

# Technical Appendix for “Transitional Dynamics of Dividend and Capital Gains Tax Cuts”

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In this appendix, we present the details of the extended model with debt in our paper “Transitional Dynamics of Dividend and Capital Gains Tax Cuts.” Section 1 presents the extended model and results. Section 2 presents the numerical algorithm to solve this model. Section 3 presents an additional figure for the simulation conducted in Section 3 of our original paper.

## 1 The Extended Model with Debt

We extend the baseline model to incorporate debt financing. To keep the model tractable, we consider risk-free debt and ignore the issue of default. Debt has a tax advantage in that interest payments are tax deductible. But debt is limited by a collateral constraint, as in Kiyotaki and Moore (1997) and Hennessy and Whited (2005). Suppose a firm issues debt  $b_t$  with interest rate  $r_t$ . We interpret the case with  $b_t < 0$  as saving. The collateral constraint is given by:

$$(1 + r_t) b_t \leq \eta k_t, \quad b_0 \text{ given}, \quad (1)$$

where  $\eta > 0$ . The firm’s flow of funds constraint becomes:

$$x_t + \frac{\psi x_t^2}{2k_t} + d_t + (1 + r_t) b_t = (1 - \tau^c) \pi(k_t, z_t; w_t) + \tau^c (\delta k_t + r_t b_t) + s_t + b_{t+1}. \quad (2)$$

Its decision problem is to choose  $\{b_{t+1}, k_{t+1}, s_t, d_t, x_t\}$  so as to maximize

$$V_t = E_t \sum_{j=0}^{\infty} \frac{1}{R_{t,t+j}} \left( \frac{1 - \tau_{t+j}^d}{1 - \tau_{t+j}^g} d_{t+j} - (1 + \lambda \mathbf{1}_{s_{t+j} > 0}) s_{t+j} \right),$$

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subject to (2), (1), and

$$d_t \geq 0, \tag{3}$$

$$s_t \geq -\bar{s}. \tag{4}$$

In this case, there are three state variables  $(k_t, b_t, z_t)$  in the firm's dynamic programming problem. As a result, firms can be differentiated by these three characteristics. In the cross section, there is a distribution  $\mu_t$  of firms over  $(k_t, b_t, z_t)$ . We use this distribution to conduct aggregation. We can then define a competitive equilibrium as in Section 2.4.

We set  $\eta = 0.3$ , which is within the range of estimates of capital resale discounts in Ramey and Shapiro (2001). Our results are robust to changes in this parameter value. In addition, this value implies that the ratio of debt to firm value is 0.14, which is within the range of empirical estimates. We take all other parameter values as in Table 1 in the original paper. Based on these parameter values, we solve the model numerically and compare the solution with that in the baseline model.

## 1.1 Steady State

We start with the steady-state properties of the extended model with debt. We conduct the policy experiment as in Section 3.2, in which dividend and capital gains tax rates are cut from 0.25 and 0.20, respectively, to the same 0.15 permanently. These tax cuts are unexpected and implemented in the initial period 1. Table 1 presents the pre-tax-cut and post-tax-cut steady states for both the baseline and extended models. We find that the impacts of the tax cuts on the economy in the two models are qualitatively similar, though there are quantitative differences. The tax cuts have a smaller effect (in percentage term) in the extended model with debt. For instance, the capital stock increases by 3.12% following the reform, whereas it is 4.05% in the baseline model without debt. This reflects that the proportional reduction in the user cost is smaller, since the user cost is smaller in the model with debt. Moreover, the productivity effect of relaxing financial frictions is smaller, since debt alleviates financial frictions somewhat.

Compared to the baseline model without debt, the flexibility of using debt and equity financing allows firms to reduce the cost of capital and thus benefits the economy. In particular, the steady-state aggregate real quantities such as investment, capital stock, consumption, employment, and output are all higher in the extended model than in the baseline model. TFP is also higher since firms' expansions can be financed with debt. However, aggregate dividends

and new equity issuance are smaller in the extended model than in the baseline model. This is because firms must use part of earnings to pay interests of debt instead of distributing dividends in the extended model. In addition, in the extended model, firms can raise debt to finance investment and distribute dividends and thus may reduce equity issuance.

[Insert Table 1 Here.]

## 1.2 Transitional Dynamics

Now, we study transitional dynamics for the policy experiments considered in Sections 3.2 and 3.3. Figures 1-4 present the results. These figures reveal that the transitional dynamics of real quantities in the baseline model and in the extended model are similar. The main difference between the two models' predictions is reflected in the financial quantities. In the extended model with debt, firms can borrow or save to transfer cash from the future to the present or from the present to the future. This flexibility allows firms to conduct intertemporal tax arbitrage so that they can take advantage of low dividend taxes. In the baseline model without debt, in order to take advantage of low dividend taxes, the only way to pay more dividends for firms is to cut back investment, *ceteris paribus*.

Figure 2 reveals that, in response to the unexpected and permanent tax cut, aggregate debts rise over time. This is because the collateral constraints are gradually relaxed as firms build up capital stock over time (see Figure 1). Because firms can borrow against their future earnings, they can distribute more dividends initially to take advantage of the dividend tax cut immediately, as revealed in the top left panel of Figure 2.

Figures 3-4 show that, when the dividend and capital gains tax cuts are unexpected and last from periods 1-8 temporarily, the economy will stay in the same steady state as that before the tax cuts in the long run. But investment decreases during periods 1-8 and jumps up in period 9. From periods 1-8 firms raise more debt. Firms use the funds raised by debt to distribute more dividends, rather than to make more investment. As in the baseline model without debt, firms still cut back investment to pay more dividends. In period 9, the dividend tax rate reverts back to the original higher level. Anticipating this policy, firms reduce dividend payments and make more investment in period 9. In addition, firms borrow more in period 8 and repay debts in period 9. Overall, the transitional dynamics of real quantities are very similar in the models with and without debt, but the dividends and equity issuance are more volatile in the extended model with debt. In particular, dividend payments rise by about 35

and 30 percent, respectively, in periods 1 and 8 (compared to about 15 and 25 percent in the model without debt), and decrease by about 20 percent in period 9 (compared to 10 percent in the model without debt).

## 2 Numerical Method

The solution method is similar to that for the model without debt. However, because we now have three state variables (capital, debt, and productivity), we need to modify the previous algorithm to make the computation faster. Our algorithm solves the value function on a relatively coarse grid, but allows the firm's choices for future capital and debt to lie on a thinner grid.

### 2.1 Steady State

We use the same three general steps as in the case without debt. However, the details differ.

**Step 1.** Starting with a guess of wage  $w$ , solve the firm's dynamic programming problem by value function iteration on a grid. We use a coarse grid with  $n_k = 25$  points for capital, and  $n_b = 15$  points for debt. The choice of capital tomorrow and debt tomorrow has to lie in a different (thinner) grid, with  $n'_k = 180$  and  $n'_b = 100$  points. To find the value outside the grid points, we use spline interpolation. We keep the same grid for  $z$  as in the case without debt.

**Step 2.** After obtaining the value function in step 1, we solve for the optimal decision rules on the thin grid by solving the dynamic programming problem once. Call these policy functions  $k' = g(k, b, z)$  and  $b' = h(k, b, z)$ . Next, we solve for the stationary distribution of firms  $\mu^*(k, b, z; w)$  by simply iterating on the following equation:

$$\mu_{t+1}(A \times B \times C) = \int 1_{g(k,b,z) \in A} 1_{h(k,b,z) \in B} Q(z, C) \mu_t(dk, db, dz),$$

starting from a uniform distribution over  $(k, b, z)$ . This equation is similar to equation (12) in the main text but is adapted to also allow for a debt choice.

**Step 3.** As in the case without debt, we obtain the aggregate labor demand  $L^d(w) = \sum_{k,b,z} \mu^*(k, b, z; w) l(k, b, z; w)$ , and then check whether the labor market clears, i.e. whether the equation  $-U_2(C, L^d(w))/U_1(C, L^d(w)) = (1 - \tau^i)w$  holds, where aggregate consumption  $C$  is deduced from the resource constraint and the stationary distribution. If the equilibrium condition is not satisfied, we use the bisection method to update the wage rate and go back to step 1.

## 2.2 Transitional Dynamics

Assume that the economy starts in the steady state associated with the tax rates  $(\tau_0^g, \tau_0^d)$ . Assume that for  $t \geq T$ , the economy reaches a new steady state with constant tax rates  $(\tau_T^g, \tau_T^d)$ . We can then solve the transitional dynamics implied by a sequence of tax rates  $\{\tau_t^d, \tau_t^g\}_{t=0}^T$ , from periods 0 to  $T$ , as follows.

**Steps 1-2.** As in the model without debt, we compute the initial and final steady-states and guess a path for the interest rate  $\{r_t\}_{t=1}^T$  and a path for the wage rate  $\{w_t\}_{t=1}^T$ .

**Step 3.** Given  $\{w_t, r_t\}$ , solve the firm's dynamic programming problem by finite backward induction, assuming that  $V_T(k, b, z)$  is the new steady-state value function  $V^*(k, b, z)$ . As we do for the steady-state, we use a coarse grid for  $k, b$  and a thin grid for future choices  $k', b'$ . We obtain the policy functions for each date by linear interpolation, so that we generate the policy functions  $k_{t+1} = g_t(k, b, z)$  and  $b_{t+1} = h_t(k, b, z)$ .

**Step 4.** Given these policy functions, we compute the evolution of the cross-sectional distribution for any time  $t$ . However, since the policy functions are interpolated, they may not fall in the (thin) grid. We proceed as follows: for any  $t, k, b, z$ , we find the unique index  $i$  such that  $k_i < g_t(k, b, z) < k_{i+1}$ , where  $\{k_i\}$  is the thin grid, and we then assume that  $g_t(k, b, z) = k_i$  with probability  $\frac{k_{i+1} - g_t(k, b, z)}{k_{i+1} - k_i}$ , and  $g_t(k, b, z) = k_{i+1}$  with probability  $\frac{g_t(k, b, z) - k_i}{k_{i+1} - k_i}$ . (This method is suggested by Rios-Rull (2000). Alternatively, we can use simulations to find the cross-sectional distribution. In practice, our method seems to work better for our problem.) We can thus find the distribution  $\mu_t(k, b, z)$ , for any  $t$ , given  $\mu_0$ , on a discrete support  $n'_k \times n'_b \times n_z$ . Then, we deduce the aggregates  $Y_t, N_t, C_t$ , for  $t = 1, \dots, T - 1$ , using aggregation and the resource constraints.

**Step 5:** As in the model without debt, we update the interest rate and wage paths. We set  $\rho = 0.94$ .

## 3 An Additional Figure

Figure 5 presents the simulated series of aggregate investment, output, consumption, employment and dividends analyzed in Section 3 of our original paper. This figure complements Figure 7 in the original paper.

**Table 1.** Steady-state results for the baseline and extended models

	Baseline model		Extended model	
	pre-tax cut	post-tax cut	pre-tax cut	post-tax cut
Investment	0.085	0.088 (4.05)	0.092	0.095 (3.12)
Capital	0.892	0.929 (4.05)	0.971	1.001 (3.12)
Consumption	0.467	0.472 (1.11)	0.485	0.489 (0.86)
Labor	0.298	0.300 (0.37)	0.301	0.302 (0.36)
Output	0.564	0.574 (1.86)	0.594	0.603 (1.50)
Dividends	0.053	0.061 (15.23)	0.047	0.052 (10.80)
Equity issuance	0.012	0.019 (55.01)	0.010	0.018 (72.45)

Notes: This table presents the pre-tax-cut and post-tax-cut steady states for both the baseline and extended models. The numbers in the brackets give the percentage changes after the tax cuts. Dividends and capital gains taxes are reduced from 25 percent and 20 percent, respectively, to the same 15 percent. The tax cuts are unexpected and permanent.

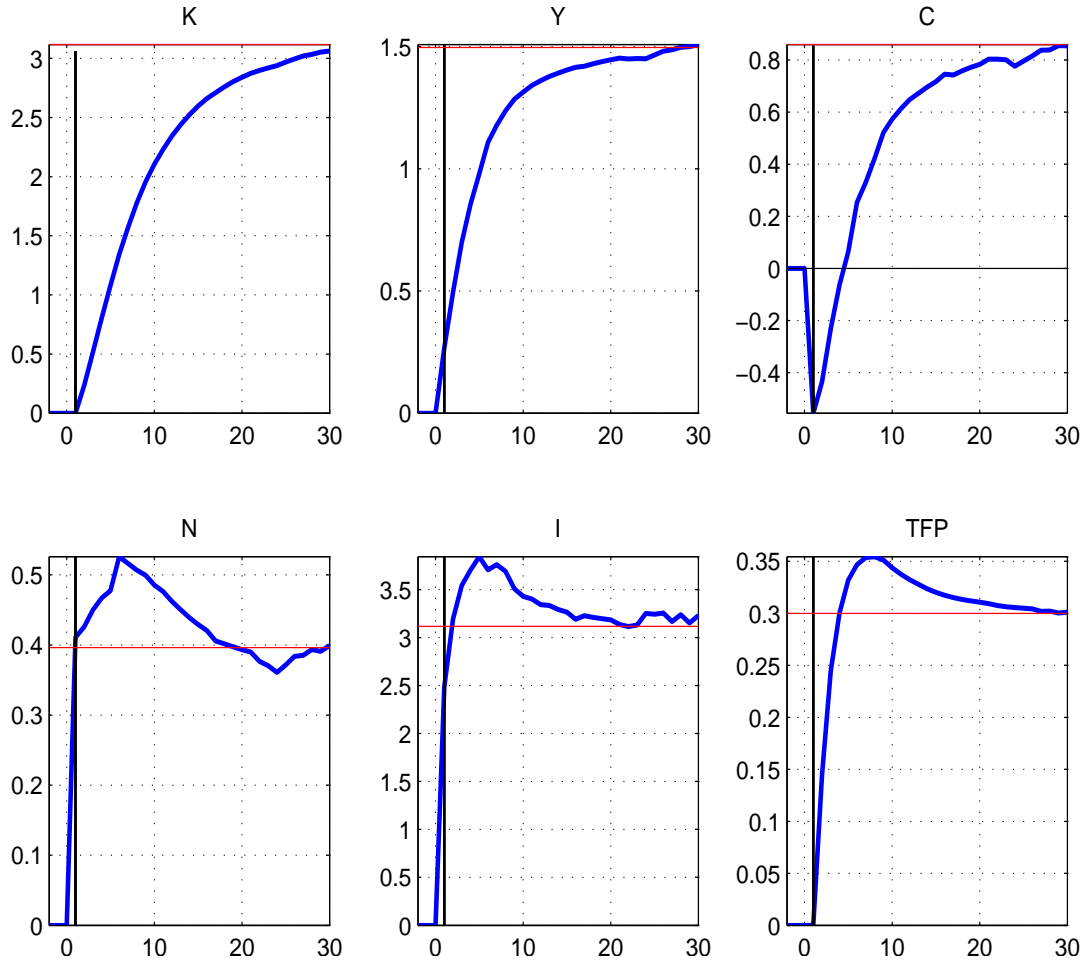


Figure 1: **Impact of unexpected permanent dividend and capital gains tax cuts in the extended model with debt.** The economy before period 1 is at the initial steady state with parameter values given in Table 1. The figure plots the responses of capital ( $K$ ), output ( $Y$ ), consumption ( $C$ ), labor ( $N$ ), investment ( $I$ ), and TFP to the unexpected permanent cuts of the dividend tax rate from 0.25 to 0.15 and of the capital gains tax rate from 0.20 to 0.15. In each panel, the horizontal axis measures time period, and the vertical axis measures percentage deviation from the initial steady state before the tax cuts.

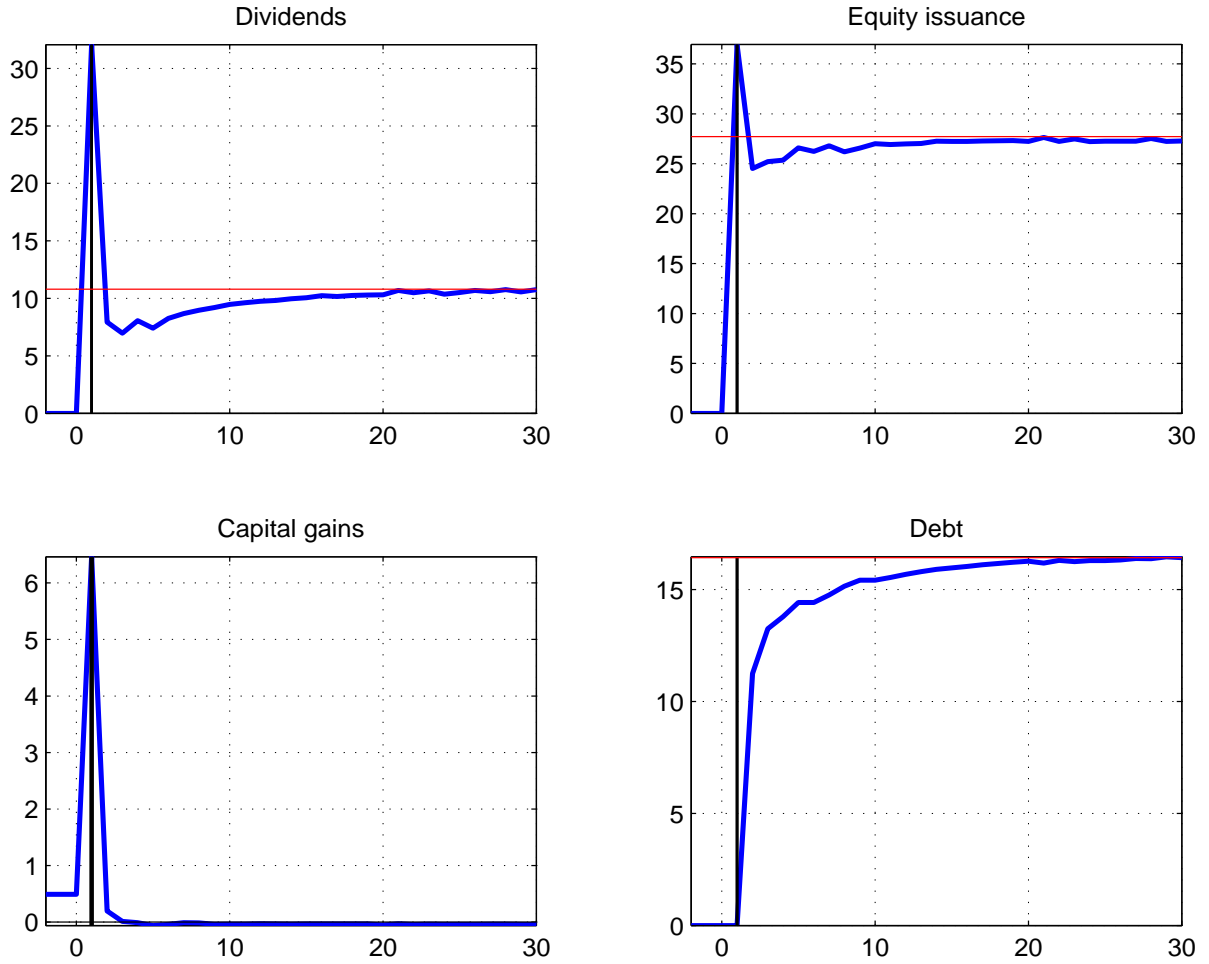


Figure 2: **Impact of unexpected permanent dividend and capital gains tax cuts in the extended model with debt.** The economy before period 1 is at the initial steady state with parameter values given in Table 1. The figure plots the responses of dividends, equity issuance, the ratio of capital gains to equity value, and finance regimes to the unexpected permanent cuts of the dividend tax rate from 0.25 to 0.15 and of the capital gains tax rate from 0.20 to 0.15. In each panel, the horizontal axis measures the time period. In the top two panels, the vertical axes measure the percentage deviation from the initial steady state before the tax cuts. The bottom left panel, the vertical axis measures the percentage of the rate of capital gains. In the bottom right panel, the vertical axis measures the share of firms in each finance regime.



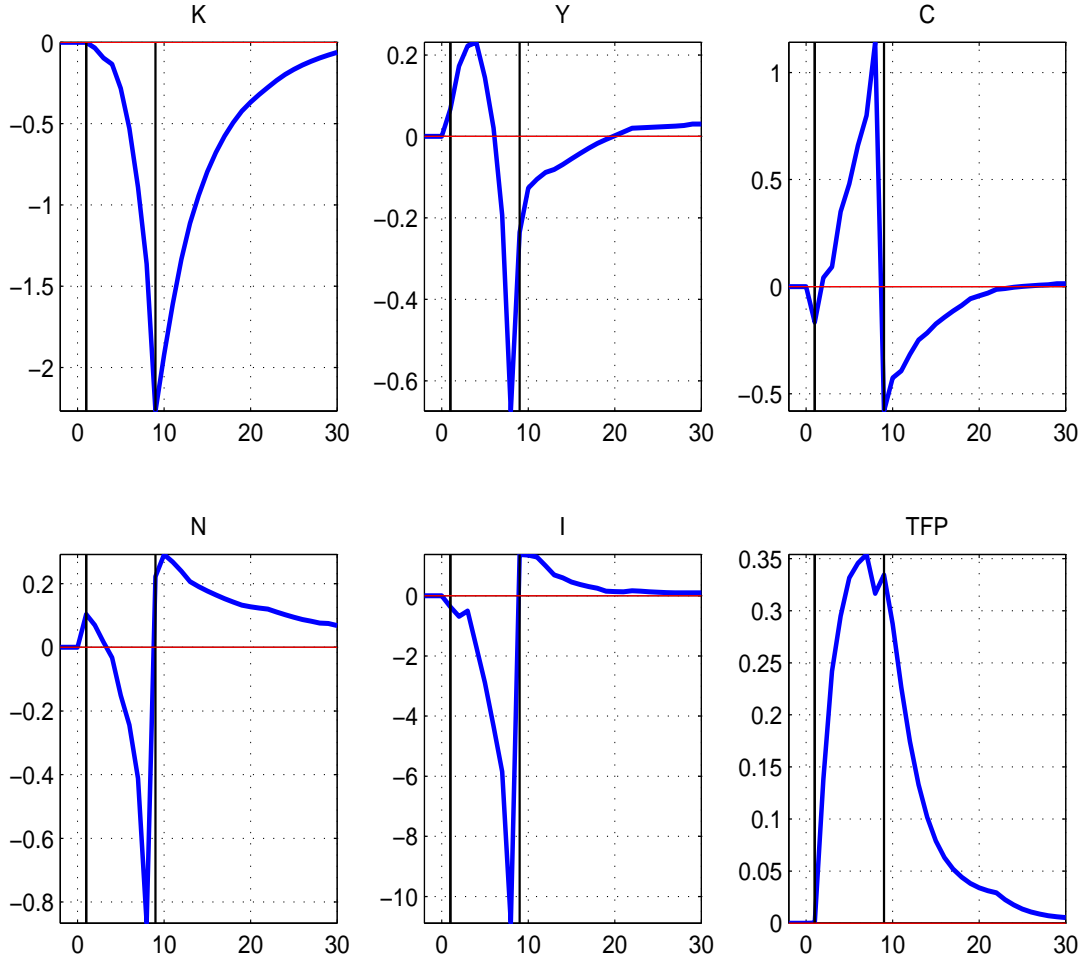


Figure 3: **Impact of unexpected temporary dividend and capital gains tax cuts in the extended model with debt.** The economy before period 1 is at the initial steady state with parameter values given in Table 1. The figure plots the responses of capital ( $K$ ), output ( $Y$ ), consumption ( $C$ ), labor ( $N$ ), investment ( $I$ ), and TFP to the unexpected temporary cuts of the dividend tax rate from 0.25 to 0.15 and of the capital gains tax rate from 0.20 to 0.15. The tax cuts last from periods 1-8. In each panel, the horizontal axis measures time period, and the vertical axis measures percentage deviation from the initial steady state before the tax cuts.

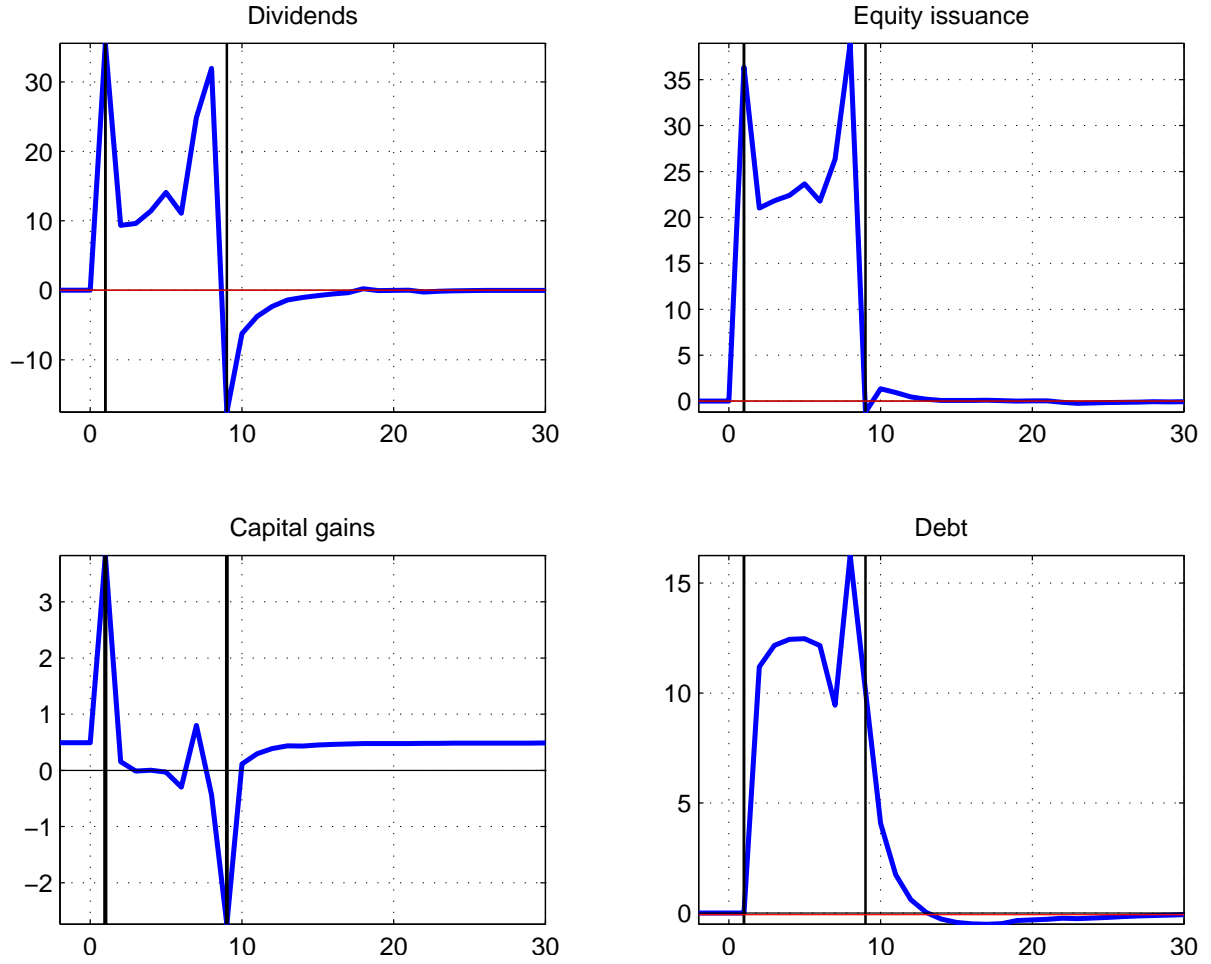


Figure 4: **Impact of unexpected temporary dividend and capital gains tax cuts in the extended model with debt.** The economy before period 1 is at the initial steady state with parameter values given in Table 1. The figure plots the responses of dividends, equity issuance, the ratio of capital gains to equity value, and finance regimes to the unexpected temporary cuts of the dividend tax rate from 0.25 to 0.15 and of the capital gains tax rate from 0.20 to 0.15. The tax cuts last from periods 1-8. In each panel, the horizontal axis measures the time period. In the top two panels, the vertical axes measure the percentage deviation from the initial steady state before the tax cuts. The bottom left panel, the vertical axis measures the percentage of the rate of capital gains. In the bottom right panel, the vertical axis measures the share of firms in each finance regime.

