

Do Managers Do Good With Other People's Money?

Online Appendix

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Abstract

This is the Online Appendix for Cheng, Hong and Shue (2013) containing details of the model.

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We start with the manager's problem is as in Chetty and Saez (2010):

$$\max_{K, D \geq 0} \alpha (1 - \tau) (1 + \eta) \left[D + \frac{f(K) + \Gamma - D}{1 + r} \right] + \frac{g(\Gamma - K - D)}{1 + r},$$

where η is the monitoring parameter, τ is the dividend tax rate, α is the ownership of the manager, Γ is total cash in the firm, K is productive capital spending, D is the dividend paid, and r is the discount rate (there is no uncertainty in this model). For simplicity, assume $\eta = 0$ for now. The function f represents the net profits of the firm; gross production may be thought of as $F(K) = f(K) + K$. Suppose that both f and g are strictly increasing and concave, with $f(0) = g(0) = 0$. Following Chetty and Saez (2010), the capital G used for investment in goodness at period 0 is returned to shareholders at period 1. One may assume that G is burned up without changing any predictions by writing the model in terms of gross production and simply replacing the second term in brackets with $\frac{F(K)}{1+r}$ and assuming that F is strictly increasing and concave, with $F(0) = 0$.

The generic first-order condition for this problem is:

$$\begin{aligned} \alpha (1 - \tau) f'(K) &= g'(\Gamma - K - D), \\ \alpha (1 - \tau) r &\leq g'(\Gamma - K - D) \text{ with strict equality if and only if } D > 0. \end{aligned}$$

Define $\bar{\alpha}$ to be the critical ownership level at which managers start paying dividends:

$$\bar{\alpha} = \frac{g'(\Gamma - K^*)}{r(1 - \tau)},$$

where K^* is the first-best investment, determined by $f'(K^*) = r$.

Let α_1 be the ownership level of any high ownership manager ($\alpha_1 > \bar{\alpha}$) and let α_2 be the ownership level of any low ownership manager ($\alpha_2 < \bar{\alpha}$). Using a subscript of "1" to denote the investment of the high ownership manager and "2" for the low ownership manager, we can re-write the optimal investment and goodness spending for the high ownership manager as:

$$\begin{aligned} f'(K_1) &= r, \\ \alpha (1 - \tau) r &= g'(G_1), \\ D &> 0, \Gamma = D + K_1 + G_1, \end{aligned}$$

and:

$$\begin{aligned} \alpha (1 - \tau) f'(K_2) &= g'(G_2), \\ g'(G_2) &> \alpha (1 - \tau) r, \\ D &= 0, G_2 = \Gamma - K_2, \end{aligned}$$

for the low ownership manager.

At a basic level, we are interested in how goodness spending G responds to changes in

the tax rate, τ . Applying the implicit function theorem reveals that:

$$\begin{aligned}\frac{\partial G_1}{\partial \tau} &= \frac{-\alpha r}{g''(G_1)} > 0, \\ \frac{\partial G_2}{\partial \tau} &= \frac{-\alpha f'(K_2)}{g''(G_2) + \alpha(1-\tau)f''(K_2)} > 0, = 0 \text{ for } \alpha_2 = 0,\end{aligned}$$

which demonstrates the most basic implication of the model: $\partial G/\partial \omega > 0$. Since $\frac{\partial G}{\partial \eta} = -\frac{\partial G}{\partial \tau} < 0$, this shows Prediction 2 as well.

For Prediction 1, we are interested in the comparative response of goodness spending to the tax cut across high and low ownership managers, $\frac{\partial G_i}{\partial \tau}$ across $i \in \{1, 2\}$. We are also interested in whether there is heterogeneity within each group, $\frac{\partial^2 G_i}{\partial \tau \partial \alpha}$ within $i \in \{1, 2\}$. We show that the prediction holds for a broad class of smooth concave production and goodness functions f and g . For intuition, we begin with the simple case where g is linear.

Linear g case. First consider the simple case where $g(G) = BG$, maintaining the assumption that f is strictly increasing and concave. Suppose $B < (1-\tau)r$ so that $\bar{\alpha} < 1$.

For the moment, rename firms with $\alpha \in (0, \bar{\alpha})$ medium ownership firms (subscripted with 2), while still calling firms with $\alpha \in (\bar{\alpha}, 1]$ high ownership firms (subscripted with 1). The case where $\alpha = 0$ gives rise to a corner solution; call this firm the zero-ownership firm, subscripted with 0.

For high ownership firms, productive capital K is the first best given by the solution to $f'(K) = r$, and anything left over is paid out as dividends since $\alpha(1-\tau)r > B$. Nothing is invested in goodness, and the firm's goodness spending does not respond to the tax cut at all.

The zero ownership firm is a corner solution who invests nothing in productive capital ($K_0 = 0$), pays no dividends and invests everything in goodness ($G_0 = \Gamma$). This firm's goodness spending does not respond to the tax cut at all either.

For medium ownership firms, no dividends are paid, and there is an interior solution for capital and goodness spending given by $\alpha(1-\tau)f'(K_2) = B$ and $G_2 = \Gamma - K_2 > 0$. Applying the implicit function theorem reveals that:

$$\begin{aligned}\frac{\partial K_2}{\partial \alpha} &= \frac{-f'}{\alpha f''} > 0, \\ \frac{\partial K_2}{\partial \tau} &= \frac{f'}{(1-\tau)f''} < 0.\end{aligned}$$

This implies:

Lemma 1. *[Prediction 1, Zero Ownership.] Suppose $B < (1-\tau)r$. Then managers with medium ownership cut goodness more than managers with zero ownership and also more than high ownership in response to the dividend tax cut.*

Proof. This immediately follows from the fact that zero and high ownership managers spend $G_0 = \Gamma$ and $G_1 = 0$, that this does not respond to the tax cut, and that $\frac{\partial G_2}{\partial \tau} = -\frac{\partial K_2}{\partial \tau} > 0$. If the tax cut is discrete and large such that a medium ownership firm becomes a high

ownership firm, goodness spending falls to zero from a positive number and the proposition is also true. ■

Suppose the production function f satisfies the regularity condition that f'/f'' is differentiable and a monotone decreasing function. As an example, any $f(K) = AK^\gamma$ with $\gamma \in (0, 1)$ and $A > 0$ satisfies this property.

Lemma 2. *Suppose f'/f'' is differentiable and a monotone decreasing function. Then $\frac{\partial K_2}{\partial \alpha \partial \tau} < 0$ for $\alpha \in (0, \bar{\alpha})$.*

Proof. Applying the implicit function theorem to $\frac{\partial K_2}{\partial \tau}$ reveals that:

$$\frac{\partial K_2}{\partial \alpha \partial \tau} = \frac{f'}{\alpha(1-\tau)f''} \left[\frac{f'f'''}{(f'')^2} - 1 \right],$$

which is negative, since $\frac{\partial}{\partial K} \left[\frac{f'}{f''} \right] = 1 - \frac{f'f'''}{(f'')^2} < 0$. ■

Now consider any $\alpha_L \in (0, \bar{\alpha})$ and re-define medium ownership managers to be those with $\alpha \in (\alpha_L, \bar{\alpha})$. Define low managers to be those with $\alpha \in (0, \alpha_L)$.

Proposition 1. *[Prediction 1, Linear g .] Suppose $B < (1-\tau)r$ and that f'/f'' is differentiable and a monotone decreasing function. In response to a tax cut, medium ownership managers cut more than low ownership managers, high ownership managers, and zero ownership managers.*

Proof. Follows directly from the previous two lemmas and the observation that $\frac{\partial G_2}{\partial \tau} = -\frac{\partial K_2}{\partial \tau} > 0$ and $\frac{\partial G_2}{\partial \alpha \partial \tau} = -\frac{\partial K_2}{\partial \alpha \partial \tau} > 0$. ■

Broader Production Functions. Returning to the general case, the trade-off between spending on K and G depends on the relative concavity of f and g . For tractability, we impose more structure within the class of increasing and concave functions. Let:

$$f(K) = AK^\gamma, \quad g(G) = BG^\gamma,$$

where $\gamma < 1$. The parameters A and B control the relative concavity of the two functions.¹

The first-best investment and critical ownership level $\bar{\alpha}$ in this problem are:

$$K^* = \left(\frac{A\gamma}{r} \right)^{\frac{1}{1-\gamma}},$$

$$\bar{\alpha} = \frac{B}{A(1-\tau)} (\bar{\Gamma} - 1)^{\gamma-1}.$$

where for convenience we denote $\bar{\Gamma} \equiv \Gamma/K^*$. To make the problem non-trivial, assume $\bar{\Gamma} > 1$, and that B is small enough such that $\frac{B}{A(1-\tau)} (\bar{\Gamma} - 1)^{\gamma-1} \in (0, 1)$. This ensures the firm will

¹If one were to assume that G is burned up in the process of investing in goodness, replacing the assumption of $f(K) = AK^\gamma$ with $F(K) = AK^\gamma$ yields identical predictions. One would replace r in all subsequent calculations with $1+r$.

have enough cash on hand to make the first-best investment if it chooses to do so, and for there to be both dividend and non-dividend-paying managers.

The high ownership ($\alpha > \bar{\alpha}$) firm's optimal investment, goodness spending, and dividends are given by:

$$\begin{aligned} K_1 &= K^* = \left(\frac{A\gamma}{r} \right)^{\frac{1}{1-\gamma}}, \\ G_1 &= \left(\frac{B\gamma}{r(1-\tau)\alpha_1} \right)^{\frac{1}{1-\gamma}}, \\ D_1 &= \Gamma - G_1 - K_1. \end{aligned}$$

Direct computation yields:

$$\begin{aligned} \frac{\partial G_1}{\partial \alpha} &= \frac{1-\tau}{\gamma-1} \left(\frac{B\gamma}{r(1-\tau)\alpha_1} \right)^{\frac{2-\gamma}{1-\gamma}} \frac{r}{B\gamma} < 0, \\ \frac{\partial G_1}{\partial \tau} &= \frac{1}{\gamma-1} \left(\frac{B\gamma}{r(1-\tau)\alpha_1} \right)^{\frac{2-\gamma}{1-\gamma}} \frac{-r\alpha_1}{B\gamma} > 0, \\ \frac{\partial G_1}{\partial \tau \partial \alpha} &= - \left(\frac{\alpha_1(1-\tau)r}{B\gamma} \right)^{\frac{2-\gamma}{\gamma-1}} \left[\left(\frac{1}{1-\gamma} \right)^2 \frac{r}{B\gamma} \right] < 0. \end{aligned}$$

For the low ownership ($\alpha < \bar{\alpha}$) firm, the capital-to-goodness spending ratio is fixed:

$$\frac{K_2}{G_2} = \underbrace{\left[\frac{A}{B} \alpha_2 (1-\tau) \right]^{\frac{1}{1-\gamma}}}_{\equiv C}.$$

This implies:

$$G_2 = \frac{\Gamma}{1+C}, K_2 = \frac{C\Gamma}{1+C}.$$

Direct computation yields:

$$\begin{aligned}
\frac{\partial G_2}{\partial \alpha} &= \frac{-\Gamma \frac{\partial C}{\partial \alpha_2}}{(1+C)^2} = \frac{-\Gamma \left(\frac{1-\tau}{1-\gamma} \frac{A}{B} \left(\frac{\alpha_2(1-\tau)A}{B} \right)^{\frac{\gamma}{1-\gamma}} \right)}{(1+C)^2} \\
&< 0 \text{ for } \alpha_2 > 0, = 0 \text{ for } \alpha_2 = 0, \\
\frac{\partial G_2}{\partial \tau} &= \frac{-\Gamma \frac{\partial C}{\partial \tau}}{(1+C)^2} = \frac{-\Gamma \left(\left[\frac{1}{1-\gamma} \frac{-A\alpha_2}{B} \left(\frac{\alpha_2(1-\tau)A}{B} \right)^{\frac{\gamma}{1-\gamma}} \right] \right)}{(1+C)^2} \\
&> 0 \text{ for } \alpha_2 > 0, = 0 \text{ for } \alpha_2 = 0, \\
\frac{\partial G_2}{\partial \alpha \partial \tau} &= \frac{(1+C)^2 \left(-\Gamma \frac{1}{1-\gamma} \frac{A}{B} \right) \left[(1-\tau) \frac{\gamma}{1-\gamma} \left(\frac{\alpha(1-\tau)A}{B} \right)^{\frac{2\gamma-1}{1-\gamma}} \left(\frac{-A\alpha}{B} \right) - \left(\frac{\alpha(1-\tau)A}{B} \right)^{\frac{\gamma}{1-\gamma}} \right] + \Gamma \left(\frac{1-\tau}{1-\gamma} \frac{A}{B} \left(\frac{\alpha(1-\tau)A}{B} \right)^{\frac{\gamma}{1-\gamma}} \right) 2(1+C) \left(\frac{1}{1-\gamma} \frac{-A\alpha}{B} \left(\frac{\alpha(1-\tau)A}{B} \right)^{\frac{\gamma}{1-\gamma}} \right)}{(1+C)^4} \\
&< 0 \text{ if and only if } \alpha > \frac{B}{(1-\tau)A}.
\end{aligned}$$

Proposition 2. *[Within High and Within Low Ownership Firms.] Within high-ownership firms, $\frac{\partial G_1}{\partial \tau}$ is positive and monotonically decreasing in α . Within low ownership firms, $\frac{\partial G_2}{\partial \tau} > 0$ for $\alpha_2 > 0$ and $\frac{\partial G_2}{\partial \tau} = 0$ for $\alpha_2 = 0$. If $\bar{\Gamma} \geq 2$, then $\frac{\partial G_2}{\partial \tau}$ is monotone increasing in α for $\alpha_2 \in (0, \bar{\alpha})$; if $\bar{\Gamma} < 2$, then $\frac{\partial G_2}{\partial \tau}$ is non-monotone in α : $\frac{\partial G_2}{\partial \tau}$ increases for $\alpha_2 \in \left(0, \frac{B}{(1-\tau)A}\right)$, achieves a maximum at $\frac{B}{(1-\tau)A}$, and decreases for $\alpha_2 \in \left(\frac{B}{(1-\tau)A}, \bar{\alpha}\right)$.*

Proof. Direct algebraic manipulation yields the result. Note that if $\bar{\Gamma} \in (1, 2)$ then $\frac{B}{(1-\tau)A} < \bar{\alpha}$. \blacksquare

Comparing across the two different regions, a general necessary and sufficient condition for $\partial G_2(\alpha_2)/\partial \tau > \partial G_1(\alpha_1)/\partial \tau$ is then

$$\frac{-\Gamma \left(\left[\frac{1}{1-\gamma} \frac{-A\alpha_2}{B} \left(\frac{\alpha_2(1-\tau)A}{B} \right)^{\frac{\gamma}{1-\gamma}} \right] \right)}{\left(1 + \left[\frac{A}{B} \alpha_2 (1-\tau) \right]^{\frac{1}{1-\gamma}} \right)^2} > \frac{1}{\gamma-1} \left(\frac{\alpha_1(1-\tau)r}{B\gamma} \right)^{\frac{2-\gamma}{\gamma-1}} \frac{-r\alpha_1}{B\gamma}. \quad (1)$$

Lemma 3. *Let $\alpha_1 > \bar{\alpha}$ and $\alpha_2 < \bar{\alpha}$ be given. We have $\partial G_2(\alpha_2)/\partial \tau > \partial G_1(\alpha_1)/\partial \tau$ if and only if*

$$\alpha_2^{\frac{-1}{1-\gamma}} + \alpha_2^{\frac{1}{1-\gamma}} \left[\frac{A(1-\tau)}{B} \right]^{\frac{2}{1-\gamma}} < \left(\frac{\alpha_1 r A}{\gamma} \right)^{\frac{1}{1-\gamma}} \Gamma \left(\frac{1-\tau}{B} \right)^{\frac{2}{1-\gamma}} - 2 \left[\frac{A(1-\tau)}{B} \right]^{\frac{1}{1-\gamma}}, \quad (2)$$

and $\partial G_2(\bar{\alpha})/\partial\tau > \partial G_1(\alpha_1)/\partial\tau$ if and only if

$$\alpha_1 > \left(\frac{B}{A(1-\tau)}\right) \left(\frac{\bar{\Gamma}}{\bar{\Gamma}-1}\right)^{1-\gamma} \equiv \hat{\alpha}. \quad (3)$$

Proof. The first part follows directly from manipulation of the necessary and sufficient condition, equation (1). Substituting $\Gamma = \bar{\Gamma}K^*$, replacing $K^* = \left(\frac{A\gamma}{r}\right)^{\frac{1}{1-\gamma}}$ and the definition of $\bar{\alpha}$ reveals that $\partial G_2(\bar{\alpha})/\partial\tau > \partial G_1(\alpha_1)/\partial\tau$ if and only if

$$\bar{\Gamma}^{-1} \left(\frac{B}{A(1-\tau)}\right)^{\frac{1}{1-\gamma}} \left[\bar{\Gamma} + 1 + \frac{1}{\bar{\Gamma}-1}\right] < \alpha_1^{\frac{1}{1-\gamma}},$$

from which the conclusion follows. ■

Proposition 3. [*Prediction 1.*] Suppose $\frac{B}{A(1-\tau)} < \left(1 - \frac{1}{\bar{\Gamma}}\right)^{1-\gamma}$. There exist cut-offs α_L and α_H with $0 < \alpha_L < \bar{\alpha} < \alpha_H < 1$ such that medium ownership firms with $\alpha \in (\alpha_L, \bar{\alpha})$ have $\partial G_2(\alpha)/\partial\tau > \partial G(\tilde{\alpha})/\partial\tau$ for any $\tilde{\alpha} \in (0, \alpha_L) \cup (\alpha_H, 1)$ and those with $\alpha \in (\bar{\alpha}, \alpha_H)$ have $\partial G_1(\alpha)/\partial\tau > \partial G(\tilde{\alpha})/\partial\tau$ for any $\tilde{\alpha}$ similarly given, where $\partial G(\tilde{\alpha})/\partial\tau$ is defined as:

$$\begin{aligned} \partial G(\tilde{\alpha})/\partial\tau &= \partial G_2(\tilde{\alpha})/\partial\tau \text{ if } \tilde{\alpha} < \bar{\alpha}, \\ &= \partial G_1(\tilde{\alpha})/\partial\tau \text{ if } \tilde{\alpha} > \bar{\alpha}. \end{aligned}$$

The cut-off α_H may be chosen to be arbitrarily close to $\hat{\alpha} \equiv \left(\frac{B}{A(1-\tau)}\right) \left(\frac{\bar{\Gamma}}{\bar{\Gamma}-1}\right)^{1-\gamma}$ on the right-side.

Proof. Suppose $\frac{B}{A(1-\tau)} < \left(1 - \frac{1}{\bar{\Gamma}}\right)^{1-\gamma}$. First note that $\bar{\alpha} < \hat{\alpha} < 1$. The first inequality follows since, by the definition of $\bar{\alpha}$,

$$\begin{aligned} \bar{\alpha} &= \frac{B}{A(1-\tau)} (\bar{\Gamma}-1)^{\gamma-1} \\ &= \frac{B}{A(1-\tau)} \left(\frac{1}{\bar{\Gamma}-1}\right)^{1-\gamma} \\ &< \hat{\alpha}, \end{aligned}$$

since $\bar{\Gamma}^{1-\gamma} > 1$. The second inequality follows by supposition.

Take any $\alpha_H \in (\hat{\alpha}, 1)$. By Lemma 3 (note the strict inequalities):

$$\frac{\partial G_2(\bar{\alpha})}{\partial\tau} > \frac{\partial G_1(\alpha_H)}{\partial\tau} > 0.$$

Recall that $\frac{\partial G_2(0)}{\partial\tau} = 0$.

Consider the case where $\bar{\Gamma} \geq 2$. By the Intermediate Value Theorem, there exists an $\alpha_L \in (0, \bar{\alpha})$ such that $\frac{\partial G_2(\alpha_L)}{\partial\tau} = \frac{\partial G_1(\alpha_H)}{\partial\tau} \equiv D$. Furthermore, this α_L must be unique in $(0, \bar{\alpha})$ (given any choice of $\alpha_H > \hat{\alpha}$) since $\partial G_2/\partial\tau$ is monotonically increasing between 0 and $\bar{\alpha}$, by Proposition 2. Note that there is no $\alpha \in (\bar{\alpha}, \alpha_H)$ such that $\partial G_1(\alpha)/\partial\tau = D$ since $\partial G_1/\partial\tau$

is monotonic, so α_L is unique in $(0, \alpha_H)$. By construction,

$$\begin{aligned} \frac{\partial G_2(\alpha)}{\partial \tau} &> D \text{ for } \alpha \in (\alpha_L, \bar{\alpha}), & D &> \frac{\partial G_2(\tilde{\alpha})}{\partial \tau} \text{ for } \tilde{\alpha} \in (0, \alpha_L), \\ \frac{\partial G_1(\alpha)}{\partial \tau} &> D \text{ for } \alpha \in (\bar{\alpha}, \alpha_H), & D &> \frac{\partial G_1(\tilde{\alpha})}{\partial \tau} \text{ for } \tilde{\alpha} \in (\alpha_H, 1), \end{aligned}$$

where the inequalities in the first row follow since $\partial G_2/\partial \tau$ is monotonically increasing on $(0, \bar{\alpha})$ and the inequalities in the second row follow since $\partial G_1/\partial \tau$ is monotonically decreasing, from Proposition 2. Therefore,

$$\begin{aligned} \frac{\partial G_2(\alpha)}{\partial \tau} &> D > \frac{\partial G(\tilde{\alpha})}{\partial \tau} \text{ for } (\alpha, \tilde{\alpha}) \in (\alpha_L, \bar{\alpha}) \times [(0, \alpha_L) \cup (\alpha_H, 1)], \\ \frac{\partial G_1(\alpha)}{\partial \tau} &> D > \frac{\partial G_1(\tilde{\alpha})}{\partial \tau} \text{ for } (\alpha, \tilde{\alpha}) \in (\bar{\alpha}, \alpha_H) \times [(0, \alpha_L) \cup (\alpha_H, 1)]. \end{aligned}$$

For $\bar{\Gamma} < 2$, note that for $\alpha \in (0, \bar{\alpha})$, $\partial G_2/\partial \tau$ increases for $\alpha_2 \in \left(0, \frac{B}{(1-\tau)A}\right)$, achieves a maximum at $\frac{B}{(1-\tau)A}$, and decreases for $\alpha_2 \in \left(\frac{B}{(1-\tau)A}, \bar{\alpha}\right)$. Since $\partial G_2/\partial \tau$ has one maximum in the interval $(0, \bar{\alpha})$ at $\frac{B}{(1-\tau)A} < \bar{\alpha}$ with $\partial G_2\left(\frac{B}{(1-\tau)A}\right)/\partial \tau > \partial G_1(\alpha_H)/\partial \tau$, it must be by the Intermediate Value Theorem that there is a $\alpha_L \in \left(0, \frac{B}{(1-\tau)A}\right)$ such that $\frac{\partial G_2(\alpha_L)}{\partial \tau} = \frac{\partial G_1(\alpha_H)}{\partial \tau} \equiv D$. Furthermore, we must have $\frac{\partial G_2(\alpha)}{\partial \tau} > D$ for $\alpha \in (\alpha_L, \bar{\alpha})$ and $\frac{\partial G_2(\tilde{\alpha})}{\partial \tau} < D$ for $\tilde{\alpha} \in (0, \alpha_L)$, by construction. Finally, since $\partial G_2\left(\frac{B}{(1-\tau)A}\right)/\partial \tau > \partial G_2(\bar{\alpha})/\partial \tau > \partial G_1(\alpha_H)/\partial \tau$, and $\partial G_2/\partial \tau$ is monotonic over $\left(0, \frac{B}{(1-\tau)A}\right)$ and $\left(\frac{B}{(1-\tau)A}, \bar{\alpha}\right)$, this α_L is unique given any choice of $\alpha_H > \hat{\alpha}$. The rest of the proof follows similarly. \blacksquare

References

Chetty, R., and E. Saez, 2010, “Dividend and Corporate Taxation in an Agency Model of the Firm,” *American Economic Journal: Economic Policy*, 2(3), 1–31.