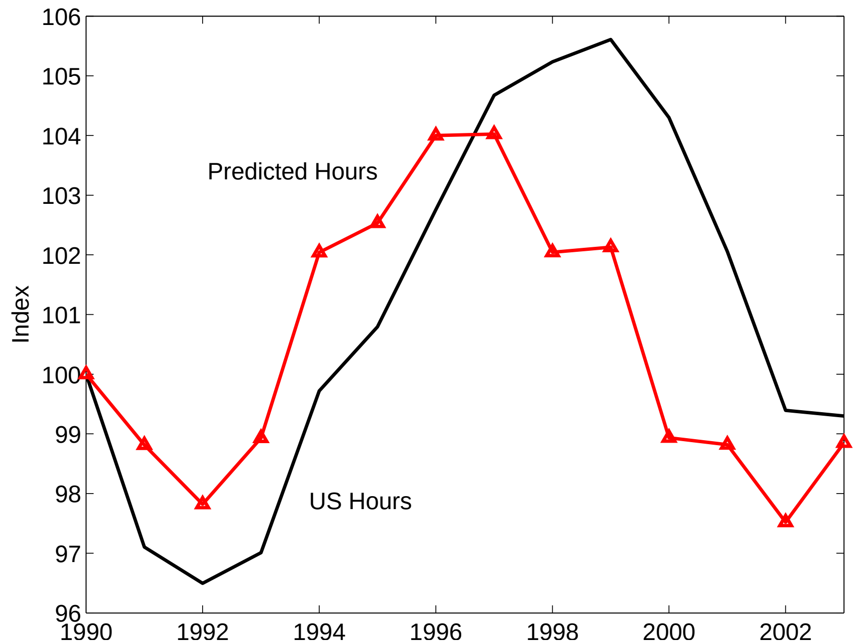

FIGURE 35. U.S. PER CAPITA BUSINESS HOURS AND PREDICTION FOR MODEL WITHOUT INTANGIBLE CAPITAL

(TFP and investment wedge constant)

35. We find them similar. Both models attribute the rise in hours to this wedge. In neither model is the boom due to TFP or to monetary shocks. So, what is the implication of this new line of research? It is that the hours boom of the 1990s is primarily due to a preference shock for leisure, which is of the same order of magnitude as the neoclassical model requires to generate the observed hours boom. In building in monetary factors necessary to evaluate the effects of monetary policy, a key puzzle has not been resolved.

Summary

Without evidence or theory for labor wedges, new Keynesian theory does not provide any better understanding of the 1990s than the neoclassical model without intangible capital and non-neutral technological change.

Chapter 5.

Checking the Sensitivity of Our Results

In this section, we report on our sensitivity analysis for the model with intangible capital and non-neutral technology. In Section 5.1, we examine the sensitivity of our findings to the values of the parameters and find that the results are robust to their specification. Consequently, the parameters are not being selected to fit the episode. In Section 5.2, we establish that the expectation assumption with regard to the total factor productivity parameters has almost no consequence for the realized path of the economy.

5.1. Varying Parameters of the Model

Here, we describe how our results are affected as we vary factor shares (θ_i, ϕ_i), the share of intangible investment financed by shareholders (χ), the depreciation rate on intangible capital (δ_I), and tax rates on labor and profits (τ_{ht}, τ_{pt}). In all cases, we follow the same procedure as described in Section 2.3 to determine the parameters that are not pre-set.

In the main paper, we imposed symmetry in technologies. In particular, we set $\theta_1 = \theta_2$ and $\phi_1 = \phi_2$. Here, we show results for a plausible alternative with an intangible sector that is intangible-capital intensive. Specifically, we set $\phi_2 = .2$ and $\theta_2 = .1$. (We could go to the extreme point of $\theta_2 = 0$, but it seems implausible that no structures or computers are being used in the sector producing intangible capital.) With these new parameters, we determine a new sequence of TFP parameters consistent with our theory. These parameters, along with the tax rates of Table 4, are input in the model.

In Figure 36, we plot hours of work in the baseline economy and the alternative

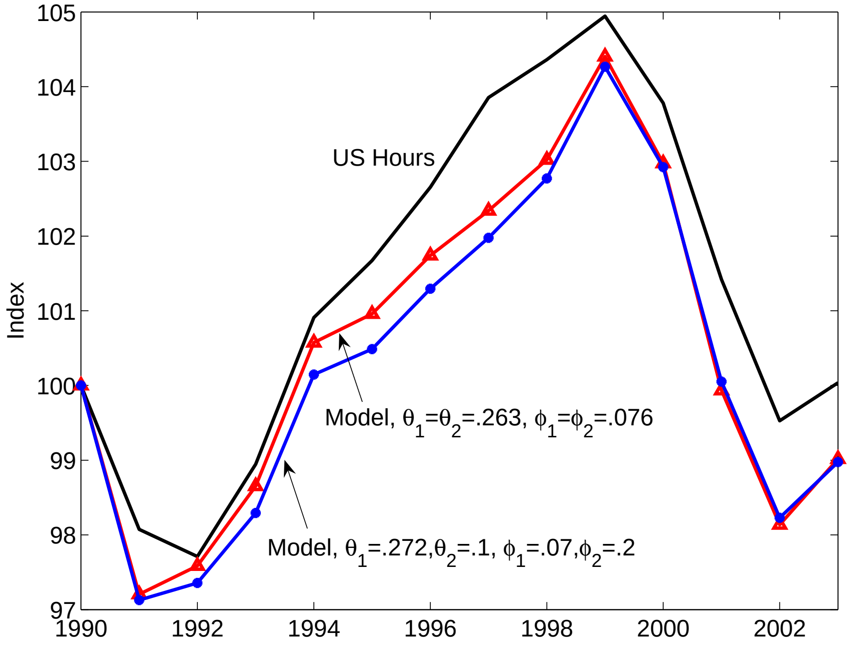

Figure 36. U.S. Per Capita Hours and Predictions of Model Intangible Capital and Non-neutral Technology

(Investment wedge constant)

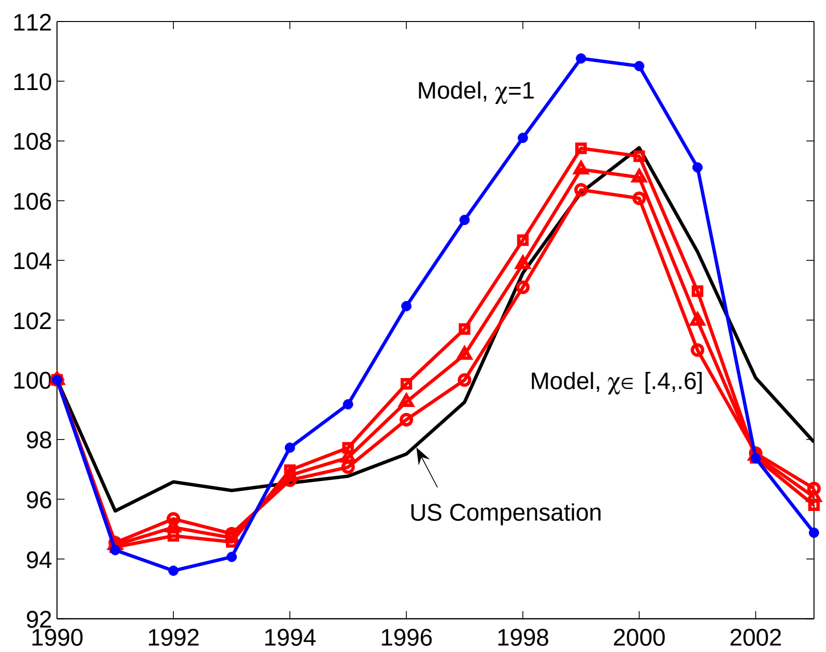

Figure 37. U.S. Per Capita NIPA Compensation and Predictions of Model with Intangible Capital and Non-neutral Technology, Series Divided by 1.02^t

(Investment wedge constant)

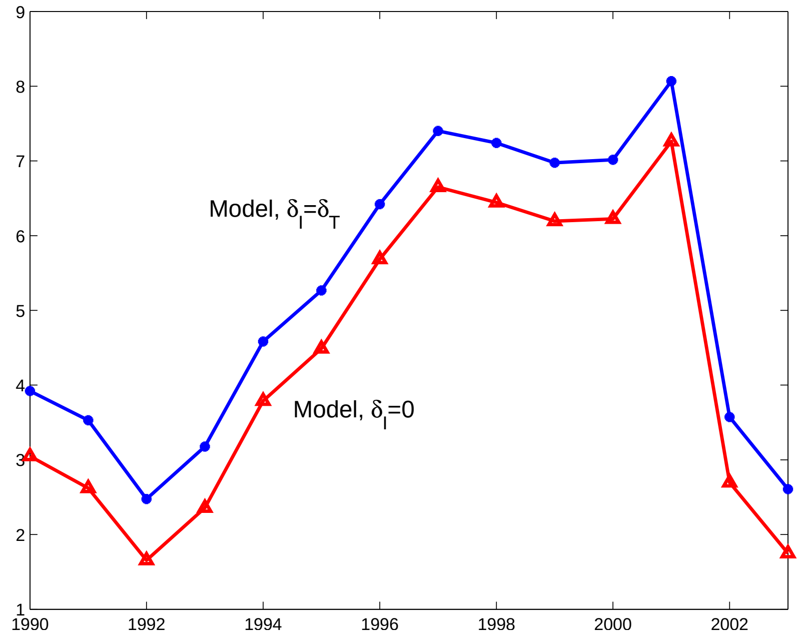

FIGURE 38. PREDICTED INTANGIBLE SHARE OF OUTPUT IN MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

economy. As the figure shows, there is only a small difference in the predictions. The same is true for the other series displayed earlier. Other variations of the technologies did not produce large deviations.

For the next experiment, we varied the parameter χ, which determines the amount of intangible capital expensed from shareholder profits versus the amount expensed from compensation of business owners. At this point, we do not have any direct estimates of this parameter. As we noted in the main paper, values for χ in a range near the baseline value of 1/2 yield similar results. This is shown in Figure 37, where we display measured (NIPA concept) compensation for $\chi = 0.4$, 0.5, and 0.6 (all in red). We then set $\chi = 1$—which is the case where all intangible capital is financed by shareholder profits. This is the assumption we made in earlier work before we realized the importance of sweat equity.

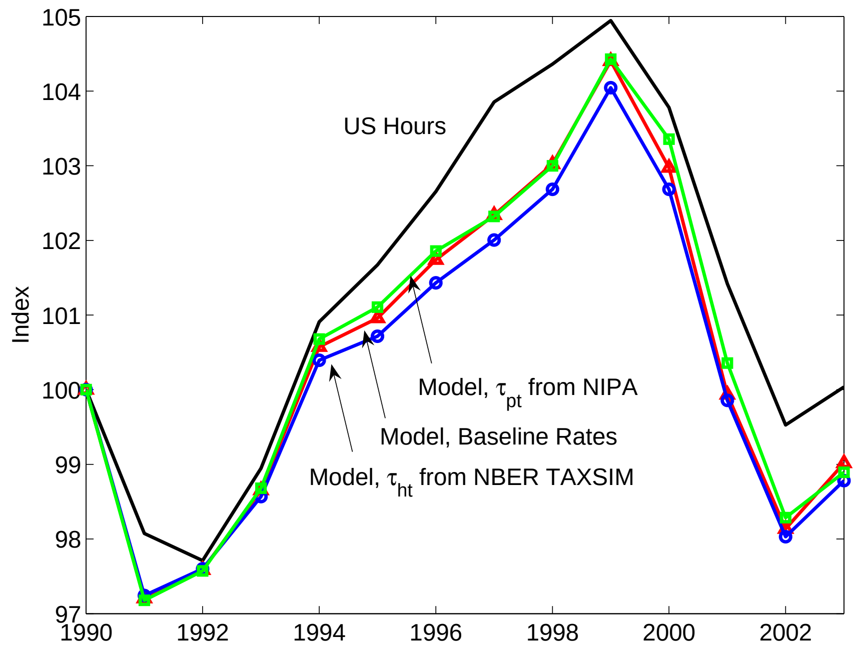

FIGURE 39. U.S. PER CAPITA HOURS AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY

(Investment wedge constant)

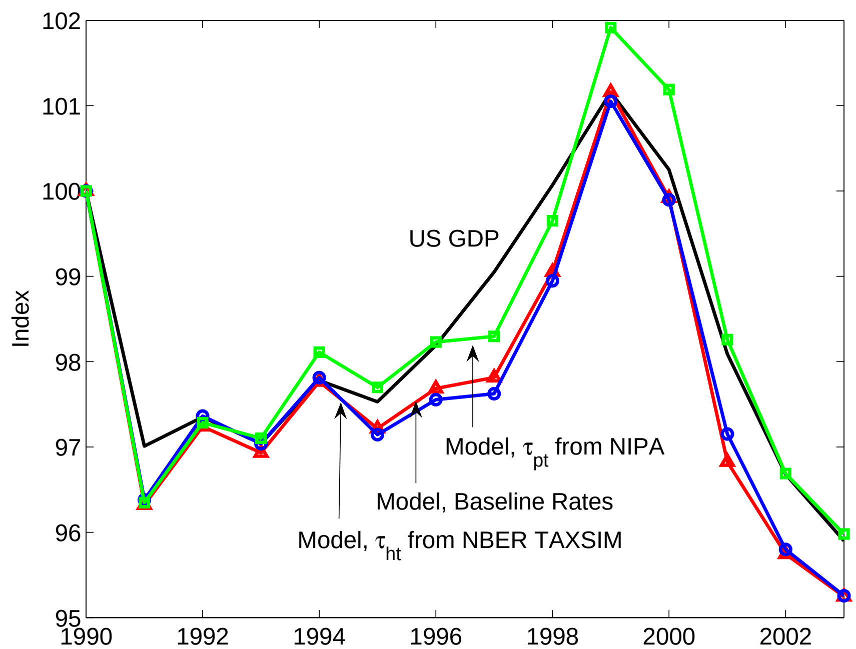

FIGURE 40. U.S. PER CAPITA REAL GDP AND PREDICTION OF MODEL WITH INTANGIBLE CAPITAL AND NON-NEUTRAL TECHNOLOGY, SERIES DIVIDED BY 1.02^t

(Investment wedge constant)

In this case, we do see that predicted (measured) compensation is quite a bit higher than compensation reported in NIPA.

Another parameter for which there is no direct estimate is δ_I. In our baseline simulations, this parameter was set to 0, in part because intangible investments include investments in organizations that are long-lived. To see how important this assumption is, we considered an alternative economy with $\delta_I = \delta_T$. With the exception of the level of the share of intangible capital in output, the figures hardly change. In Figure 38, we show the share of intangible investment in output. Notice that the curve shifts up roughly the same amount at each date. The growth in the share is unaffected.

We turn next to the tax rates. Given the fact that most current models rely on large preference shocks to leisure—which are isomorphic to large declines in the tax on labor—we thought it important to try other measures of the labor tax. In Figures 39 and 40, we display the predictions for per capita hours and real GDP for the baseline rate τ_{ht} in Table 4 and the estimates of the NBER TAXSIM model available at www.nber.org. (See Feenberg and Coutts 1993 for details.) It is hard to distinguish the predictions for these alternative measures in the figures.

We also examined how our results change if we relax the assumption of a constant profit tax rate. We use the following measure for τ_{pt}: NIPA corporate tax liabilities divided by corporate income (with Federal Reserve profits subtracted from both the numerator and the denominator). The difference in the hours predictions for this case and the baseline case is hard to see in Figure 39. There is some noticeable difference in the predictions for GDP because the constructed τ_{pt} series has some effect on our prediction for tangible capital. One could reasonably argue that adding variable profits taxes improves the fit of the model.

5.2. Varying Expectations

In this section, we demonstrate that the perfect-foresight assumption is innocuous. We do this by comparing results for two versions of our model that differ only in the assumption of expectations.

To simplify our analysis, we set all tax rates and non-business variables equal to their 1990 levels and ignore the investment wedge. Another simplification involves the TFP processes. We want to generate a non-neutrality in A_t^2 relative to A_t^1. We do this by modeling the two TFPs as functions of the same AR(1) process. Thus, the problem boils down to one with a one-dimensional shock process.

More specifically, we use Tauchen's (1986) discrete approximation for AR processes to construct a Markov chain for an AR(1) process $s_{t+1} = \rho s_t + \epsilon_t$ with $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. We set A_t^1 equal to its steady-state value plus s_t. We set A_t^2 equal to its steady-state value plus 1.5 times s_t. The transition matrix is the matrix computed using Tauchen's method for a specific value of ρ.

Given the processes for the TFP parameters, we find an equilibrium stochastic process for the economy. We pick a particular realization of the exogenous stochastic TFPs, making sure that our choice implies increases in hours and output that are of similar magnitude as their observed counterparts. We then find the equilibrium realization of the endogenous variables for the realization of exogenous stochastic elements of the economy. Finally, we compare this to the realization if households had perfect foresight about the paths of the TFP parameters.

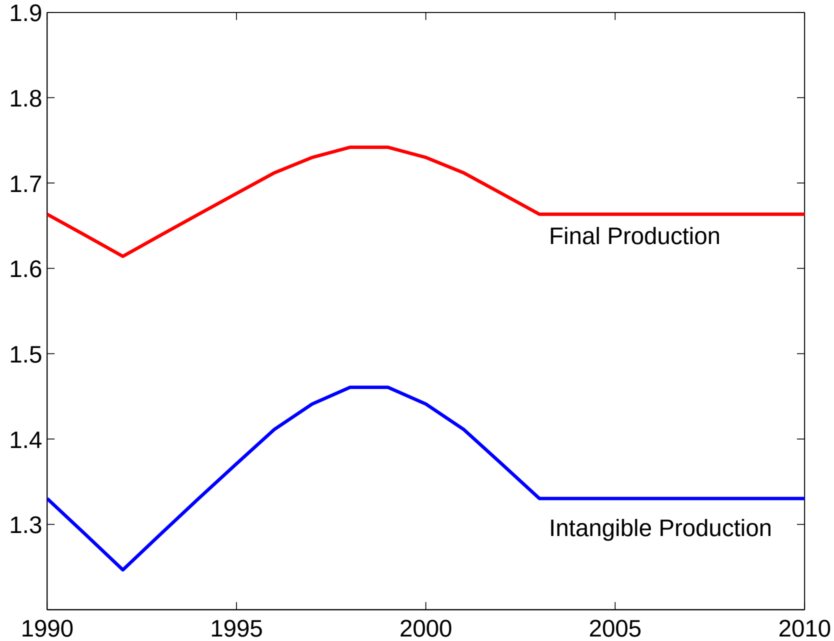

Figure 41. Realizations of TFP Inputs in Stochastic Model with Intangible Capital and Non-neutral Technology

(All other exogenous inputs constant)

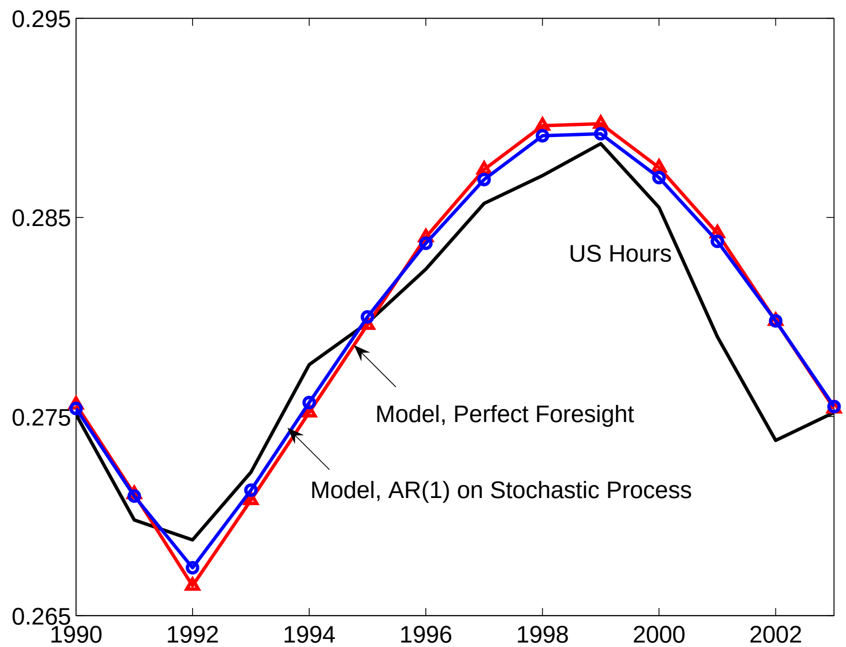

Figure 42. U.S. Per Capita Hours and Predictions of Stochastic Model with Intangible Capital and Non-neutral Technology

(All other exogenous inputs constant)

In Figure 41, we display the realizations of the paths of TFP in final production A_t^1 and TFP in intangible production A_t^2 that we use in our experiment. These are not the same paths used in generating the results of the paper because of the restrictions on the exogenous inputs. However, these values of TFPs do imply a technology boom of the same magnitude as we saw in the United States during the 1990s.

In Figure 42, we display the household's realized hours in the case that TFPs are governed by an AR(1) process and in the perfect-foresight case. The difference in predicted paths for hours is small. Similarly, the difference in terms of other endogenous variables is small. The reason is that the realizations of A_t^1 and A_t^2 are what is important, not the choice of household expectations. For this experiment, we use $\rho = .95$, but varying ρ does not change the result. Also plotted are U.S. hours. As we noted earlier, this model does generate a boom of the right magnitude.

Summary

We find that our conclusions are robust to plausible variations in parameters and expectations.

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