Internet Appendix for:
Waves in Ship Prices and Investment

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A: Model Solution and Omitted Proofs

We begin by deriving equation (14) in the main text. Each firm chooses current net investment to maximize the expected net present value of earnings. Each firm's Bellman equation is

$$\begin{aligned} J\left(q_t, A_t, Q_t\right) &= \max_{i_t}\left\{V(q_t, i_t, A_t, Q_t) + \frac{E_f\left[J\left(q_t + i_t, A_{t+1}, Q_{t+1}\right) \mid A_t, Q_t\right]}{1+r}\right\} \\ &= \sum_{j=0}^{\infty} \frac{E_f\left[V(q_{t+j}, i_{t+j}^*, A_{t+j}, Q_{t+j}) \mid A_t, Q_t\right]}{(1+r)^{j-1}}, \end{aligned} \tag{A1}$$

where $r > 0$ is the constant discount rate or required return used by firms. The first order condition for firm net investment is

$$0 = -P_r - ki_t^* + \frac{1}{1+r} E_f\left[\frac{\partial J\left(q_t + i_t^*, A_{t+1}, Q_{t+1}\right)}{\partial q_{t+1}} \mid A_t, Q_t\right] \tag{A2}$$

The Envelope Theorem implies that[1]

$$\frac{\partial J\left(q_t, A_t, Q_t\right)}{\partial q_t} = \Pi_t + \frac{1}{1+r} E_f\left[\frac{\partial J\left(q_t + i_t^*, A_{t+1}, Q_{t+1}\right)}{\partial q_{t+1}} \mid A_t, Q_t\right]. \tag{A3}$$

Assuming the standard "no bubbles" condition holds,[2] we can iterate (A3) forward to obtain

[1] To see this, suppose that i_t^* is the optimal policy action so that

$$J\left(q_t, A_t, Q_t\right) = V(q_t, i_t^* A_t, Q_t) + (1+r)^{-1} E_f\left[J\left(q_t + i_t^*, A_{t+1}, Q_{t+1}\right) \mid A_t, Q_t\right],$$

We then have

$$\partial J\left(q_t, A_t, Q_t\right)/\partial q_t = \Pi_t + (1+r)^{-1} E_f\left[\partial J\left(q_t + i_t^*, A_{t+1}, Q_{t+1}\right)/\partial q_{t+1} \mid A_t, Q_t\right]$$

$$+ \overbrace{\left(-P_r - ki_t^* + (1+r)^{-1} E_f\left[\partial J\left(q_t + i_t^*, A_{t+1}, Q_{t+1}\right)/\partial q_{t+1} \mid A_t, Q_t\right]\right)}^{=0 \text{ by first order condition}} (\partial i_t^* / \partial q_t).$$

[2] The "no bubbles" or "transversality" condition is $\lim_{j\to\infty}\left(\frac{1}{1+r}\right)^j E_f\left[\Pi_{t+j} | A_t, Q_t\right] = 0$.

$$\frac{1}{1+r}E_f\left[\frac{\partial J\left(q_t+i_t^*,A_{t+1},Q_{t+1}\right)}{\partial q_{t+1}}\mid A_t,Q_t\right]=\sum_{j=1}^{\infty}\frac{E_f\left[\Pi_{t+j}\mid A_t,Q_t\right]}{(1+r)^j}. \tag{A4}$$

This shows that firm net investment is given by the familiar *q*-theory type investment equation

$$i_t^*=\frac{P(A_t,Q_t)-P_r}{k}, \tag{A5}$$

where P_r is the replacement cost of a ship and

$$\begin{aligned} P(A_t,Q_t) &= \frac{E_f\left[\Pi_{t+1}+P(A_{t+1},Q_{t+1})\mid A_t,Q_t\right]}{1+r} \\ &= \sum_{j=1}^{\infty}\frac{E_f\left[\Pi_{t+j}\mid A_t,Q_t\right]}{(1+r)^j} \\ &= \sum_{j=1}^{\infty}\frac{E_f\left[A_{t+j}-BQ_{t+j}-C-\delta P_r\mid A_t,Q_t\right]}{(1+r)^j}, \end{aligned} \tag{A6}$$

is the market price of a ship.

The Bellman operator for this problem satisfies Blackwell's Sufficient Conditions and is a Contraction Mapping. Therefore, the Contraction Mapping Theorem implies that there is a unique solution to the Bellman Equation. Thus, if we can guess and verify a solution to the Bellman equation, then this must be the unique solution. Specifically, using equations (A5) and (A6) it is easy to check that the following function solves the Bellman equation in (A1):

$$\begin{aligned} J\left(q_t,A_t,Q_t\right) &= q_t\left(A_t-BQ_t-C-\delta P_r+P(A_t,Q_t)\right) \\ &\quad +\frac{1}{2k}\left(P(A_t,Q_t)-P_r\right)^2+\frac{1}{2k}\sum_{j=1}^{\infty}E_t\left[\left(P(A_{t+j},Q_{t+j})-P_r\right)^2\mid A_t,Q_t\right]. \end{aligned} \tag{A7}$$

Proposition 1 (Equilibrium investment and prices): *There exists a unique equilibrium such that the net investment of the representative firm is* $i_t^*=x_i^*+y_i^*A_t+z_i^*Q_t$ *and equilibrium ship prices are* $P_t^*=P_r+kx_i^*+ky_i^*A_t+kz_i^*Q_t$. *The two slope coefficients (i.e.,* y_i^* *and* z_i^**) are a function of five exogenous parameters: k, r,* ρ_f*,* θ*, and B. In addition to these five parameters, the intercept term (i.e.,* x_i^**) also depends on* $\bar{A}$*, C,* δ*, and* P_r*.*

Investment and ship prices are decreasing in the current fleet size ($z_i^*<0$*). Furthermore, (i) investment and prices react more aggressively to the current fleet size when firms underestimate the* *competition (i.e., $\partial z_i^* / \partial\theta > 0$); (ii) firms' response to the current fleet size independent of the perceived persistence of demand (i.e., $\partial z_i^* / \partial\rho_f = 0$); (iii) investment and prices react more aggressively to the current fleet size when the demand curve is more inelastic (i.e., $\partial z_i^* / \partial B < 0$); (iv) investment and prices react less aggressively to current fleet size when required returns are higher (i.e., $\partial z_i^* / \partial r > 0$); and (v) when adjustment costs are higher, investment reacts less aggressively to current fleet size but prices react more aggressively (i.e., $\partial z_i^* / \partial k > 0$ and $\partial(z_i^* k) / \partial k = z_i^* + k \cdot (\partial z_i^* / \partial k) < 0$).*

Investment and ship prices are increasing in current demand ($y_i^* > 0$). Furthermore, (i) investment and prices react more aggressively to current demand when firms underestimate the competition (i.e., $\partial y_i^* / \partial\theta < 0$); (ii) investment and prices reacts more aggressively to current demand when demand is more perceived to be persistent (i.e., $\partial y_i^* / \partial\rho_f > 0$); (iii) investment and prices react less aggressively to current demand when demand curve is more inelastic (i.e., $\partial y_i^* / \partial B < 0$); (iv) investment and prices react less aggressively to current demand when required returns are higher (i.e., $\partial y_i^* / \partial r < 0$); and (v) when adjustment costs are higher, investment reacts less aggressively to current demand but prices react more aggressively ($\partial y_i^* / \partial k < 0$ and $\partial(y_i^* k) / \partial k = y_i^* + k \cdot (\partial y_i^* / \partial k) > 0$).

Proof: We first solve for the equilibrium coefficients. We then prove the comparative statics discussed above and then characterize the system dynamics.

Solve for the equilibrium coefficients: We conjecture that

$$\begin{aligned} i_t &= x_i + y_i A_t + z_i Q_t \\ P_t(A_t, Q_t) &= P_r + kx_i + ky_i A_t + kz_i Q_t \end{aligned} \tag{A8}$$

Using (10), (12), (15), and (20), the equilibrium must satisfy

$$
\begin{aligned}
&P_r + kx_i + ky_iA_t + kz_iQ_t \\
&= \frac{E_f\left[\left(A_{t+1} - BQ_{t+1} - C - \delta P_r\right) + \left(P_r + kx_i + ky_iA_{t+1} + kz_iQ_{t+1}\right) \mid A_t, Q_t\right]}{1+r} \\
&= \frac{\begin{array}{l}\left((1-\rho)\bar{A} + \rho A_t - B\left(Q_t + \theta(x_i + y_iA_t + z_iQ_t)\right) - C - \delta P_r\right) \\ +\left(P_r + kx_i + ky_i((1-\rho)\bar{A} + \rho A_t) + kz_i\left(Q_t + \theta(x_i + y_iA_t + z_iQ_t)\right)\right)\end{array}}{1+r}
\end{aligned}
\tag{A9}
$$

Both the left-hand and right-hand size are linear in A_t, and Q_t. Thus, by matching coefficients in (A8), we can then solve for the fixed-point values of x_i, y_i, and z_i.

Specifically, matching coefficients on Q_t, shows that the equilibrium value of z_i satisfies $0 = f(z_i^*)$ where

$$
f(z) = -k\theta \cdot z^2 + (kr + B\theta) \cdot z + B. \tag{A10}
$$

We want the negative root of the quadratic in (A10) which is

$$
z_i^* = \frac{kr + B\theta}{2k\theta} - \sqrt{\left(\frac{kr + B\theta}{2k\theta}\right)^2 + \frac{B}{k\theta}}, \tag{A11}
$$

when $\theta > 0$ and $z_i^* = -B/(kr)$ when $\theta = 0$. Given this solution for z_i^*, matching coefficients on A_t and the constant shows that the equilibrium values of y_i and x_i are given by

$$
y_i^* = \frac{\rho_f}{k(1-\rho_f) + kr + \theta(B - kz_i^*)} = \frac{\rho_f}{k(1-\rho_f) - B/z_i^*} > 0, \tag{A12}
$$

and

$$
x_i^* = \frac{\bar{A}(1-\rho_f)(1+ky_i^*) - C - (r+\delta)P_r}{kr + \theta(B - kz_i^*)} = \frac{\bar{A}(1-\rho_f)(1+ky_i^*) - C - (r+\delta)P_r}{-B/z_i^*}. \tag{A13}
$$

Uniqueness of the equilibrium: We now show that the linear equilibrium described in *Proposition 1* is the unique stationary equilibrium of the model. To show this is the case, we conjecture an alternate stationary equilibrium of the form

$$
\begin{aligned}
i_t &= x_i + y_iA_t + z_iQ_t + g(A_t, Q_t) \\
P_t(A_t, Q_t) &= P_r + kx_i + ky_iA_t + kz_iQ_t + kg(A_t, Q_t),
\end{aligned}
\tag{A14}
$$

where $g(A_t, Q_t)$ is an arbitrary function of the state variables. Proceeding as above, we again obtain conditions (A11), (A12), and (A13) as well as a functional equation characterizing $g(A_t, Q_t)$:

$$g(A_t, Q_t) = \frac{1}{1 - B/(kz_i^*)} E_t\left[g(A_{t+1}, Q_{t+1}) \mid A_t, Q_t\right]. \tag{A15}$$

Iterating on (A15) and making use of the law of iterated expectations, we have

$$g(A_t, Q_t) = \lim_{k\to\infty}\left(\frac{1}{1 - B/(kz_i^*)}\right)^k \cdot \lim_{k\to\infty} E_t\left[g(A_{t+k}, Q_{t+k}) \mid A_t, Q_t\right]. \tag{A16}$$

Since the conjectured equilibrium is stationary, we have $\lim_{k\to\infty} E_t\left[g(A_{t+k}, Q_{t+k}) \mid A_t, Q_t\right] = c$ for some constant *c* irrespective of the initial values of A_t and Q_t. Thus, since $1 - B/(kz_i^*) > 1$ equation (A16) shows that we must have $g(A_t, Q_t) = 0$ in any stationary equilibrium.

Comparative statics for z_i^****:*** Before proceeding we first show that

$$\theta z_i^* + 1 > 0. \tag{A17}$$

Using equation (A11), one can show that equation (A17) is equivalent to

$$\frac{k(1+r) + B\theta}{2k\theta} + \frac{1}{2\theta} > \sqrt{\left(\frac{k(1+r) + B\theta}{2k\theta} - \frac{1}{2\theta}\right)^2 + \frac{B}{k\theta}}.$$

This holds since $4XY > Z \Leftrightarrow X + Y > \sqrt{(X-Y)^2 + Z}$ and using $X = \left(k(1+r) + B\theta\right)/\left(2k\theta\right)$, $Y = 1/(2\theta)$, and $Z = B/(k\theta)$, we have

$$\frac{1+r}{\theta^2} + \frac{B}{k\theta} = 4XY > Z = \frac{B}{k\theta}.$$

With (A17) in hand, we now proceed to the comparative statics for z_i^*. Since $0 = a \cdot z_i^{*2} + b \cdot z_i^* + c$ where $a = -k\theta < 0$, $b = kr + B\theta > 0$, and $c = B$. Thus, by the Implicit Function theorem, we have

$$\frac{\partial z_i^*}{\partial \phi} = -\frac{z_i^{*2} \cdot \frac{\partial a}{\partial \phi} + z_i^* \cdot \frac{\partial b}{\partial \phi} + \frac{\partial c}{\partial \phi}}{2az_i^* + b} \propto -\left(z_i^{*2} \cdot \frac{\partial a}{\partial \phi} + z_i^* \cdot \frac{\partial b}{\partial \phi} + \frac{\partial c}{\partial \phi}\right), \tag{A18}$$

for any primitive model parameter. Thus, we have:

- $\partial z_i^* / \partial \theta \propto -\left(-z_i^{*2} \cdot k + z_i^* \cdot B\right) > 0$;
- $\partial z_i^* / \partial \rho_f = 0$;
- $\partial z_i^* / \partial B = -\left(\theta z_i^* + 1\right) < 0$;

- $\partial z_i^* / \partial r \propto -kz_i^* > 0$;
- $\partial z_i^* / \partial k = -\left(-z_i^{*2} \cdot \theta + z_i^* \cdot r\right) > 0$;
- $\partial (kz_i^*) / \partial k = z_i^* + k \cdot (\partial z_i^* / \partial k) = \dfrac{\theta z_i^* (B - kz_i^*)}{-2k\theta z_i^* + kr + B\theta} < 0$;

Comparative statics for y_i^****:*** Recall that $y_i^* = \dfrac{\rho_f}{k(1-\rho_f) + kr + \theta(B - kz_i^*)} = \dfrac{\rho_f}{k(1-\rho_f) - B / z_i^*}$.

Thus, we have

- $\partial y_i^* / \partial \theta < 0$: Obvious since $\partial z_i^* / \partial \theta > 0$;
- $\partial y_i^* / \partial \rho_f > 0$;
- $\partial y_i^* / \partial B < 0$: Obvious since $\partial z_i^* / \partial B < 0$;
- $\partial y_i^* / \partial r < 0$: Obvious since $\partial z_i^* / \partial r > 0$;
- $\partial y_i^* / \partial k < 0$: Obvious since $\partial (kz_i^*) / \partial k < 0$;
- $\partial (ky_i^*) / \partial k = y_i^* + k \cdot (\partial y_i^* / \partial k) > 0$: Note that $ky_i^* = \dfrac{\rho_f}{(1-\rho_f) - B / (kz_i^*)}$. Since $\partial (kz_i^*) / \partial k < 0$

  , the denominator is decreasing in *k* which shows that $\partial (ky_i^*) / \partial k > 0$. ■

System dynamics and stability: The true system dynamics perceived by the econometrician can be summarized using a vector auto-regression:

$$\begin{bmatrix} A_{t+1} \\ Q_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\rho_0)\bar{A} \\ x_i^* \end{bmatrix} + \begin{bmatrix} \rho_0 & 0 \\ y_i^* & 1+z_i^* \end{bmatrix} \begin{bmatrix} A_t \\ Q_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{A19}$$

Therefore, the true steady state is given by

$$\begin{bmatrix} A^* \\ Q^* \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \rho_0 & 0 \\ y_i^* & 1+z_i^* \end{bmatrix} \right)^{-1} \begin{bmatrix} (1-\rho_0)\bar{A} \\ x_i^* \end{bmatrix} = \begin{bmatrix} \bar{A} \\ \dfrac{x_i^* + y_i^* \bar{A}}{-z_i^*} \end{bmatrix} = \begin{bmatrix} \bar{A} \\ \dfrac{\bar{A} - C - (r+\delta)P_r}{B} \end{bmatrix}, \tag{A20}$$

assuming that $\rho_0 \neq 1$ and $z_i^* \neq -1$ so the matrix is invertible.

The system dynamics are governed by sign and magnitude of the second eigenvalue of the 2×2 matrix in (A19). Specifically, the two eigenvalues are $\lambda_1 = \rho_0$ and $\lambda_2 = 1 + z_i^*$. Specifically, the dynamics are oscillatory if $\lambda_2 < 0$ and non-oscillatory if $\lambda_2 > 0$; and the system has convergent dynamics which return it to the steady state if $|\lambda_2| < 1$ and divergent dynamics if $|\lambda_2| > 1$.

Above we showed that $0 < 1 + \theta z_i^* < 1$. Thus, in the rational expectation case ($\theta = 1$) we always have non-oscillatory and convergent dynamics. When $0 \le \theta < 1$, we can have either non-oscillatory dynamics about the steady-state if $1 + z_i^* > 0$ or oscillatory dynamics if $1 + z_i^* < 0$. When $0 \le \theta < 1$, we can have $1 + z_i^* < -1$, corresponding to divergent, oscillatory dynamics if (i) θ is sufficient close to 0 and (ii) *B* is sufficiently large or *k* is sufficiently small. Obviously, in our model simulations and estimation we focus on the empirically relevant case with convergent dynamics.

Steady-state distribution induced by model: We can rewrite equation (A19) as

$$\begin{bmatrix} A_{t+1} - \bar{A} \\ Q_{t+1} - Q^* \end{bmatrix} = \begin{bmatrix} \rho_0 & 0 \\ y_i^* & 1 + z_i^* \end{bmatrix} \begin{bmatrix} A_t - \bar{A} \\ Q_t - Q^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{A21}$$

Taking variances of both sides of (A21), the variance of the system about the steady-state satisfies

$$\begin{bmatrix} \sigma_{A,0}^2 & \sigma_{AQ,0} \\ \sigma_{AQ,0} & \sigma_{Q,0}^2 \end{bmatrix} = \begin{bmatrix} \rho_0 & 0 \\ y_i^* & 1 + z_i^* \end{bmatrix} \begin{bmatrix} \sigma_{A,0}^2 & \sigma_{AQ,0} \\ \sigma_{AQ,0} & \sigma_{Q,0}^2 \end{bmatrix} \begin{bmatrix} \rho_0 & y_i^* \\ 0 & 1 + z_i^* \end{bmatrix} + \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & 0 \end{bmatrix}. \tag{A22}$$

Solving (A24) for the 3-unknown parameters, we obtain

$$\sigma_{A,0}^2 = \rho_0^2 \sigma_{A,0}^2 + \sigma_\varepsilon^2 \Rightarrow \sigma_{A,0}^2 = \frac{\sigma_\varepsilon^2}{1 - \rho_0^2} > 0, \tag{A23}$$

$$\sigma_{AQ,0} = \rho_0 (y_i^* \sigma_{A,0}^2 + (1 + z_i^*) \sigma_{AQ,0}) \Rightarrow \sigma_{AQ,0} = \frac{\rho_0 y_i^* \sigma_{A,0}^2}{1 - \rho_0 (1 + z_i^*)} > 0, \tag{A24}$$

and

$$\begin{aligned} \sigma_{Q,0}^2 &= (y_i^*)^2 \sigma_{A,0}^2 + (1 + z_i^*)^2 \sigma_{Q,0}^2 + 2 y_i^* (1 + z_i^*) \sigma_{AQ,0} \\ &\Rightarrow \sigma_{Q,0}^2 = \frac{(y_i^*)^2 \sigma_{A,0}^2}{1 - (1 + z_i^*)^2} \frac{1 + \rho_0 (1 + z_i^*)}{1 - \rho_0 (1 + z_i^*)} > 0, \end{aligned} \tag{A25}$$

since $(1 + z_i^*) > -1$ in any stationary distribution induced by the model.

Straightforward algebra shows that the variance of earnings is

$$\begin{aligned} \sigma_{\Pi,0}^2 &= \sigma_{A,0}^2 + B^2 \sigma_{Q,0}^2 - 2B \sigma_{AQ,0} \\ &= \frac{\sigma_{A,0}^2}{1 - \rho_0 (1 + z_i^*)} \left((1 - \rho_0 (1 + z_i^* + 2B y_i^*)) + B^2 (y_i^*)^2 \frac{1 + \rho_0 (1 + z_i^*)}{1 - (1 + z_i^*)^2} \right), \end{aligned} \tag{A26}$$

and the variance of prices is

$$\sigma_{P,0}^2 = (ky_i^*)^2\sigma_{A,0}^2 + (kz_i^*)^2\sigma_{Q,0}^2 + 2(ky_i^*)(kz_i^*)\sigma_{AQ,0} = 2\frac{(ky_i^*)^2\sigma_{A,0}^2}{1+(1+z_i^*)}\frac{1-\rho_0}{1-\rho_0(1+z_i^*)} \tag{A27}$$

We can also use (A21) to characterize the path and auto-covariance of earnings in the model. Specifically, we have

$$\begin{aligned}
E_0[\Pi_{t+j} \mid A_t, Q_t] &= \Pi^* + \begin{bmatrix}1 & -B\end{bmatrix}\begin{bmatrix}\rho_0 & 0 \\ y_i^* & 1+z_i^*\end{bmatrix}^j \begin{bmatrix}A_t - \bar{A} \\ Q_t - Q^*\end{bmatrix} \\
&= \Pi^* + \rho_0^j(A_t - \bar{A}) - By_i^*\left(\sum_{l=0}^{j-1}\rho_0^{j-1-l}(1+z_i^*)^l\right)(A_t - \bar{A}) - B(1+z_i^*)^j(Q_t - Q^*) \quad \text{(A28)} \\
&= \Pi^* + \left(\rho_0^j - By_i^*\frac{\rho_0^j - (1+z_i^*)^j}{\rho_0 - (1+z_i^*)}\right)(A_t - \bar{A}) - B(1+z_i^*)^j(Q_t - Q^*).
\end{aligned}$$

Thus, the auto-covariance of ship earnings is

$$\begin{aligned}
Cov_0[\Pi_{t+j}, \Pi_t] &= Cov_0\left[\left(\rho_0^j - By_i^*\frac{\rho_0^j - (1+z_i^*)^j}{\rho_0 - (1+z_i^*)}\right)(A_t - \bar{A}) - B(1+z_i^*)^j(Q_t - Q^*), (A_t - \bar{A}) - B(Q_t - Q^*)\right] \\
&= \left(\rho_0^j - By_i^*\frac{\rho_0^j - (1+z_i^*)^j}{\rho_0 - (1+z_i^*)}\right)\left(\sigma_{A,0}^2 - B\sigma_{AQ,0}\right) - B(1+z_i^*)^j\left(\sigma_{AQ,0} - B\sigma_{Q,0}^2\right) \\
&= \left(\rho_0^j - By_i^*\frac{\rho_0^j - (1+z_i^*)^j}{\rho_0 - (1+z_i^*)}\right)\left(\sigma_{A\Pi,0}\right) - B(1+z_i^*)^j\left(\sigma_{Q\Pi,0}\right) \quad \text{(A29)}
\end{aligned}$$

And the auto-correlations of earnings are given by

$$Corr_0[\Pi_{t+j}, \Pi_t] = \left(\rho_0^j - By_i^*\frac{\rho_0^j - (1+z_i^*)^j}{\rho_0 - (1+z_i^*)}\right)\left(\frac{\sigma_{A\Pi,0}}{\sigma_{\Pi,0}^2}\right) - B(1+z_i^*)^j\left(\frac{\sigma_{Q\Pi,0}}{\sigma_{\Pi,0}^2}\right).$$

System dynamics and steady-state distribution perceived by firms: The system dynamics perceived by firms are

$$\begin{bmatrix}A_{t+1} \\ Q_{t+1}\end{bmatrix} = \begin{bmatrix}(1-\rho_f)\bar{A} \\ \theta x_i^*\end{bmatrix} + \begin{bmatrix}\rho_f & 0 \\ \theta y_i^* & 1+\theta z_i^*\end{bmatrix}\begin{bmatrix}A_t \\ Q_t\end{bmatrix} + \begin{bmatrix}\varepsilon_{t+1} \\ 0\end{bmatrix}, \tag{A30}$$

so the perceived steady state is given by

$$\begin{bmatrix}A^* \\ Q^*\end{bmatrix} = \left(\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} - \begin{bmatrix}\rho_f & 0 \\ \theta y_i^* & 1+\theta z_i^*\end{bmatrix}\right)^{-1}\begin{bmatrix}(1-\rho_0)\bar{A} \\ \theta x_i^*\end{bmatrix} = \begin{bmatrix}\bar{A} \\ \dfrac{x_i^* + y_i^*\bar{A}}{-z_i^*}\end{bmatrix}. \tag{A31}$$

Thus, firms perceive the same steady-state as the econometrician. However, since $0 < 1+\theta z_i^* < 1$ firms expect the dynamics to be convergent and non-oscillatory.

We can rewrite equation (A30) as

$$\begin{bmatrix} A_{t+1}-\bar{A} \\ Q_{t+1}-Q^* \end{bmatrix} = \begin{bmatrix} \rho_f & 0 \\ \theta y_i^* & 1+\theta z_i^* \end{bmatrix} \begin{bmatrix} A_t-\bar{A} \\ Q_t-Q^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{A32}$$

Taking variances of (A32), the perceived variance of the system about the steady-state is

$$\begin{bmatrix} \sigma_{A,f}^2 & \sigma_{AQ,f} \\ \sigma_{AQ,f} & \sigma_{Q,f}^2 \end{bmatrix} = \begin{bmatrix} \rho_f & 0 \\ \theta y_i^* & 1+\theta z_i^* \end{bmatrix} \begin{bmatrix} \sigma_{A,f}^2 & \sigma_{AQ,f} \\ \sigma_{AQ,f} & \sigma_{Q,f}^2 \end{bmatrix} \begin{bmatrix} \rho_f & 0 \\ \theta y_i^* & 1+\theta z_i^* \end{bmatrix} + \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & 0 \end{bmatrix}. \tag{A33}$$

Thus, we obtain

$$\sigma_{A,f}^2 = \frac{\sigma_\varepsilon^2}{1-\rho_f^2} > 0, \tag{A34}$$

$$\sigma_{AQ,f} = \frac{\rho_f \theta y_i^* \sigma_{A,f}^2}{1-\rho_f(1+\theta z_i^*)} > 0, \tag{A35}$$

and

$$\sigma_{Q,f}^2 = \frac{(\theta y_i^*)^2 \sigma_{A,f}^2}{1-(1+\theta z_i^*)^2} \frac{1+\rho_f(1+\theta z_i^*)}{1-\rho_f(1+\theta z_i^*)} > 0. \tag{A36}$$

The perceived variance of earnings is

$$\begin{aligned} \sigma_{\Pi,f}^2 &= \sigma_{A,f}^2 + B^2\sigma_{Q,f}^2 - 2B\sigma_{AQ,f} \\ &= \frac{\sigma_{A,f}^2}{1-\rho_f(1+\theta z_i^*)}\left((1-\rho_f(1+\theta z_i^* + 2B\theta y_i^*)) + B^2(\theta y_i^*)^2 \frac{1+\rho_f(1+\theta z_i^*)}{1-(1+\theta z_i^*)^2} \right), \end{aligned} \tag{A37}$$

and the perceived variance of prices is

$$\sigma_{P,f}^2 = (k\theta y_i^*)^2\sigma_{A,f}^2 + (k\theta z_i^*)^2\sigma_{Q,f}^2 + 2(k\theta y_i^*)(k\theta z_i^*)\sigma_{AQ,f} = 2\frac{(k\theta y_i^*)^2\sigma_{A,f}^2}{1+(1+\theta z_i^*)} \frac{1-\rho_f}{1-\rho_f(1+\theta z_i^*)} \tag{A38}$$

We can use (A32) to characterize the path of earnings expected perceived by firms.

$$E_f[\Pi_{t+j} \mid A_t, Q_t] = \Pi^* + \left(\rho_f^j - B\theta y_i^* \frac{\rho_f^j - (1+\theta z_i^*)^j}{\rho_f - (1+\theta z_i^*)} \right)(A_t - \bar{A}) - B(1+\theta z_i^*)^j (Q_t - Q^*). \tag{A39}$$

Thus, the perceived auto-covariance of ship earnings is

$$\begin{aligned} Cov_f[\Pi_{t+j}, \Pi_t] &= \left(\rho_f^j - B\theta y_i^* \frac{\rho_f^j - (1+\theta z_i^*)^j}{\rho_f - (1+\theta z_i^*)} \right)\left(\sigma_{A,f}^2 - B\sigma_{AQ,f}\right) - B(1+\theta z_i^*)^j \left(\sigma_{AQ,f} - B\sigma_{Q,f}^2\right) \\ &= \left(\rho_f^j - B\theta y_i^* \frac{\rho_f^j - (1+\theta z_i^*)^j}{\rho_f - (1+\theta z_i^*)} \right)\left(\sigma_{A\Pi,f}\right) - B(1+z_i^*)^j \left(\sigma_{Q\Pi,f}\right). \end{aligned} \tag{A40}$$

And the auto-correlations of earnings perceived by firms are given by

$$Corr_f[\Pi_{t+j},\Pi_t]=\left(\rho_f^j - B\theta y_i^* \frac{\rho_f^j-(1+\theta z_i^*)^j}{\rho_f-(1+\theta z_i^*)}\right)\left(\frac{\sigma_{A\Pi,f}}{\sigma_{\Pi,f}^2}\right)-B(1+z_i^*)^j\left(\frac{\sigma_{Q\Pi,f}}{\sigma_{\Pi,f}^2}\right).$$

Special case where $\delta=0, \rho_f=1, \sigma_\varepsilon=0$ and $C=0$.***:*** We now focus on the special case discussed in the text. As above, z_i^* is given in equation (A11). In this case, it is also easy to see that $y_i^*=-z_i^*/B$ and $x_i^*=z_i^*(rp_r)/B$, which implies that aggregate investment is $I_t=-(z_i^*/B)(A_t-BQ_t-rP_r)=-(z_i^*/B)(\Pi_t-rP_r)$. Thus, steady-state earnings are $\Pi^*=rP_r$ and the initial steady-state fleet size is $Q^*(A_0)=(A_0-rP_r)/B$. Thus, following the shock at time 0, the equilibrium price at time $t>0$ $P_t=P_r-k(z_i^*/B)(\Pi_t-rP_r)$ and $\Pi_{t+1}=\Pi_t-BI_t=\Pi_t+z_i^*(\Pi_t-rP_r)$. Thus, the realized return from owning and operating a ship between time *t* and *t*+1 along the equilibrium path following the initial shock is

$$\begin{aligned}1+R_{t+1}&=\frac{\Pi_{t+1}+P_r-((kz_i^*)/B)(\Pi_{t+1}-rP_r)}{P_r-((kz_i^*)/B)(\Pi_t-rP_r)}\\&=\frac{(1+r)P_r+(1+z_i^*)((B-kz_i^*)/B)(\Pi_t-rP_r)}{P_r-((kz_i^*)/B)(\Pi_t-rP_r)}\\&=(1+r)-\frac{(1-\theta)(B-kz_i^*)(-z_i^*/B)(\Pi_t-rP_r)}{P_r+k(-z_i^*/B)(\Pi_t-rP_r)}\end{aligned} \qquad \text{(A30)}$$

It is easy to show that $1+R_{t+1}=1+r$ when $\theta=1$ and that for $\theta\in[0,1)$ $R_{t+1}<r$ when $\Pi_t>rP_r$ and $R_{t+1}>r$ when $\Pi_t<rP_r$. Thus, expected returns are below *r* when $\Pi_t>rP_r$—i.e., when prices and earnings are above their steady state and investment is positive.

***Invariance*:** Note that

$$z_i^*=\frac{1}{2}\frac{r}{\theta}+\frac{1}{2}\frac{B}{k}-\sqrt{\left(\frac{1}{2}\frac{r}{\theta}+\frac{1}{2}\frac{B}{k}\right)^2+\frac{1}{\theta}\frac{B}{k}},$$

only depends on *B*/*k*. Furthermore, kx_i^* and ky_i^* only depend on *B*/*k*. Thus, if we change *B* and *k* proportionately holding *B/k* constant, it is easy to see that this has a proportional effect on I_t and Q_t, but has no effect on I_t/Q_t, Π_t, P_t, or R_{t+1}, and thus no effect on any of the moment conditions used in our SMM estimation exercise.

It is also straightforward to see that a change in $\overline{A}$ or C has an additive effect on the fleet size Q_t, but has no effect on I_t, Π_t, P_t, or R_{t+1}. As a result, a change in these parameters has a small impact on I_t/Q_t.

***Equilibrium expected returns*:** Expected returns will generally not equal firms' required returns. Specifically, equation (A9) insures that

$$\begin{aligned} &E_f\left[1+R_{t+1} \mid A_t, Q_t\right] \\ &= \frac{E_f\left[\begin{array}{l}\left(A_{t+1}-BQ_{t+1}-C-\delta P_r\right) \\ +\left(P_r+k(x_i^*+y_i^*A_{t+1}+z_i^*Q_{t+1})\right) \mid A_t, Q_t\end{array}\right]}{P_r+kx_i^*+ky_i^*A_t+kz_i^*Q_t} \\ &= \frac{\left(\begin{array}{l}\left(P_r(1-\delta)+kx_i^*-C\right)+\left((1+ky_i^*)((1-\rho_f)\overline{A}+\rho_f A_t)\right) \\ -(B-kz_i^*)\left(Q_t+\theta(x_i^*+y_i^*A_t+z_i^*Q_t)\right)\end{array}\right)}{P_r+kx_i^*+ky_i^*A_t+kz_i^*Q_t} \\ &= 1+r, \end{aligned} \tag{A31}$$

by construction. In words, the representative firm expects that holding period returns will equal the required return on capital, r. However, the true expected return perceived by the econometrician—who does not suffer from competition neglect and does not overestimate the persistence of demand shocks—will generally differ from $1 + r$. Specifically, we have

$$\begin{aligned} &E_0\left[1+R_{t+1} \mid A_t, Q_t\right] \\ &= \frac{E_0\left[\begin{array}{l}\left(A_{t+1}-BQ_{t+1}-C-\delta P_r\right) \\ +\left(P_r+k(x_i^*+y_i^*A_{t+1}+z_i^*Q_{t+1})\right) \mid A_t, Q_t\end{array}\right]}{P_r+kx_i^*+ky_i^*A_t+kz_i^*Q_t} \\ &= \frac{\left(\begin{array}{l}\left(P_r(1-\delta)+kx_i^*-C\right)+\left((1+ky_i^*)((1-\rho_0)\overline{A}+\rho_0 A_t)\right) \\ -(B-kz_i^*)\left(Q_t+(x_i^*+y_i^*A_t+z_i^*Q_t)\right)\end{array}\right)}{P_r+kx_i^*+ky_i^*A_t+kz_i^*Q_t} \end{aligned} \tag{A32}$$

Subtracting (A31) from (A32) shows that

$$E_0\left[R_{t+1} \mid A_t, Q_t\right] = r - \frac{(1-\theta)(B-kz_i^*)(x_i^*+y_i^*A_t+z_i^*Q_t)+(\rho_f-\rho_0)(1+ky_i^*)(A_t-\overline{A})}{P_r+kx_i^*+ky_i^*A_t+kz_i^*Q_t} \tag{A33}$$

Equation (A33) gives the general expression for expected returns when $\theta \neq 1$ and $\rho_f \neq \rho_0$. Although (A33) shows that expected returns can be decomposed into a term that vanishes when there is full competition awareness ($\theta = 1$) and a term that vanishes when there is no demand over-extrapolation ($\rho_f = \rho_0$), these two biases do interact in our model. Specifically, since $\partial y_i^* / \partial\theta < 0$ and $\partial y_i^* / \partial\rho_f > 0$, demand over-extrapolation naturally amplifies the return predictability due to competition neglect and vice versa.

Since the latent demand process, A_t, is not readily observable, it is useful to recast equation (A33) in terms of observables, namely, industry net investment (I_t^N) and operating profits (π_t) which contain the same information as A_t and Q_t. Note that

$$\begin{bmatrix} \Pi_t + (C+\delta P_r) \\ I_t - x_i^* \end{bmatrix} = \begin{bmatrix} 1 & -B \\ y_i^* & z_i^* \end{bmatrix} \begin{bmatrix} A_t \\ Q_t \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A_t \\ Q_t \end{bmatrix} = \frac{1}{z_i^* + By_i^*} \begin{bmatrix} B(I_t - x_i^*) + z_i^*\Pi_t + z_i^*(C+\delta P_r) \\ (I_t - x_i^*) - y_i^*\Pi_t - y_i^*(C+\delta P_r) \end{bmatrix}.$$

(A12) implies $z_i^* + By_i^* = (1+ky_i^*)z_i^*(1-\rho_f)$. Thus, using our expression for A_t and (A13), yields

$$\begin{aligned}
(1+ky_i^*)(A_t - \overline{A}) &= \frac{1+ky_i^*}{z_i^* + By_i^*}\left(B(I_t - x_i^*) + z_i^*\Pi_t + z_i^*(C+\delta P_r) - \overline{A}(z_i^* + By_i^*)\right) \\
&= \frac{1}{(1-\rho_f)}\left((B/z_i^*)I_t + \Pi_t - \left((B/z_i^*)x_i^* + \overline{A}(1-\rho_f)(1+ky_i^*) - (C+\delta P_r)\right)\right) \\
&= \frac{1}{(1-\rho_f)}\left((B/z_i^*)I_t + (\Pi_t - rP_r)\right) \\
&= \frac{1}{(1-\rho_f)}\left((B/z_i^*)I_t + (\Pi_t - \Pi^*)\right).
\end{aligned}$$

Therefore, we obtain

$$E_0[R_{t+1} \mid I_t^N, \Pi_t] = r - (1-\theta)\left[\frac{(B-kz_i^*)I_t}{P_r + kI_{t\,t}}\right] - \frac{(\rho_f - \rho_0)}{1-\rho_f}\left[\frac{(\Pi_t - \Pi\ ) + (B/z_i^*)I_t}{P_r + kI_t}\right]. \qquad \text{(A34)}$$

Proposition 2 (Forecasting regressions): *In a neighborhood of the steady-state:*

(a) Consider a multivariate regression of returns on demand (A_t) and fleet size (Q_t). If $\theta < 1$ or $\rho_0 < \rho_f$, then $\partial E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t < 0$. If $\theta < 1$, then $\partial E_0[R_{t+1} \mid A_t, Q_t] / \partial Q_t > 0$.

(b) If $\theta < 1$ or $\rho_0 < \rho_f$, then investment (I_t), prices (P_t), and profits (Π_t) will each negatively forecast returns in a univariate regression.

(c) Consider a multivariate regression of returns on investment (I_t) and profits (Π_t).

(i) *If there is competition neglect but no demand over-extrapolation (* $\theta < 1$ *and* $\rho_f = \rho_0$*), then* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial I_t < 0$ *and* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial \Pi_t = 0$*;*

(ii) *If there is demand over-extrapolation but no competition neglect (* $\theta = 1$ *and* $\rho_f > \rho_0$*), then* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial \Pi_t < 0$ *and* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial I_t > 0$*;*

(iii) *If there is both competition neglect and over-extrapolation, (i.e.,* $\theta < 1$ *and* $\rho_f > \rho_0$*), then we always have* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial \Pi_t < 0$*. Furthermore, if competition neglect is* <u>*relatively important*</u> *in the sense that* $\left((\rho_f - \rho_0)/(1-\rho_f)\right)\left(B/(-z_i^*(B - kz_i^*))\right) < (1-\theta)$*, then* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial I_t < 0$*. Otherwise, if competition neglect is relatively unimportant, then* $\partial E_0[R_{t+1} \mid \Pi_t, I_t]/\partial I_t > 0$*.*

Proof: We have

$$\frac{\partial E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t} = -(1-\theta)\left[\frac{P_r y_i^*(B-kz_i^*)}{(P_r + kx_i^* + ky_i^* A_t + kz_i^* Q_t)^2}\right] - (\rho_f - \rho_0)\left[\frac{P_r(1+ky_i^*)(P_r + kz_i^*(Q_t - Q^*))}{(P_r + kx_i^* + ky_i^* A_t + kz_i^* Q_t)^2}\right],$$

and

$$\frac{\partial E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial Q_t} = -(1-\theta)\left[\frac{P_r z_i^*(B-kz_i^*)}{(P_r + kx_i^* + ky_i^* A_t + kz_i^* Q_t)^2}\right] + (\rho_f - \rho_0)\left[\frac{P_r(1+ky_i^*)kz_i^*(A_t - \overline{A})}{(P_r + kx_i^* + ky_i^* A_t + kz_i^* Q_t)^2}\right]$$

Thus, in a neighborhood of the steady state, $(A_t, Q_t) = (\overline{A}, Q^*)$, we have

$$\frac{\partial E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t} \approx \frac{\partial E_0[R_{t+1} \mid \overline{A}, Q^*]}{\partial A_t} = -(1-\theta)P_r^{-1} y_i^*(B - kz_i^*) - (\rho_f - \rho_0)P_r^{-1}(1+ky_i^*) < 0, \tag{A35}$$

and

$$\frac{\partial E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial Q_t} \approx \frac{\partial E_0[R_{t+1} \mid \overline{A}, Q^*]}{\partial Q_t} = -(1-\theta)P_r^{-1} z_i^*(B - kz_i^*) > 0. \tag{A36}$$

Part (a) follows from inspecting (A35) and (A36).

To prove part (b), we approximate expected returns as

$$\begin{aligned} E_0\left[1 + R_{t+1} \mid A_t, Q_t\right] \approx r + P_r^{-1}\left[-(1-\theta)y_i^*(B - kz_i^*) - (\rho_f - \rho_0)(1+ky_i^*)\right](A_t - \overline{A}) \\ + P_r^{-1}\left[-(1-\theta)z_i^*(B - kz_i^*)\right](Q_t - Q^*). \end{aligned} \tag{A37}$$

Therefore

$$
\begin{aligned}
Cov[R_{t+1}, I_t] &= Cov[E_0[R_{t+1} \mid A_t, Q_t], I_t] \\
&\approx -P_r^{-1}(1-\theta)(B-kz_i^*)[(y_i^*)^2\sigma_A^2 + (z_i^*)^2\sigma_Q^2 + 2y_i^* z_i^* \sigma_{AQ}] \\
&\quad - P_r^{-1}(\rho_f - \rho_0)(1+ky_i^*)[y_i^*\sigma_A^2 + z_i^*\sigma_{AQ}]
\end{aligned} \tag{A38}
$$

(A38) is negative since $[(y_i^*)^2\sigma_A^2 + (z_i^*)^2\sigma_Q^2 + 2y_i^* z_i^* \sigma_{AQ}] = Var[I_t] > 0$ and $y_i^*\sigma_A^2 + z_i^*\sigma_{AQ} = y_i^*\sigma_A^2[(1-\rho_0)/(1-\rho_0(1+z_i^*))] > 0$. Similarly, we have

$$
\begin{aligned}
Cov[R_{t+1}, \Pi_t] &= Cov[E_0[R_{t+1} \mid A_t, Q_t], \Pi_t] \\
&\approx -P_r^{-1}(1-\theta)(B-kz_i^*)[y_i^*\sigma_A^2 - Bz_i^*\sigma_Q^2 - (By_i^* - z_i^*)\sigma_{AQ}] \\
&\quad - P_r^{-1}(\rho_f - \rho_0)(1+ky_i^*)[\sigma_A^2 - B\sigma_{AQ}].
\end{aligned} \tag{A39}
$$

(A39) is negative since algebra shows $[y_i^*\sigma_A^2 - Bz_i^*\sigma_Q^2 - (By_i^* - z_i^*)\sigma_{AQ}] > 0$ and $[\sigma_A^2 - B\sigma_{AQ}] > 0$.

To prove part (c), note that

$$
\frac{\partial E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t} = -(1-\theta)(B-kz_i^*)\frac{P_r}{(P_r + kI_t)^2} - \frac{(\rho_f - \rho_0)}{1-\rho_f}\left[\frac{(B/z_i^*)P_r - k(\Pi_t - \Pi^*)}{(P_r + kI_t)^2}\right],
$$

and

$$
\frac{\partial E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial \Pi_t} = -\frac{(\rho_f - \rho_0)}{1-\rho_f}\frac{1}{P_r + kI_t}.
$$

Thus, in a neighborhood of the steady state, $(I_t, \Pi_t) = (0, \Pi^*)$, we have

$$
\frac{\partial E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t} \approx \frac{\partial E_0[R_{t+1} \mid 0, \Pi^*]}{\partial I_t} = -(1-\theta)P_r^{-1}(B-kz_i^*) - \frac{\rho_f - \rho_0}{1-\rho_f}P_r^{-1}(B/z_i^*). \tag{A40}
$$

and

$$
\frac{\partial E_0[R_{t+1} \mid I_t^N, \Pi_t]}{\partial \Pi_t} \approx \frac{\partial E_0[R_{t+1} \mid 0, \Pi^*]}{\partial \Pi_t} = -\frac{\rho_f - \rho_0}{1-\rho_f}P_r^{-1}. \tag{A41}
$$

Part (c) follows from inspecting (A40) and (A41). ■

Proposition 3 characterizes how these predictability results vary with the underlying model parameters. Specifically, we take comparative statics on the forecasting results near the model's steady-state. However, when computing these comparative statics, we allow the steady-state of the model to change where relevant.

Proposition 3 (The role of competition neglect, demand over-extrapolation, inelastic demand, and elastic supply): *(a) Return predictability becomes stronger when competition neglect is more severe (i.e.,* $\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right] / \partial A_t \partial(1-\theta) < 0$ *and* $\partial^2 E_0\left[1 + R_{t+1} \mid A_t, Q_t\right] / \partial Q_t \partial(1-\theta) > 0$ *) or demand over-extrapolation is more severe (i.e.,* $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial \rho_f < 0$, *but* $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial Q_t \partial \rho_f = 0$ *).*

(b) The predictability due to competition neglect becomes stronger when demand is more inelastic and weaker when supply is more inelastic. Formally, when $\theta < 1$ *and* $\rho_f = \rho_0$, $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial B < 0$, $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial Q_t \partial B > 0$, $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial k > 0$, *and* $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial Q_t \partial k < 0$.

(c) The predictability due to demand over-extrapolation becomes weaker when demand is more inelastic and stronger when supply is more inelastic. Formally, when $\theta = 1$ *and* $\rho_f > \rho_0$, $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial B > 0$ *and* $\partial^2 E_0[R_{t+1} \mid A_t, Q_t] / \partial A_t \partial k < 0$.

(d) In a multivariate regression of returns on earnings and investment, the coefficient on earnings becomes more negative when demand extrapolation is more severe (i.e., $\partial^2 E_0[R_{t+1} \mid \Pi_t, I_t] / \partial \Pi_t \partial \rho_f < 0$ *); the coefficient on investment falls when competition neglect is more severe and rises when demand extrapolation is more severe (i.e.,* $\partial^2 E_0[R_{t+1} \mid \Pi_t, I_t] / \partial I_t \partial(1-\theta) < 0$ *and* $\partial^2 E_0[R_{t+1} \mid \Pi_t, I_t] / \partial I_t \partial \rho_f > 0$ *). Finally, when competition neglect is relatively important, the coefficient on investment becomes more negative when either demand or supply is more inelastic.*

Proof: Part (a) follows from differentiating (A35) and (A36). Specifically, we have

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t \partial \theta} = P_r^{-1} y_i^*(B - kz_i^*) - (1-\theta) P_r^{-1}(B - kz_i^*) \frac{\partial y_i^*}{\partial \theta} + (1-\theta) P_r^{-1} k y_i^* \frac{\partial z_i^*}{\partial \theta} - (\rho_f - \rho_0) P_r^{-1} k \frac{\partial y_i^*}{\partial \theta} > 0,$$

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t \partial \rho_f} = -(1-\theta) P_r^{-1}(B - kz_i^*) \frac{\partial y_i^*}{\partial \rho_f} - P_r^{-1}(1 + k y_i^*) - (\rho_f - \rho_0) P_r^{-1} k \frac{\partial y_i^*}{\partial \rho_f} < 0,$$

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial Q_t \partial \theta} = P_r^{-1} z_i^*(B - kz_i^*) - (1-\theta) P_r^{-1}(B - 2kz_i^*) \frac{\partial z_i^*}{\partial \theta} < 0,$$

and

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial Q_t \partial \rho_f} = 0.$$

Part (b) follows from differentiating (A35) and (A36) after setting $\rho_f = \rho_0$. Specifically,

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t \partial B} = -(1-\theta)P_r^{-1}\left[y_i^*\left(1-k\frac{\partial z_i^*}{\partial B}\right)+\left(B-kz_i^*\right)\frac{\partial y_i^*}{\partial B}\right]$$

$$= -(1-\theta)P_r^{-1}\left(1-k\frac{\partial z_i^*}{\partial B}\right)(y_i^*)^2 k(1-\rho_f+r)/\rho_f < 0,$$

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial Q_t \partial B} = -(1-\theta)P_r^{-1}\left[(B-kz_i^*)\frac{\partial z_i^*}{\partial B}+z_i^*\left(1-k\frac{\partial z_i^*}{\partial B}\right)\right] > 0,$$

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t \partial k} = (1-\theta)P_r^{-1}\left[y_i^*\frac{\partial (kz_i^*)}{\partial k}-(B-kz_i^*)\frac{\partial y_i^*}{\partial k}\right]$$

$$= (1-\theta)P_r^{-1}\left[(y_i^*)^2\frac{(1-\rho_f+r)(B-kz_i^*)}{\rho_f}\frac{kr+\theta(B-kz_i^*)}{kr+\theta(B-2kz_i^*)}\right] > 0,$$

and

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial Q_t \partial k} = -(1-\theta)P_r^{-1}\left[(B-kz_i^*)\frac{\partial z_i^*}{\partial k}-z_i^*\frac{\partial (kz_i^*)}{\partial k}\right]$$

$$= -(1-\theta)P_r^{-1}\left[\frac{-rz_i^*(B-kz_i^*)}{kr+\theta(B-2kz_i^*)}\right] < 0.$$

Part (c) follows from differentiating (A35) after setting $\theta = 1$. Specifically, we have

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t \partial k} \approx -(\rho_f-\rho_0)P_r^{-1}\frac{\partial (ky_i^*)}{\partial k} < 0,$$

and

$$\frac{\partial^2 E_0\left[R_{t+1} \mid A_t, Q_t\right]}{\partial A_t \partial B} \approx -(\rho_f-\rho_0)P_r^{-1}k\frac{\partial y_i^*}{\partial B} > 0.$$

Finally, part (d) follows from differentiating (A40) and (A41). Specifically, we have

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t \partial \theta} = P_r^{-1}(B-kz_i^*)+(1-\theta)P_r^{-1}k\frac{\partial z_i^*}{\partial \theta}+\frac{\rho_f-\rho_0}{1-\rho_f}P_r^{-1}\frac{B}{(z_i^*)^2}\frac{\partial z_i^*}{\partial \theta} > 0,$$

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t \partial \rho_f} = -\frac{1-\rho_0}{(1-\rho_f)^2} P_r^{-1}(B / z_i^*) > 0,$$

and

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial \Pi_t \partial \rho_f} = -\frac{1-\rho_0}{(1-\rho_f)^2} P_r^{-1} < 0.$$

Finally, differentiating (A40) after setting $\rho_f = \rho_0$, we have

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t \partial B} = -(1-\theta) P_r^{-1}\left(1 - k\frac{\partial z_i^*}{\partial B}\right) < 0,$$

and

$$\frac{\partial^2 E_0[R_{t+1} \mid I_t, \Pi_t]}{\partial I_t \partial k} = (1-\theta) P_r^{-1} \frac{\partial (k z_i^*)}{\partial k} < 0. \qquad \blacksquare$$

Model extensions

We have made a number of assumptions to keep the model tractable so as to transparently model the logic of competition neglect. For instance, we have assumed that demand follows a stationary process. One could easily add a deterministic time trend to demand and instead assuming that deviations of demand from trend are stationary.

We have also assumed that ships have a constant replacement cost of P_r. However, it is straightforward to extend the model so ship replacement costs follows an $AR(1)$ process. In that case, firm investment and ship prices are a linear function of the three state variables: the time-varying replacement cost, $P_{r,t}$, as well as A_t and Q_t.[3]

We could also extend the model so that firms optimally choose $i_t^N = 0$ for a non-degenerate set of values for A_t and Q_t. For instance, we could add a fixed adjustment cost that is incurred whenever $|i_t^N| > 0$. Alternately, we could introduce a wedge between the scrap value realized when

[3] Thus, if we had time-series data on the real price of raw materials used to construct a ship—this is not the same as the price of a new ship which would reflect a potentially time-varying mark-up over cost—we could extend the model and consider this extension in our estimation below.

$i_t^N < 0$ and the replacement cost of a new ship incurred when $i_t^N > 0$. To keep the model tractable, we have also assumed adjustment costs are symmetric. Obviously, any differences between the costs of increasing or decreasing the capital stock would generate asymmetries in the speed of adjustment to positive or negative demand shocks.

B: Simulated Method of Moments

We use the Simulated Method of Moments (SMM) procedure to estimate the parameters of our model of industry cycles. We are interested in a $L \times 1$ vector of parameters $\boldsymbol{\theta}$. Assume that we have $M \geq L$ moment conditions of the form

$$\mathbf{m}_T(\boldsymbol{\theta}) = (\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta})), \tag{B1}$$

where the $\boldsymbol{\mu}_T$ are functions of our sample of T time-series observations (e.g., time-series means, time-series variances, time-series regression coefficients, etc.) and the $\mathbf{g}(\boldsymbol{\theta})$ are the corresponding functions of our simulated time-series data. By simulating a sufficiently long time series we can eliminate simulation noise. We can thus regard the simulated moments as deterministic and continuously differentiable function of the unknown parameters $\mathbf{g}(\boldsymbol{\theta})$.

Now define the estimator

$$\hat{\boldsymbol{\theta}}_T = \arg\min_{\boldsymbol{\theta}} (\mathbf{m}_T(\boldsymbol{\theta}))' \mathbf{W} (\mathbf{m}_T(\boldsymbol{\theta})) = \arg\min_{\boldsymbol{\theta}} (\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta}))' \mathbf{W} (\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta})) \tag{B2}$$

If we assume

- *Compactness*: $\boldsymbol{\theta}_0 \in \text{int}(\boldsymbol{\Theta})$ where $\boldsymbol{\Theta}$ is a compact subset of $\mathbf{R}^L$.
- *Identification*: $\mathbf{0} = E[\mathbf{m}_T(\boldsymbol{\theta})] = E[\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta})]$ implies $\boldsymbol{\theta} = \boldsymbol{\theta}_0$
- *Limiting Behavior*: A Central Limit Theorem implies that $\sqrt{T}(\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta}_0)) \xrightarrow{d} N(\mathbf{0}, \mathbf{S})$
- *Full Rank:* $\boldsymbol{\Gamma}(\boldsymbol{\theta}) = \mathbf{D}_{\boldsymbol{\theta}} \mathbf{g}(\boldsymbol{\theta})$ exists, has full rank L, and is a continuous function of $\boldsymbol{\theta}$.

Then we will have

$$\hat{\boldsymbol{\theta}}_T \xrightarrow{p} \boldsymbol{\theta}_0, \tag{B3}$$

i.e., our estimator will be consistent for the true population parameter of interest.

We now prove asymptotic normality. The first order condition for $\hat{\boldsymbol{\theta}}_T$ is

$$\mathbf{0} = -(\boldsymbol{\Gamma}(\hat{\boldsymbol{\theta}}_T))'\mathbf{W}(\boldsymbol{\mu}_T - \mathbf{g}(\hat{\boldsymbol{\theta}}_T)). \tag{B4}$$

By the intermediate value theorem, this implies

$$\begin{aligned} &\mathbf{0} = -(\boldsymbol{\Gamma}(\hat{\boldsymbol{\theta}}_T))'\mathbf{W}(\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta}_0) - \boldsymbol{\Gamma}(\overline{\boldsymbol{\theta}}_T)(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0)) \\ &\Rightarrow \sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) = [(\boldsymbol{\Gamma}(\hat{\boldsymbol{\theta}}_T))'\mathbf{W}\boldsymbol{\Gamma}(\overline{\boldsymbol{\theta}}_T)]^{-1}(\boldsymbol{\Gamma}(\hat{\boldsymbol{\theta}}_T))'\mathbf{W}\sqrt{T}(\boldsymbol{\mu}_T - \mathbf{g}(\boldsymbol{\theta}_0)), \end{aligned} \tag{B5}$$

for some $\overline{\boldsymbol{\theta}}_T$ that is a convex combination of $\hat{\boldsymbol{\theta}}_T$ and $\boldsymbol{\theta}_0$. Thus, since $\hat{\boldsymbol{\theta}}_T \xrightarrow{p} \boldsymbol{\theta}_0$ and since $\boldsymbol{\Gamma}(\boldsymbol{\theta}) = \mathbf{D}_{\boldsymbol{\theta}}\mathbf{g}(\boldsymbol{\theta})$ is continuous, Slutsky's Theorem implies that

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} N(\mathbf{0}, (\boldsymbol{\Gamma}'\mathbf{W}\boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}'\mathbf{WSW}\boldsymbol{\Gamma}(\boldsymbol{\Gamma}'\mathbf{W}\boldsymbol{\Gamma})^{-1}), \tag{B6}$$

where $\boldsymbol{\Gamma} = \mathbf{D}_{\boldsymbol{\theta}}\mathbf{g}(\boldsymbol{\theta}_0)$. It is easy to show that asymptotically efficient GMM estimates can be obtained by using $\mathbf{W} = \hat{\mathbf{S}}^{-1}$ where $\hat{\mathbf{S}} \xrightarrow{p} \mathbf{S}$. Specifically, in that case, equation (B6) implies that

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \xrightarrow{d} N(\mathbf{0}, (\boldsymbol{\Gamma}'\mathbf{S}^{-1}\boldsymbol{\Gamma})^{-1}), \tag{B7}$$

which has the smallest asymptotic variance in the class of GMM estimators. Because $\hat{\mathbf{S}}^{-1}$ is not always well-conditioned, in our estimation we use $\mathbf{W} = [diag(\hat{\mathbf{S}})]^{-1}$. That is, we use a diagonal weighting matrix that weights each moment inversely to its estimated variance, so we solve

$$\hat{\boldsymbol{\theta}}_T = \arg\min_{\boldsymbol{\theta}} \sum_{m=1}^{M} [(\mu_{m,DATA} - g_{m,SIM}(\boldsymbol{\theta})) / \sigma_{m,DATA}]^2. \tag{B8}$$

Our estimator for the variance of the SMM estimator takes the form

$$T^{-1}\hat{\mathbf{V}}(\hat{\boldsymbol{\theta}}_T) = T^{-1}(\hat{\boldsymbol{\Gamma}}'\mathbf{W}\hat{\boldsymbol{\Gamma}})^{-1}\hat{\boldsymbol{\Gamma}}'\mathbf{W}\hat{\mathbf{S}}\mathbf{W}\hat{\boldsymbol{\Gamma}}(\hat{\boldsymbol{\Gamma}}'\mathbf{W}\hat{\boldsymbol{\Gamma}})^{-1}, \tag{B9}$$

where $\hat{\boldsymbol{\Gamma}} = \mathbf{D}_{\boldsymbol{\theta}}\mathbf{g}(\hat{\boldsymbol{\theta}}_T)$ is a consistent estimator of $\boldsymbol{\Gamma}$ and $\hat{\mathbf{S}}$ is a consistent estimator of $\mathbf{S}$.

Intuitively, a moment $g_m(\boldsymbol{\theta}_0)$ is informative about a parameter l if the partial derivative of the moment with respect to the parameter is large in absolute magnitude—i.e., if $\Gamma_{ml} = \partial g_m(\boldsymbol{\theta}_0) / \partial \theta_l$ is large in magnitude. If we have lots of uninformative moments, then $\boldsymbol{\Gamma}'\mathbf{W}\boldsymbol{\Gamma}$ will be an ill-conditioned matrix with lots of near-zero values. Consequently, $(\boldsymbol{\Gamma}'\mathbf{W}\boldsymbol{\Gamma})^{-1}$ will be very large in magnitude, implying that the standard errors on our SMM estimates will be very large—i.e., they will be estimated quite imprecisely.

More formally, we follow Gentzkow and Shaprio (2013) and examine the elements of the influence matrix $\boldsymbol{\Lambda} = (\boldsymbol{\Gamma}'\mathbf{W}\boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}'\mathbf{W}$ and $\boldsymbol{\Lambda}_{SCALE} = [diag(\mathbf{V})]^{-1/2}(\boldsymbol{\Gamma}'\mathbf{W}\boldsymbol{\Gamma})^{-1}\boldsymbol{\Gamma}'\mathbf{W}[diag(\mathbf{S})]^{1/2}$. The elements of the influence matrix, Λ, are simply the partial derivatives of the SMM estimator with respect to the sample moments, evaluated at the true parameter vector (see (B5)). The elements of the scaled influence matrix, Λ_{SCALE}, are a natural unit-free measure of identification influence: $[\mathbf{\Lambda}_{SCALE}]_{lm}$ is the standard deviation response of parameter *l* to a one standard deviation increase in moment *m*. Thus, as Gentzkow and Shaprio (2013) argue, examining Λ_{SCALE} is a natural way to assess sources of identification in non-linear models source as ours.

To obtain a consistent estimator for **S**, we use a system OLS approach (or seemingly-unrelated regression framework) using Newey-West (1987) standard errors that account for the serial correlation of residuals to estimate the joint variance of the empirical moments. Specifically, we can interpret our moment conditions as consisting of a system of linear equations

$$\mathbf{y}_t = \mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\varepsilon}_t \tag{B10}$$

where

$$\mathbf{y}_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{M,t} \end{bmatrix}, \mathbf{X}_t = \begin{bmatrix} \mathbf{x}'_{1,t} & \mathbf{0}' & \cdots & \mathbf{0}' \\ \mathbf{0}' & \mathbf{x}'_{1,t} & \cdots & \mathbf{0}' \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}' & \mathbf{0}' & \cdots & \mathbf{x}'_{M,t} \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_M \end{bmatrix}, \boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{M,t} \end{bmatrix}. \tag{B11}$$

The system OLS estimator for **β** is

$$\mathbf{b}_{OLS} = \left(\sum_{t=1}^{T} \mathbf{X}'_t \mathbf{X}_t \right)^{-1} \left(\sum_{t=1}^{T} \mathbf{X}'_t \mathbf{y}_t \right), \tag{B12}$$

and, letting $\mathbf{e}_t = \mathbf{y}_t - \mathbf{X}_t \mathbf{b}_{OLS}$, the Newey-West (1987) style variance estimator for $\mathbf{b}_{OLS}$ is

$$T^{-1}\hat{\mathbf{S}}_{NW} = \frac{T}{T-L} \left(\sum_{t=1}^{T} \mathbf{X}'_t \mathbf{X}_t \right)^{-1} \left[\sum_{t=1}^{T} \mathbf{X}'_t \mathbf{e}'_t \mathbf{e}_t \mathbf{X}_t + \sum_{j=1}^{J} \left(1 - \frac{j}{J+1} \right) \sum_{t=1}^{T-j} \left(\mathbf{X}'_t \mathbf{e}'_t \mathbf{e}_{t+j} \mathbf{X}_{t+j} + \mathbf{X}'_{t+j} \mathbf{e}'_{t+j} \mathbf{e}_t \mathbf{X}_t \right) \right] \left(\sum_{t=1}^{T} \mathbf{X}'_t \mathbf{X}_t \right)^{-1} \tag{B13}$$

We use (B12) allowing for serial correlation of residuals at up to *J* = 36 months.

C: Model-Implied Impulse Response Functions

Figures A1, A2, and A3 show the model-implied impulse response functions allowing for competition neglect only, allowing for demand over-extrapolation only, and allowing for both biases.

Figure A1
Model-Implied Impulse Response Functions: Competition Neglect Only

This figure shows the model-implied impulse response functions following a one-time shock to demand. The figure corresponds to the estimates in column (1) of Table VI which allows for competition neglect ($\theta < 1$), but does not allow for demand over extrapolation (we impose $\rho_f = \rho_0$). Following the demand shock at t = 1, the figures contrast the impulse response under rational expectations (imposing θ = 1) with the impulse response *anticipated* by firms who suffer from competition neglect and the actual impulse respond under competition neglect.

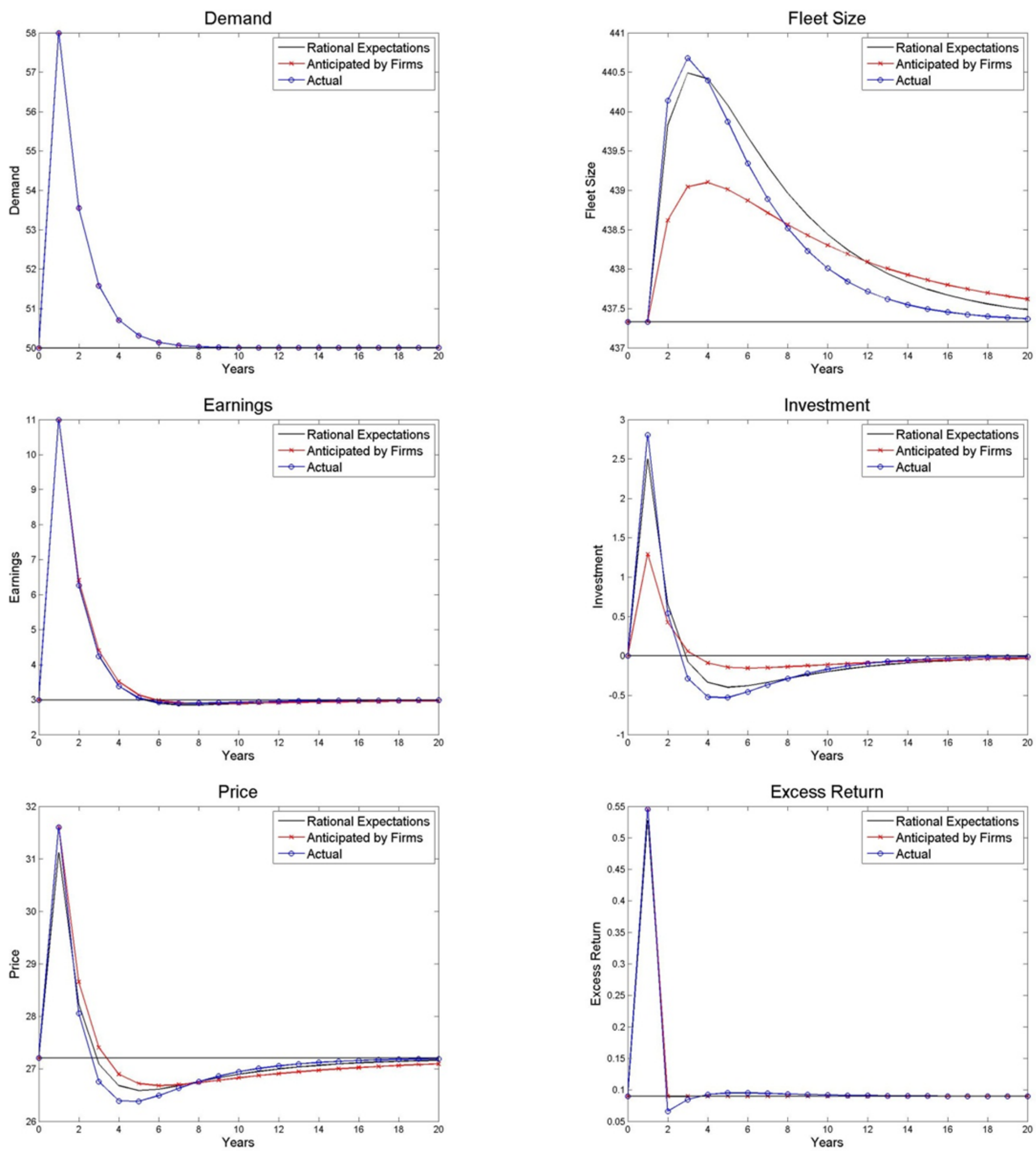

Figure A2
Model-Implied Impulse Response Functions: Demand Over-Extrapolation Only

This figure shows the model-implied impulse response functions following a one-time shock to demand. The figure corresponds to the estimates in column (2) of Table VI which allows for demand over extrapolation ($\rho_f > \rho_0$), but does not allow for competition neglect (we impose $\theta = 1$). Following the demand shock at t = 1, the figures contrast the impulse response under rational expectations ($\rho_f = \rho_0$) with the impulse response *anticipated* by firms who suffer over-extrapolation demand and the actual impulse response under demand over-extrapolation.

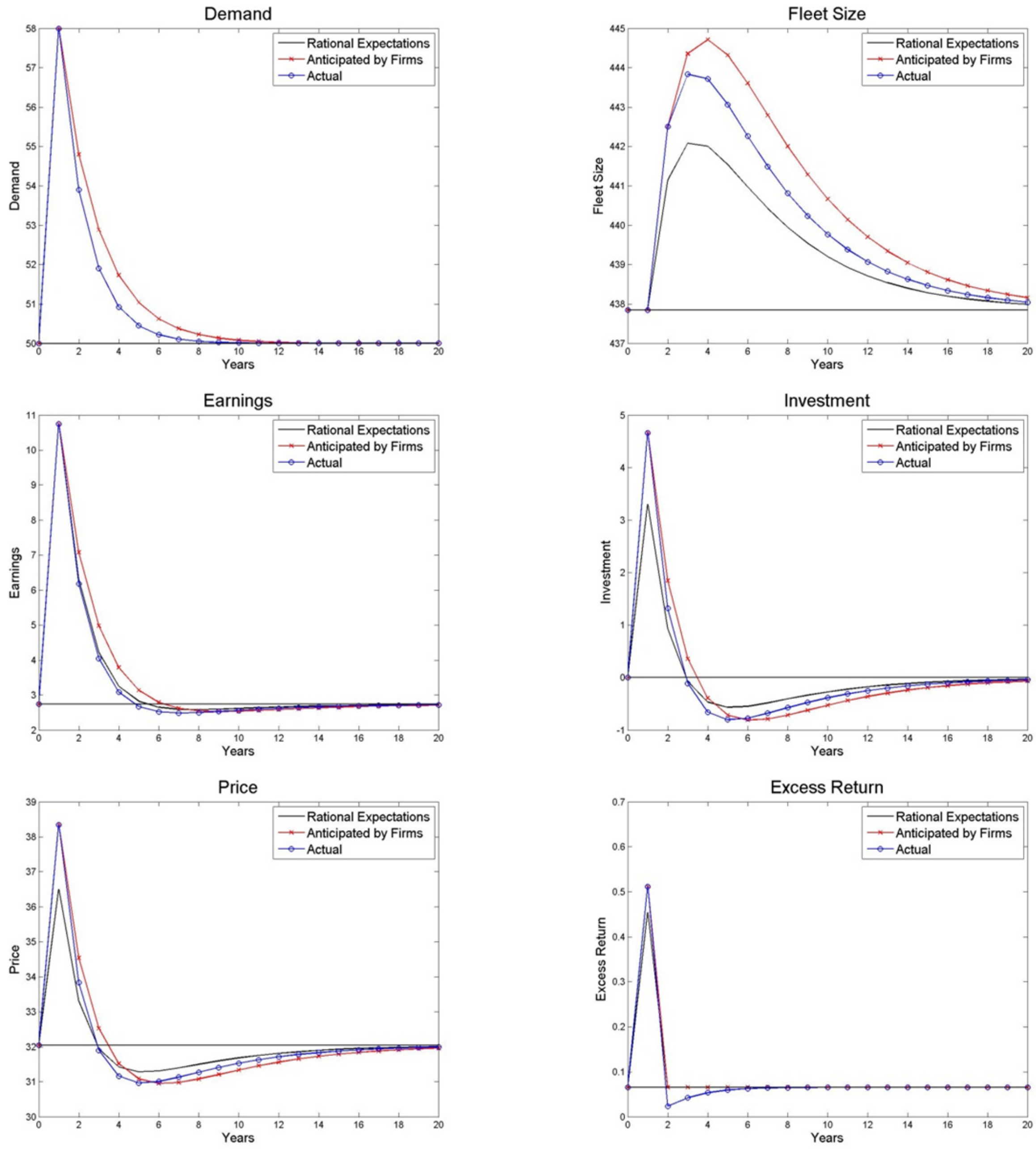

Figure A3
Model-Implied Impulse Response Functions: Both Biases

This figure shows the model-implied impulse response functions following a one-time shock to demand. The figures corresponds to the estimates in column (3) of Table VI which allows for both competition neglect ($\theta < 1$) and demand over extrapolation ($\rho_f > \rho_0$). Following the demand shock at $t = 1$, the figures contrast the impulse response under rational expectations ($\rho_f = \rho_0$ and $\theta = 1$) with the impulse response *anticipated* by firms who suffer from both biases and the actual impulse response when firms suffer from both biases.