Appendix for Demand Analysis using Strategic Reports: An application to a school choice mechanism (Not for Publication)

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The appendix follows the organization of the paper. Appendix A describes the data sources and the cleaning process, appendix B presents technical details relevant for section 5. These include details on condition 1 and limit equilibria, details on RSP+C mechanisms and proofs of results presented in that section. Appendix C presents technical details relevant for section 6, including proofs and additional results on identification and testable restrictions of equilibrium behavior. Appendix D proves consistency of our two-step approach and details the Gibbs' sampler used in section 7.

A Data Appendix

The primary data for the study come from the Cambridge Public Schools. Under a non-disclosure agreement, we use data from student registration records, assignment files, and data on student characteristics.

The student registration records contain the school/program the student is registered at, student's grade, language spoken at home, and the paid-lunch status at registration.

The assignment files include the rank-order list of the student, sibling or proximity priority at the ranked school, the randomly generated tie-breaker used in the assignment as well paid-lunch/free-lunch status of the student. Cambridge pre-assigns about 40% of the students to public elementary schools via arrangements with pre-kindergarten schools. The assignment files provide detail on whether the student is pre-assigned and if the student participated in the school choice process (the Cambridge mechanism) studied in this paper.

We also obtained reports from the school district containing the overall capacity of each school/program in each year and the numbers assigned through each process. We use these reports as the primary source for computing the number of seats available at various schools

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and programs in the mechanism. In rare cases, the rank order lists, the random tie-breaker and the priority codes indicated an inconsistency in the capacity data. We used the knowledge of the mechanism to adjust these capacities and were able to compute the correct assignment for almost all students with these modified capacities.

The student characteristics file duplicates several of the variables in the registration and school choice ranking and assignment file. Importantly, it also includes the home address of the student. The Network Analyst Toolbox in ArcGIS and information in ESRI's Datamaps 10.1 on the US road network was used to compute the distance by road between the student's home and the school address based on brochures from the relevant years. This computation ignores one-way restrictions because Cambridge uses walking distance to compute proximity priority.

These files were merged using a unique student identifier.[1] Schools and programs are also uniquely identified in the dataset.

B Limits: Equilibrium, Mechanisms and Convergence

B.1 Convergence of Equilibrium Probabilities

Since we will be considering the properties of a sequence of equilibrium strategies, it is useful to define equilibrium strategies for the limit case, ϕ^∞, when each agent is playing against a continuum of other agents. We say that σ^* is a **Limit Equilibrium** if $\sigma^*_R(v,t) > 0$ implies that $v \cdot \phi^\infty((R,t), m^{\sigma^*}) \geq v \cdot \phi^\infty((R',t), m^{\sigma^*})$ for all $R' \in \mathcal{R}$. Our next will show that condition 1 allows for several useful conclusions in this section.

Corollary B.1. *Assume that ϕ^n satisfies condition 1 at m^{σ^*} for some strategy σ^*.*

1. *If $\sigma^{*,n}$ is a sequence BNE such that $\|\sigma^{*,n} - \sigma^*\|_F \to 0$, the strategy σ^* is a limit equilibrium.*
2. *If σ^* is a limit equilibrium, then for each $\varepsilon > 0$, and large enough n, $\sigma^*_R(v,t) > 0$ implies that for all $R' \in \mathcal{R}$,*

$$|v \cdot \left(\mathbb{E}_{\sigma^*}[\phi^n((R,t), m^{n-1}) - \phi^n((R',t), m^{n-1})]\right)| \leq \varepsilon \|v\|.$$

The result shows that a convergent sequence of Bayesian Nash Equilibria converge to a limit equilibrium, and that all limit equilibria are approximate BNE for large enough n.

[1]We are grateful to Parag Pathak for sharing the dataset for this project.

The result is similar in spirit to Kalai (2004), which shows that equilibria in limit games are approximate BNE in large games.

Proof. **Part 1:**
We will show that $\sigma^*_R(v,t) > 0$ for all $(v,t) \in \text{int}(\text{supp}F_{V,T})$ only if $v \cdot (\phi^\infty((R,t), m^{\sigma^*}) - \phi^\infty((R',t), m^{\sigma^*}) \geq 0$ for all $R' \in \mathcal{R}$. We treat two strategies as equivalent if they only differ outside the support of $F_{V,T}$.

Fix $(v,t) \in \text{int}(\text{supp}F_{V,T})$. Towards a contradiction, suppose that $\sigma^*_R(v,t) > 0$, and $v \cdot (\phi^\infty((R,t), m^{\sigma^*}) - \phi^\infty((R',t), m^{\sigma^*})) < -2\varepsilon$ for some $R' \in \mathcal{R}$ and $\varepsilon > 0$. Since $(v,t) \in \text{int}(\text{supp}F_{V,T})$, there exists a $\delta > 0$, such that for all v' with $\|v - v'\| < \delta$, we have $v' \in \text{int}(\text{supp}F_{V,T})$, and $v' \cdot (\phi^\infty((R,t), m^{\sigma^*}) - \phi^\infty((R',t), m^{\sigma^*})) < -\varepsilon$. Let m^{n-1} be an empirical measure of $n-1$ samples from $m^{\sigma^*,n}$. Since $|\phi^n((R,t), m^{n-1}) - \phi^\infty((R',t), m^{\sigma^*})| \xrightarrow{p} 0$ (theorem 1), and ϕ^n is bounded, there exists an N, such that for all $n > N$ and all $R' \in \mathcal{R}$,

$$\left|\mathbb{E}_{\sigma^*,n}[\phi^n((R',t), m^{n-1})] - \phi^\infty((R',t), m^{\sigma^*})]\right| \leq \frac{\varepsilon}{2(\|v\| + \delta)}.$$

Hence, for all v' in the δ neighborhood of v, we have that

$$\begin{aligned} & v' \cdot (\mathbb{E}_{\sigma^*,n}[\phi^n((R,t), m^{n-1}) - \mathbb{E}_{\sigma^*,n}[\phi^n((R',t), m^{n-1})]) \\ \leq\ & v' \cdot (\phi^\infty((R,t), m^{\sigma^*})] - \phi^\infty((R',t), m^{\sigma^*})) + \varepsilon \\ <\ & 0. \end{aligned}$$

Since $\sigma^{*,n}$ is a Bayesian Nash Equilibrium strategy, it must be that for all $n > N$, $\sigma^{*,n}_R(v',t) = 0$. Therefore, $\|\sigma^{*,n} - \sigma^*\|_F \to 0$ implies that $\sigma^*(v',t) = 0$ for all v' in the δ neighborhood of v. This conclusion contradicts the hypothesis that $\sigma^*_R(v,t) > 0$ for any R such that $v \cdot (\phi^\infty((R,t), m^{\sigma^*}) - \phi^\infty((R',t), m^{\sigma^*})) < 0$. Hence, σ^* is a limit equilibrium.

Part 2:
For a strategy σ^*, a particular realization of the reports of the other agents is given by the empirical measure m^{n-1} from $n-1$ iid draws from m^{σ^*} where $m^{\sigma^*}(R,t) = f_T(t) \times \int \sigma^*(v,t;R) dF_{V|T}$. Condition 1 implies that $\phi^n((R_i,t_i), m^{n-1}) \xrightarrow{p} \phi^\infty((R_i,t_i), m^{\sigma^*})$. Fix $\varepsilon > 0$ and pick n_0 such that for all $n > n_0$,

$$P\left(\sup_{R,t} \|\phi^n((R,t), m^{n-1}) - \phi^\infty((R,t), m)\|_\infty > \frac{\varepsilon}{8|S|}\right) < \frac{\varepsilon}{8|S|}.$$

Since $\|\phi^n((R,t), m^{n-1}) - \phi^\infty((R,t), m^{\sigma^*})\|_\infty$ is bounded by 1, we have that

$$\mathbb{E}\left[\|\phi^n((R,t), m^{n-1}) - \phi^\infty((R,t), m^{\sigma^*})\|_\infty\right] < \frac{\varepsilon}{4|S|}.$$

Note that the choice of n_0 did not depend on v_i.

Now, we show that no agent of type t_i and utility v_i can expect a gain of more than $\varepsilon\|v_i\|_\infty$ by deviating from σ^*. For $n > n_0$ and each (R_i, t_i), let

$$V_i^n((R_i,t_i),m^\sigma) = \mathbb{E}_\sigma\left[\sum_j \phi^n((R_i,t_i),m^{\sigma,n-1})v_{ij}\right]$$

be the expected utility from report R_i given priority type t_i and type-symmetric strategy σ followed by other agents. For any strategy σ such that condition 1 is satisfied at m^σ, we have that

$$\begin{aligned}
& |V_i^n((R_i,t_i),m^\sigma) - V_i^\infty((R_i,t_i),m^\sigma)| \\
\leq\ & \mathbb{E}_\sigma\left|\sum_j \phi_j^n((R_i,t_i),m^{\sigma,n-1})v_{ij} - \sum_j \phi_j^\infty((R_i,t_i),m^\sigma)v_{ij}\right| \\
\leq\ & 2|S|\|v_i\|_\infty\,\mathbb{E}\left[\|\phi^n((R_i,t_i),m^{\sigma,n-1}) - \phi^\infty((R_i,t_i),m^\sigma)\|_\infty\right] \\
\leq\ & \frac{\varepsilon}{2}\|v_i\|_\infty
\end{aligned}$$

Since σ^* is a limit equilibrium, $\sigma^*(v_i,t_i;R_i) > 0$ implies that for all R_i',

$$\begin{aligned}
& V_i^\infty((R_i,t_i),m^{\sigma^*}) && \geq V_i^\infty((R_i',t_i),m^{\sigma^*}) \\
\Rightarrow\quad & V_i^n((R_i,t_i),m^{\sigma^*}) && \geq V_i^n((R_i',t_i),m^{\sigma^*}) - \varepsilon\|v_i\|_\infty
\end{aligned}$$

for all $n > n_0$. □

B.2 RSP+C Mechanisms: Existence and (Generic) Uniqueness of Cutoffs

We introduce two definitions before discussing existence and uniqueness. The first definition is a notion of substitutes in a neighborhood around the market clearing price. This borrows from the notion of connected substitutes introduced in Berry et al. (2013) and Berry and Haile (2010) to show conditions when demand is invertible.

Definition B.1. *$D_j(p|\eta)$ satisfies **local connected substitutes** at $p^* \in [0,1]^J$ if there exists an $\varepsilon > 0$, such that for all $p \in [0,1]^J$ with $\|p - p^*\| < \varepsilon$, we have that*

1. *for all $j \in \{0,1,\ldots,J\}$ and $k \notin \{1,\ldots,J\}\backslash\{j\}$, $D_j(p|\eta)$ is nondecreasing in p_k*
2. *for all non-empty subsets $K \subset \{1,\ldots,J\}$, there exist $k \in K$ and $l \notin K$ such that $D_l(p|\eta)$ is strictly increasing in p_k*

Definition B.2 (Azevedo and Leshno (2013)). *$D(p|\eta)$ is **regular** if the image $D(\bar{P}|\eta)$, where*

$$\bar{P} = \{p \in [0,1]^J : D(\cdot|\eta) \text{ is not continuously differentiable at } p\}$$

has Lebesgue measure 0.

We now observe that assumption 1.2 is satisfied (generically satisfied) if $D_j(p|\eta)$ satisfies local connected substitutes at any market clearing cutoff (is regular).

Proposition B.1. *Every economy (η, q) admits at least one market clearing cutoff.*

Further, for a fixed η, let Q be the set of capacities such that (η, q) has multiple market clearing cutoffs. Then,

1. *$Q \cap \{q : \sum_{j=1}^{J} q_j < \eta(\mathcal{R} \times [0,1]^J \times T)\}$ has Lebesgue measure zero if $D_j(p|\eta)$ is regular*
2. *Q is empty if $D(p|\eta)$ satisfies local connected substitutes for any market clearing cutoff p^*. In particular, Q is empty if $D(p|\eta)$ satisfies local connected substitutes at every cutoff p.*

Proof. Existence of cutoffs follows from corollary A1 and lemma 1 of Azevedo and Leshno (2013). Statement 1 is a consequence of Azevedo and Leshno (2013), theorem 1(2) and lemma 1. Statement 2 is a strengthening of Azevedo and Leshno (2013), theorem 1(1). By the Lattice Theorem (Azevedo and Leshno, 2013), there exist minimum and maximum market clearing cutoffs $p^- \leq p^+$. Note that the measure of students matched with program j at cutoff p is given by $D_j(p|\eta)$, and the measure of students unmatched is given by $D_0(p|\eta)$. Hence, by the Rural Hospitals Theorem (Azevedo and Leshno, 2013), for all $C \subseteq S$,

$$\sum_{j \in C} D_j(p^+|\eta) = \sum_{j \in C} D_j(p^-|\eta). \tag{B.1}$$

Let p^* be a market clearing cutoff such that $D(p|\eta)$ satisfies local connected substitutes at p^*. Let $C^+ = \{j \in S : p_j^* < p_j^+\}$ and $C^- = \{j \in S : p_j^* > p_j^-\}$. We will show that $C^+ = \emptyset$ i.e. $p^+ = p^*$. The proof to show that $C^- = \emptyset$ is symmetric and together, these claims imply that $p^+ = p^- = p^*$.

Towards a contradiction, assume that $C^+ \neq \emptyset$. Since $D(p|\eta)$ satisfies local connected substitutes at p^* (Definition B.1), there exist $\varepsilon \in (0,1)$, $k \in C^+$, and $l \notin C^+$ such that

$$D_l(p^*|\eta) < D_l(p^\varepsilon|\eta),$$

where $p_k^\varepsilon = \varepsilon p_k^+ + (1-\varepsilon)p_k^*$ for $j \neq k$ and $p_j^\varepsilon = p_j^*$. Hence, we have that

$$\sum_{j\in S\setminus C^+} D_j(p^*|\eta) < \sum_{j\in S\setminus C^+} D_j(p^\varepsilon|\eta) \leq \sum_{j\in S\setminus C^+} D_j(p^+|\eta),$$

where the implication on the summation and the second inequality are implied by the definition of $D(p|\eta)$. Since this inequality contradicts equation (B.1), it must be that $C^+ = \emptyset$. $\square$

Remark B.1. *The condition that $D(p|\eta)$ satisfies local connected substitutes for all $p \in [0,1]$ is testable. Note that local connected substitutes is implied by strict gross substitutes.*

B.3 Proof of theorem 2

We begin by showing a few preliminaries.

The first result shows that for any (R,e), and iid draws of the reports and priority types of the other $n-1$ agents from η, the associated market clearing cutoffs $p^n(R,e)$ converge to the limit market clearing cutoff p for (η, q).

Lemma B.1. *Suppose (η, q) satisfies assumption 1. If $p^n(R,e)$ is a sequence of market clearing cutoffs for the market (η^n, q^n) where*

$$\eta^n = \frac{n-1}{n}\eta^{n-1} + \frac{1}{n}\delta_{(R,e)}$$

and η^{n-1} are a sequence of empirical measures that converges in probability to η and $q^n \to q$, then

$$\sup_{(R,e)} \|p^n(R,e) - p^*\|_\infty \xrightarrow{p} 0.$$

Proof. The result is similar in spirit to Azevedo and Leshno (2013), theorem 2. It differs from their results in that we are considering a sequence of randomly drawn economies.

Define the class $\mathcal{B} = \{\{(e_i, R_i) : e_{ij} \geq p_j, R_i = R\} : p_j, j, R\}$. Note that $\mathcal{B}$ is a VC class since it is collection of half-spaces, which are VC classes. Hence, the class of sets

$$\mathcal{V} = \left\{ v_{pj} = \{(e_i, R_i) : e_{ij} \geq p_j,\ jR_i0\} \bigcap_{j'\neq j} \left(\{(e_i, R_i) : jR_ij'\} \cup \{(e_i, R_i) : e_{ij'} < p_{j'}\}\right) : p, j \right\}$$

is a VC-class since it is a subset of finite unions and intersections of sets in $\mathcal{B}$ and their complements. Hence, for any (R,e) and j,

$$\begin{aligned}
\sup_p |D_j(p|\eta) - D_j(p\,|\eta^n)| &= \sup_p |\eta^n(v_{pj}) - \eta(v_{pj})| \\
&\leq \sup_{V\in\mathcal{V}} \left| \frac{n-1}{n}\eta^{n-1}(V) + \frac{1}{n}1\{(R,e)\in V\} - \eta(V) \right| \\
&\leq \sup_{V\in\mathcal{V}} \left| \frac{n-1}{n}\eta^{n-1}(V) - \eta(V) \right| + \frac{1}{n} \\
&\xrightarrow{p} 0,
\end{aligned}$$

by the Glivenko-Cantelli theorem. Hence, $D(p|\eta^n) - q^n \xrightarrow{p} D(p|\eta) - q$ uniformly in p and (R,e). Similarly, we also have that $D(p|\eta^{n-1}) - q^n \xrightarrow{p} D(p|\eta) - q$ uniformly in p.

Let the unique market clearing cutoff for (η, q) be p^*. Define for each (R,e)

$$Q_n(p;R,e) = \left\| \begin{bmatrix} \max\{z(p\,|\eta^n, q^n), 0\} \\ p * z(p\,|\eta^n, q^n) \end{bmatrix} \right\|,$$

where $*$ represents the Hadamard product. Note that $p^n(R,e)$ is a market clearing cutoff iff $Q_n(p;R,e) = 0$. Let Q_0 be the limiting objective function,

$$Q_0(p) = \left\| \begin{bmatrix} \max\{z(p|\eta,q), 0\} \\ p * z(p|\eta,q) \end{bmatrix} \right\|,$$

and note that it does not depend on (R,e). By the continuous mapping theorem, $\sup_{p,R,e} |Q_n(p;R,e) - Q_0(p)| \xrightarrow{p} 0$. Also, $Q_0(p)$ is continuous since assumption 1.1 implies that $D(p|\eta)$ is continuous. Assumption 1.2 implies that $Q_0(p)$ is uniquely minimized at p^*. For $\varepsilon > 0$, let $\delta_\varepsilon = \inf_{p:\|p-p^*\|>\varepsilon} Q_0(p)$. Since Q_0 is continuous, p is an element of a compact space and $Q_0(p) = 0$ only at p^*, $\delta_\varepsilon > 0$. Pick N such that for all $n > N$, $\mathbb{P}(\sup_{p,R,e} |Q_0(p) - Q_n(p;R,e)| > \delta_\varepsilon) < \varepsilon$. For any market clearing cutoff $p^n(R,e)$, $Q_n(p^n(R,e);R,e) = 0$. Note that

$$\begin{aligned}
&|Q_0(p^n(R,e)) - Q_0(p^*)| \\
\leq\ & |Q_0(p^n(R,e)) - Q_n(p^n(R,e);R,e)| + |Q_n(p^n(R,e);R,e) - Q_0(p^*)| \\
\leq\ & \sup_{p,R,e} |Q_0(p) - Q_n(p;R,e)| + 0.
\end{aligned} \tag{B.2}$$

Hence, we have that for all $n > N$,

$$\begin{aligned}\mathbb{P}\left(\sup_{R,e}|p^n(R,e)-p^*|>\varepsilon\right) &\leq \mathbb{P}\left(\sup_{R,e}|Q_0(p^n(R,e))-Q_0(p^*)|>\delta_\varepsilon\right)\\ &\leq \mathbb{P}\left(\sup_{p,R,e}|Q_0(p)-Q_n(p;R,e)|>\delta_\varepsilon\right)<\varepsilon\end{aligned}$$

where the first inequality follows from set inclusion, the second from equation (B.2), and the third by our choice of N. □

Theorem 2 is a corollary to showing condition 1 for the following simpler class of mechanisms.

Definition B.3. *A mechanism ϕ^n is a **random tie-breaker + cutoff** mechanism if there is a measure $\gamma_{\nu|t} \in \Delta[0,1]^J$ for each t such that*

(i) $\phi^n((R_i,t_i), m(R_{-i},t_{-i})) = \int \ldots \int D^{(R_i,\nu_i)}(p^n)\ \mathrm{d}\gamma_{\nu_1|t_1}\ldots \mathrm{d}\gamma_{\nu_n|t_n}$

(ii) p^n are market clearing cutoffs for capacity q^n and each profile of reports and random tie-breakers $((R_1,\nu_1),\ldots,(R_n,\nu_n))$

Lemma B.2. *Suppose (η,q) satisfies assumption 1 where*

$$\eta(\{R,\nu<p\}) = \sum_t m(R,t)\gamma_{\nu|t}(\{\nu<p\}).$$

If ϕ^n is a random tie-breaker + cutoff mechanism, then ϕ^n satisfies condition 1.

Proof. It is enough to show that $\phi^n_j((R,t),m^{n-1}) \xrightarrow{p} \phi^\infty_j((R,t),m))$ for a fixed report R, priority type t and j since there are finitely many elements in $\mathcal{R}\times T\times S$. Since ϕ^n is a random tie-breaker + cutoff mechanism,

$$\phi^n_j((R,t),m^{n-1}) \quad = \quad \int \mathbb{E}\left[D^{(R,\nu)}_j(p^n(R,\nu))\Big| R,\nu,m^{n-1}\right]\mathrm{d}\gamma_{\nu|t}$$

where the expectation is taken with respect to the random draws of the tie-breaker for the other $n-1$ agents conditional on t.

Let the unique market clearing cutoff at (η,q) be p^*. Fix $\varepsilon>0$. Let $U = \left\{\nu : \min_j|\nu_j - p^*_j| \leq \dfrac{\varepsilon}{4\kappa|S|}\right\}$, where κ is defined in assumption 1.1. Note that assumption 1.1 implies that $\gamma_{\nu|t}(U) \leq \dfrac{\varepsilon}{2}$.

For any j

$$
\begin{aligned}
& |\phi_j^n((R,t),m^{n-1}) - \phi_j^\infty((R,t),m)| \\
= & \left| \int \mathbb{E}\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \mathrm{d}\gamma_{\nu|t} \right| \\
\leq & \int \left| \mathbb{E}\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \right| \mathrm{d}\gamma_{\nu|t} \\
\leq & \sup_{\nu \notin U} \left| \mathbb{E}\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \right| (1-\gamma_{\nu|t}(U)) \\
& + \sup_{\nu \in U} \left| \mathbb{E}\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \right| \gamma_{\nu|t}(U) \\
\leq & \sup_{\nu \notin U} \left| \mathbb{E}\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \right| (1-\gamma_{\nu|t}(U)) + \frac{\varepsilon}{2}
\end{aligned}
$$

where the last inequality follows from the fact that

$$\left| E\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \right| \leq 1.$$

We now show that there exists an N such that for all $n > N$,

$$\mathbb{P}\left(\sup_{\nu \notin U} \left| E\left[D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \middle| R,\nu,m^{n-1} \right] \right| > \frac{\varepsilon}{2} \right) < \varepsilon.$$

This would complete the proof since it implies that for all $n > N$,

$$\mathbb{P}\left(|\phi_j^n((R,t),m^{n-1}) - \phi_j^\infty((R,t),m)| > \varepsilon \right) < \varepsilon.$$

Pick an N such that for all $n > N$,

$$\mathbb{P}\left(\sup_{\nu} \|p^n(R,\nu) - p^*\|_\infty > \frac{\varepsilon}{4\kappa|S|} \right) < \frac{\varepsilon^2}{2}.$$

Such an N exists by lemma B.1. Further, note that for any p, if $\nu \notin \{\nu : \exists j, p_j \vee p_j^* < \nu_j < p_j \wedge p_j^*\}$ then $D^{(R,\nu)}(p) = D^{(R,\nu)}(p^*)$. Hence, if $\|p^n(R,\nu) - p^*\|_\infty < \dfrac{\varepsilon}{4\kappa|S|}$ and $\nu \notin U$, then $D_j^{(R,\nu)}(p^n(R,\nu)) = D_j^{(R,\nu)}(p^*)$. Therefore, for all $n > N$,

$$\mathbb{P}\left(\sup_{\nu \notin U} \left| D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*) \right| \neq 0 \right) < \frac{\varepsilon^2}{2}.$$

Since $\left|D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*)\right| \leq 1$, we have that for all $n > N$,

$$\begin{aligned}
\frac{\varepsilon^2}{2} &> \mathbb{E}\left[\sup_{\nu \notin U}\left|D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*)\right|\middle| R,\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[\sup_{\nu \notin U}\left|D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*)\right|\middle| R, m^{n-1}\right]\right] \\
&\geq \mathbb{E}\left[\sup_{\nu \notin U}\mathbb{E}\left[\left|D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*)\right|\middle| R, \nu, m^{n-1}\right]\right]
\end{aligned}$$

where the equality follows from the law of iterated expectations, and the weak inequality follows from a well-known property of expectations and supremums. Finally, Markov's inequality implies that

$$\mathbb{P}\left(\sup_{\nu \notin U}\mathbb{E}\left[\left|D_j^{(R,\nu)}(p^n(R,\nu)) - D_j^{(R,\nu)}(p^*)\right|\middle| R, \nu, m^{n-1}\right] > \frac{\varepsilon}{2}\right) < \varepsilon,$$

proving the desired result. □

We now show that theorem 2 is a corollary to lemma B.2 by observing that ϕ^n is a random tie-breaker + cutoff mechanism with a distribution $\gamma_{\nu|t}$ that depends on f. To see this, note that p^n is a market clearing cutoff for the economy $((R_1, f(R_1,\nu_1)), \ldots, (R_n, f(R_n,\nu_n)))$ and that

$$\begin{aligned}
\phi_j^n((R_i,t_i), m(R_{-i},t_{-i})) &= \int \cdots \int D^{(R_i, f(R_i,\nu_i))}(p^n)\mathrm{d}\gamma_{\nu_1|t_1}\ldots\mathrm{d}\gamma_{\nu_n|t_n} \\
&= \int \cdots \int D^{(R_i,e_i)}(p^n)\mathrm{d}\eta^f_{R_1,\mathbf{e}_1|t_1}(R_1,\cdot)\ldots\mathrm{d}\eta^f_{R_n,\mathbf{e}_n|t_n}(R_n,\cdot).
\end{aligned}$$

B.4 Proof of proposition 1

Deferred Acceptance:

Let $\underline{\nu}_j$ be supremum of the priority scores of the rejected students. We claim that $p^n = \underline{e}$ are the cutoffs with the desired properties (if a school does not reject any students, set $p_j = 0$).

Let $\underline{\nu}_j^r$ be the supremum the priority scores of students that were rejected in round r. Set $\underline{e}_j^r = 0$ if no students are rejected. Observe that for each school, $\underline{\nu}_j^r \leq \underline{\nu}_j^{r+1}$. If the algorithm terminates in round k, then $\underline{\nu}_j^k = \underline{\nu}_j$. The algorithm terminates in finitely many rounds for every n.

Assume that student i is assigned to school j' and consider any school j with jR_jj'. Let r be round in which student i was rejected by j. By definition, it must be that $\nu_{ij} < \underline{\nu}_j^r$. Therefore, $\nu_{ij} < \underline{\nu}_j$ and we have that each student is assigned to $D^{(R_i,\nu_i)}(p^n)$.

Finally, the aggregate demand cannot exceed q_j by construction of p^n.

Boston Mechanism:
We show that the Boston Mechanism is report-specific priority + cutoff mechanisms for

$$f_j(R,\nu) = \frac{\nu_j - \#\{k : kR_i j\}}{J} + \frac{J-1}{J}$$

by constructing market cutoffs p^n for each profile $((R_1,\nu_1),\ldots,(R_N,\nu_N))$ such that (i) the assignment of each agent is given by $D^{(R_i,f(R_i,\nu_i))}(p^n)$ and (ii) p^n clears the market for the economy $((R_1, f(R_1,\nu_1)),\ldots,(R_N, f(R_N,\nu_N)))$.

Note that if a school rejects a student in round k, then it rejects students in all further rounds since it is full at the end of that round. Let k_j denote that round for school j, and let $\underline{\nu}_j$ be supremum of the random priorities of the rejected students in round k_j. We claim that $p_j^n = 1 - \dfrac{k_j - \underline{\nu}_j}{J}$ are the cutoffs with the desired properties (if a school does not reject any students, set $k_j = J$ and $p_j = 0$).

We first show that the assignment of each student in the Boston mechanism is given by $D^{(R_i,f(R_i,\nu_i))}(p^n)$. Assume that student i is assigned to school j' and consider any school j with jR_ij'. Since jR_ij', it must be that the student was rejected at j, and could not have applied to j before round k_j. If student applied to k_j after round j, then $\nu_{ij} - \#\{k : kR_ij\} < \underline{\nu}_j - k_j$ since $|\nu_{ij} - \underline{\nu}_j| \leq 1$. If $\#\{k : kR_ij\} = k_j$, then $\nu_{ij} < \underline{\nu}_j$. In either case, $f_j(R_i,\nu_i) < p_j$. Therefore, the student is assigned to $D^{(R_i,f(R_i,\nu_i))}(p^n)$.

Next, we show that p^n clears the market for economy $((R_1, f(R_1,\nu_1)),\ldots,(R_N, f(R_N,\nu_N)))$. As noted earlier, each agent is assigned to $D^{(R_i,f(R_i,\nu_i))}(p^n)$. By construction of p^n, the aggregate demand must be less than q_j, and $p_j^n = 0$ if aggregate demand is strictly less than q_j.

Serial Dictatorship:
The Serial Dictatorship Mechanism orders the students according to a single priority and then assigns the top student to her top ranked choice. The k-th student is then assigned to her top ranked choice that has remaining seats. It is straightforward to show that this mechanism is equivalent to a Deferred Acceptance mechanism in which all students have identical priorities at all schools. Hence, it is a report-specific priority + cutoff mechanism.

First Preferences First:
The First Preferences First mechanism assigns students to their top ranked choice if seats are available, with tie-breaking according to priorities and a random number. Rejected students

are then processed for the remaining seats according to the Deferred Acceptance mechanism. Arguments identical to the ones above show that the First Priority First mechanism is a report-specific priority + cutoff mechanism for

$$f_j(R,\nu) = \frac{\nu_j + 1\{jRj' \;\; \forall j' \neq j\}}{2}.$$

Chinese Parallel (Chen and Kesten, 2013):

The chinese parallel mechanism operates in t rounds, each with t_c-subchoices. In each round, rejected students applies to the next t_c highest choices that have not yet rejected her. Within each round, the algorithm implements a deferred acceptance procedure in which applications are held tentatively until no new proposals are made. Assignments are finalized after all t_c choices have been considered. It is straightforward to show that the Chinese Parallel mechanism is a report-specific priority + cutoff mechanism for

$$f_j(R,\nu) = \frac{\nu_j - \left\lfloor \frac{\#\{k : kR_i j\}}{t_c} \right\rfloor}{\left\lfloor \frac{J}{t_c} \right\rfloor} + \frac{\left\lfloor \frac{J-1}{t_c} \right\rfloor}{\left\lfloor \frac{J}{t_c} \right\rfloor}.$$

Pan London Admissions (Pennell et al., 2006):

The Pan London Admissions system uses the Student Proposing Deferred Acceptance Mechanism except that a subset of schools upgrade the priority of students that rank the school highly. Suppose school j upgrades students that rank it first. For such schools, we set

$$f_j(R,\nu) = \frac{\nu_j + 1\{jRj' \;\; \forall j' \neq j\}}{2},$$

and $f_j(R,\nu) = \nu$ otherwise. With this modification, the Pan London Admissions scheme is a report-specific priority + cutoff mechanism.

B.5 Verifying condition 1 for the Cambridge Mechanism

We first find a representation of the Cambridge Mechanism as a function

$$\phi^n : (\mathcal{R} \times T) \times \Delta\left(\mathcal{R} \times T\right) \to \Delta S$$

B.5.1 Representation

Priorities and Tie-breakers

Each student receives an independent priority draw ν_i from a uniform $[0,1]$ distribution. We modify this random priority by the sibling and proximity priority t_i. Let $f : [0,1] \times \mathcal{T} \to [0,1]^J$, such that for each $j = 1, ..., J$:

$$e_{ij} = f_j(\nu_i, t_i) = \frac{\nu_i + t_{ij}}{T} \in [0,1]$$

where T is the maximum priority points a student can have. In Cambridge, $t_{ij} = 1$ if student i has only proximity priority at program j, $t_{ij} = 2$ if student i has only sibling priority at program j, and $t_{ij} = 3$ if student i has both proximity and sibling priority at program j.

Economy

Let Π be a partition of the programs in Cambridge into a set of schools in Cambridge and let $q \in \mathbb{R}_+^{J+|\Pi|}$ be a vector of program and school capacities. Typically, for any $\pi \in \Pi$, $\sum_{j\in\pi} q_j < q_\pi$.

Consider a n-student economy where the vector of capacities is represented by $q^n \in \mathbb{R}_+^{J+|\Pi|}$, the measure of report-priority shares of all but the focal student is given by

$$m^{n-1} = \frac{1}{n-1}\sum_{i=1}^{n-1} \delta_{R_i,t_i}$$

and η^{n-1} includes the realization of random priority draws of the $n-1$ students

$$\eta^{n-1} = \frac{1}{n-1}\sum_{i=1}^{n-1} \delta_{R_i,t_i,e_i}$$

where η^{n-1} agrees with m^{n-1} on the marginals on R and t.

Sub-Economies in Rounds $k \in \{1,2,3\}$

With a slight abuse of notation, let $R_{[k]}$ be the program in position k in report R. We will use a map $s(\eta, q|k) \mapsto (\eta', q')$ that takes a measure over reports, priority types, random priorities, and a capacity in each round and maps it to a measure over remaining reports, priority-types and random priorities in the next round.

To define $s(\eta, q|k)$, we introduce some additional notation. Let

$$D_{j,k}(p|\eta) = \eta\left(\left\{(R,e) : R_{[k]} = j, e_j \geq p\right\}\right)$$

be the measure of types that ranked school j in the k-th round and have eligibility score of at least p in that round. Note that $D_{j,k}(p|\eta)$ is non-increasing. Define the excess capacity

z_j for school j at eligibility score p as:

$$\begin{aligned}\tilde{z}_j(p;\eta,q|k) &= q_j - D_{j,k}(p|\,\eta)\\ \tilde{z}_{\pi_j}(p;\eta,q|k) &= q_{\pi_j} - q_j - \sum_{l\in\pi_j/\{j\}} \min\{q_l, D_{l,k}(p|\,\eta)\}\\ z_j(p;\eta,q|k) &= \tilde{z}_j(p;\eta,q|k) + \min\left(0, \tilde{z}_{\pi_j}(p;\eta,q|k)\right).\end{aligned}$$

Note that z_j is non-decreasing in p.

In the Cambridge mechanism, a student is not assigned to a school in round k if the measure of students that have (weakly) higher eligibility exceeds the school or the program's capacity. Therefore, the set of students that are not assigned in step k can be written as

$$r(\eta,q|k) = \left\{(R,e) : R = R(k), z_{R(k)}\left(e_{R(k)};\eta,q|k\right) < 0\right\}.$$

Define η' as the restriction of η to $r(\eta,q|k)$.

The capacities that remain after step k, are given by:

$$q'_j = \max\{q_j - D_{j,k}(0|\eta), 0\}$$

since all students, i.e. measure $D_{j,k}(0|\eta)$, are assigned if there are seats available.

Cambridge Mechanism

Let $(\eta_1, q_1) = (\eta, q)$ and $(\eta_k, q_k) = s(\eta_{k-1}, q_{k-1}|k)$. Define the function,

$$\varphi_{(R,t)}(\nu;\eta,q,k) = 1\left\{\left(R, \frac{\nu+t}{T}\right) \in r(\eta_k, q_k|k)^c \cap_{k'<k} r(\eta_{k'}, q_{k'}|k')\right\}.$$

This function returns 1 if a student that reports R and has priority (ν, t) is assigned to program $R(k)$ when the measure over reports and priorities is given by ν and the vector of capacities is q.

For a fixed student priority-type, report and draw of the tie-breaker, (R,t,ν) define

$$\eta^n = \frac{1}{n}\left[(n-1)\,\eta^{n-1} + \delta_{R,t,e}\right].$$

Note that the finite economy and limit economy mechanisms are given by

$$\begin{aligned}
\phi^n_{R(k)}\left((R,t), m^{n-1}, q^n\right) &= \int \mathbb{E}\left[\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)\middle| m^{n-1},\nu\right] d\nu \\
\phi^\infty_{R(k)}\left((R,t), m, q\right) &= \int \varphi_{(R,t)}\left(\nu;\eta,q,k\right) d\nu
\end{aligned}$$

where the limit measure η is given by

$$\eta\left(\{R, e < p\}\right) = \sum_{t=0}^{T} m\left(R,t\right) \min_j \left(p_j T - t_j\right). \tag{B.3}$$

B.5.2 Main Result: Condition 1 for the Cambridge Mechanism

We make the following assumption about the genericity of vacancies:

Assumption B.1 (Generic Vacancies)**.** *For $k = 1,2,3$, let m_k be the marginal of η_k on $\mathcal{R} \times T$ where $(\eta_k, q_k) = s(\eta_{k-1}, q_{k-1}|k-1)$ and $(\eta_1, q_1) = (\eta, q)$. If $m\left(R,t\right) = 0$ then for each k,*

$$\min \left[\begin{array}{c} q_{k,R(k)} - \sum_{R',t'} m_k(\{R'_{[k]} = R(k), t_{R'_{[k]}} > t_{R(k)}\}), \\ q_{k,\pi_{R(k)}} - \sum_{l\in\pi_{R_{[k]}}} \min\left\{q_{k,l}, \sum_{R',t'} m_k(\{R'_{[k]} = l, t_{R'_{[k]}} > t_{R(k)}\})\right\} \end{array} \right] \neq 0$$

For each (R,t), there is no open set in $[0,1]^{J+|\Pi|}$ such that every q in that set violates assumption B.1. Fix a q such that this assumption is satisfied. We now show that condition 1 is satisfied for the Cambridge Mechanism.

Proposition B.2. *Assume that (m,q) satisfies assumption B.1 above. If m^{n-1}, q^n are empirical sequences such that $m^{n-1} \xrightarrow{p} m$, and $q^n \xrightarrow{p} q$, then for each $k \in \{1,2,3\}$ and (R,t)*

$$\phi^n_{R(k)}\left((R,t), m^{n-1}, q^n\right) \xrightarrow{p} \phi^\infty_{R(k)}\left((R,t), m, q\right).$$

The proof requires two preliminary results. Let $\triangle$ be the symmetric difference operator. Consider the VC class of sets

$$\mathcal{V} = \left\{V : \exists\left(R,p,k\right) \in \mathcal{R} \times [0,1]^J \times \{1,2,3\}, V = v\left(R,p,k\right)\right\},$$

where $v\left(R,p,k\right) = \left\{(R,e) : e_{R(k)} < p\right\}$.

Lemma B.3. *If $\sup_{V\in\mathcal{V}} \left|\eta^n\left(V\right) - \eta\left(V\right)\right| \xrightarrow{p} 0$, $\sup_j \left|q^n_j - q_j\right| \xrightarrow{p} 0$ and $D_{j,k}(p|\eta)$ is continuous in p for all j and k, then*

(i) $\sup_{p,j,k} \left|D_{j,k}(p|\eta^n) - D_{j,k}(p|\eta)\right| \xrightarrow{p} 0,$

(ii) $\sup_{\nu,j,k}|z_j(\nu;t,\eta^n,q^n|k) - z_j(\nu;t,\eta,q|k)| \xrightarrow{p} 0$ *where each* $z_j(\nu;t,\eta,q|k)$ *is continuous and non-decreasing in* ν*,*

(iii) $r(\eta,q|k) = \bigcup_{R\in\mathcal{R}} V_R$ *where each* $V_R \in \mathcal{V}$*,*

(iv) $\eta^n\left(r(\eta^n,q^n|k) \triangle r(\eta,q|k)\right) \xrightarrow{p} 0$*, and*

(v) if η' *is the restriction of* η *to* $r(\eta,q|k)$ *then* $D_{j,k}(p|\eta')$ *is continuous in* p *for all* j *and* k*.*

Proof. **Parts (i - iii):** For every $p \in [0,1]$,

$$\begin{aligned} D_{j,k}(p|\eta) &= \eta\left(\{(R,e) : R(k) = j, e_j \geq p\}\right) \\ &= \sum_{R:R(k)=j} \eta\left(v(R,0,k)\right) - \eta\left(v(R,p,k)\right). \end{aligned}$$

Hence, part (i) follows from uniform convergence in probability of η^n over sets in $\mathcal{V}$. Part (ii) follows from the continuous mapping theorem: $z_j(\cdot;\eta,q|k)$ is continuous with respect to functions $D_{l,k}(\cdot|\,\eta)$, where both types of functions belong to vector spaces endowed with the sup-norm. Continuity of $z_j(\nu;t,\eta,q|k)$ follows directly continuity of the min function and of $D_{l,k}(\cdot|\,\eta)$ for every l. Part (iii) is easily verified noting that $r(\eta,q|k) = \bigcup_j \bigcup_{R:R(k)=j} v(R,p_j,k)$ where $p_j = 0$ if $z_j(0;\eta,q|k) \geq 0$ and otherwise,

$$p_j = \sup\left\{e \in [0,1] : z_j(e;\eta,q|k) < 0\right\}.$$

Part (iv): The definitions of $r(\eta,q|k)$ and $r(\eta^n,q^n|k)$ imply:

$$\begin{aligned} &\eta^n\left(r(\eta^n,q^n|k) \triangle r(\eta,q|k)\right) \\ = &\sum_j \eta^n\left(\{R(k) = j, (e_j < p_j \vee z_j(e;\eta^n,q^n|k) \geq 0) \wedge (e_j \geq p_j \vee z_j(e;\eta^n,q^n|k) < 0)\}\right), \end{aligned} \tag{B.4}$$

where $\vee$ and $\wedge$ are logical AND and OR respectively. It is enough to show convergence in probability for each term in the summation.

Pick an N such that for all $n > N$ with probability greater than $1-\varepsilon$,

$$\sup_{k,e}|z_j(e;\eta,q|k) - z_j(e;\eta^n,q^n|k)| \leq \frac{\varepsilon}{2} \tag{B.5}$$

and

$$\sup_{p_1\leq p_2,R'} \eta^n\left(\{(R,t,\nu) : R = R', p_1 \leq e_j \leq p_2\}\right) \leq T\,|p_1 - p_2| + \frac{\varepsilon}{8}. \tag{B.6}$$

Existence of such an N is guaranteed by part (ii) of the lemma above and since

$$\sup_{p_1 \leq p_2, R'} \eta\left(\{(R,t,\nu) : R = R', p_1 \leq e_j \leq p_2\}\right) \leq T\left|p_1 - p\right|.$$

We first show that equation (B.6) implies that

$$\eta^n\left(\{R(k) = j, z_j\left(e; \eta^n, q|k\right) \in [a,b]\}\right) \leq \frac{\varepsilon}{4} + b - a. \tag{B.7}$$

Let $\underline{e}_n = \inf\{e : z_j\left(e;\eta^n, q|k\right) > a\}$, $\overline{e}_n = \sup\{e : z_j\left(e;\eta^n, q|k\right) < b\}$. We have that

$$\begin{aligned}
& \eta^n\left(\{R(k) = j, z_j\left(e;\eta^n, q^n|k\right) \in [a,b]\}\right) \\
\leq\ & \eta^n\left(\{R(k) = j, e \in [\underline{e}_n, \overline{e}_n]\}\right) \\
=\ & \eta^n\left(\{R(k) = j, e \in (\underline{e}_n, \overline{e}_n)\}\right) + \eta^n\left(\{R(k) = j, e \in \{\overline{e}_n, \overline{e}_n\}\}\right) \\
\leq\ & \lim_{e \downarrow \underline{e}_n} D_{j,k}\left(e|\eta^n\right) - \lim_{e \uparrow \overline{e}_n} D_{j,k}\left(e|\eta^n\right) + \eta^n\left(\{R(k) = j, e \in \{\overline{e}_n, \overline{e}_n\}\}\right) \\
\leq\ & \lim_{e \downarrow \underline{e}_n} D_{j,k}\left(e|\eta^n\right) - \lim_{e \uparrow \overline{e}_n} D_{j,k}\left(e|\eta^n\right) + \frac{\varepsilon}{4} \\
=\ & \lim_{e \uparrow \overline{e}_n} \tilde{z}_j\left(e;\eta^n, q^n|k\right) - \lim_{e \downarrow \underline{e}_n} \tilde{z}_j\left(e|;\eta^n, q^n|k\right) + \frac{\varepsilon}{4} \\
\leq\ & \lim_{e \uparrow \overline{e}_n} z_j\left(e;\eta^n, q^n|k\right) - \lim_{e \downarrow \underline{e}_n} z_j\left(e|;\eta^n, q^n|k\right) + \frac{\varepsilon}{4} \\
\leq\ & b - a + \frac{\varepsilon}{4}
\end{aligned}$$

where the first inequality follows by the definition of $\underline{e}_n$ and $\overline{e}_n$; the second inequality follows from the definition of $D_{j,k}\left(e|\eta^n\right)$ and because it is decreasing; the third inequality follows from equation (B.6); the last inequality follows from the definition of $\tilde{z}_j$ and the final inequality follows from the fact that for all $e \in (\underline{e}_n, \overline{e}_n)$, $z_j\left(e;\eta^n, q^n|k\right) \in (a,b)$ and that $z_j\left(e;\eta^n, q^n|k\right)$ is monotonically increasing.

Now consider the term corresponding to program j in the summation in equation (B.4). If $z_j\left(p_j;\eta^n, q^n|k\right) < 0$, this term is bounded by

$$\eta^n\left(\{e_j \geq p_j, z_j\left(e_j;\eta^n, q^n|k\right) \in \left[z_j\left(p_j;\eta^n, q^n|k\right), 0\right]\}\right).$$

If $z_j\left(p_j;\eta^n, q^n|k\right) \geq 0$, the term is bounded by

$$\eta^n\left(\{e_j < p_j, z_j\left(e;\eta^n, q^n|k\right) \in \left[0, z_j\left(p_j;\eta^n, q^n|k\right)\right]\}\right).$$

Hence, equations (B.7) and (B.5) imply that

$$\begin{aligned} & \eta^n\left(\{R(k)=j,(e_j<p_j \vee z_j(e;\eta^n,q^n|k)\geq 0)\wedge(e_j\geq p_j\vee z_j(e;\eta^n,q^n|k)<0)\}\right) \\ \leq\ & |z_j(p_j;\eta,q|k)-z_j(p_j;\eta^n,q^n|k)|+2\times\frac{\varepsilon}{4} \\ \leq\ & \varepsilon. \end{aligned}$$

Since equations (B.5) and (B.6) (consequently, equation (B.7)), hold for all $n>N$ with probability at least $1-\varepsilon$, we have the desired result.

Part (v): Follows because

$$\begin{aligned} D_{j,k}(p|\eta') &= \eta'\left(\{R(k)=j,e_j\geq p\}\right) \\ &= \eta\left(\{R(k)=j,e_j\geq p\}\cap r(\eta,q|k)\right) \\ &= \eta\left(\{R(k)=j,p_j>e_j\geq p\}\right) \\ &= \begin{cases} D_{j,k}(p|\eta)-D_{j,k}(p_j|\eta) & \text{if } p_j<p \\ 0 & \text{if } p_j\geq p \end{cases} \end{aligned}$$

and $D_{j,k}(p|\eta)$ is continuous. □

Before stating the second preliminary result, we first define the function

$$\zeta_{(R,t)}(\nu;\eta,q,k)=\min\left\{z_{R(k')}\left(\frac{\nu+t_{R(k)}}{T};\eta_k,q_k\middle|\,k\right),-\max_{k'<k}z_{R(k')}\left(\frac{\nu+t_{R(k')}}{T};\eta_{k'},q_{k'}\middle|\,k'\right)\right\}.$$

If $\zeta_{(R,t)}(\nu;\eta,q,k)>0$ both terms are positive. Program $R(k)$ could enroll every unassigned student that ranked it in position k and that has a priority score higher than $\frac{\nu+t_{R(k)}}{T}$ without exhausting program or school capacity. At the same time, if for some $k'<k$, program $R(k')$ had enrolled every unassigned student that ranked it in position k' and had a priority score higher than $\frac{\nu+t_{R(k')}}{T}$, it would have exceeded the total program or school capacity. Therefore a student with report and priority (R,t,ν) such that $\zeta_{(R,t)}(\nu;\eta,q,k)>0$ is assigned to school $R_{[k]}$ in round k. Notice that $\zeta_{(R,t)}(\nu;\eta,q,k)>0$ implies $\varphi_{(R,t)}(\nu;\eta,q,k)=1$ and $\zeta_{(R,t)}(\nu;\eta,q,k)<0$ implies $\varphi_{(R,t)}(\nu;\eta,q,k)=0$.

Lemma B.4. *If* $\sup_{V\in\mathcal{V}}|\eta^n(V)-\eta(V)|\xrightarrow{p}0$ *and* $\sup_j\left|q_j^n-q_j\right|\xrightarrow{p}0$*, where* η *is defined as in (B.3), then* $\sup_{\nu,R,t,k}\left|\zeta_{(R,t)}(\nu;\eta^n,q^n,k)-\zeta_{(R,t)}(\nu;\eta,q,k)\right|\xrightarrow{p}0$.

Proof. We first show that if $D_{j,k}(p|\eta)$ is continuous in p for all j and k, then

$$\|s(\eta^n,\ q^n|k)-s(\eta,\ q|k)\|_\infty=\max\left\{\sup_j\left|q_j'^n-q_j'\right|,\sup_{V\in\mathcal{V}}|\eta'^n(V)-\eta'(V)|\right\}\xrightarrow{p}0.$$

Since q'_j is jointly continuous in q_j and $D_{j,k}(0|\eta)$, $q'^{,n}_j \xrightarrow{p} q'_j$ by the continuous mapping theorem. Consider,

$$\begin{aligned} & \sup_{V\in\mathcal{V}} \left|\eta'^n(V) - \eta'(V)\right| \\ = & \sup_{V\in\mathcal{V}} \left|\eta^n\left(r(\eta^n, q^n|k)\cap V\right) - \eta\left(r(\eta, q|k)\cap V\right)\right| \\ \leq & \sup_{V\in\mathcal{V}} \left|\eta^n\left(r(\eta, q|k)\cap V\right) - \eta\left(r(\eta, q|k)\cap V\right)\right| \\ & + \sup_{V\in\mathcal{V}} \left|\eta^n\left(r(\eta^n, q^n|k)\cap V\right) - \eta^n\left(r(\eta, q|k)\cap V\right)\right|. \end{aligned}$$

The first term converges in probability to zero because $r(\eta, q|k) \in \mathcal{V}$ (lemma B.3, part iii) since $\mathcal{V}$ is closed under finite intersections. The second term is bounded by: $\eta^n\left(r(\eta^n, q^n|k) \triangle r(\eta, q|k)\right)$, which is shown to converge in probability to zero (lemma B.3, part iv). Moreover, for all j and k, $D_{j,k}(p|\eta')$ is continuous in p (lemma B.3, part v).

Notice that $D_{j,k}(p|\eta_1)$ is continuous in p for all j and k. By mathematical induction, $\sup_{V\in\mathcal{V}}\left|\eta^n_{k-1}(V) - \eta_{k-1}(V)\right| \xrightarrow{p} 0$ and $\sup_j\left|q^n_{k-1,j} - q_{k-1,j}\right| \xrightarrow{p} 0$ implies that for all $k = 2, 3$, we have $\sup_{V\in\mathcal{V}}\left|\eta^n_k(V) - \eta_k(V)\right| \xrightarrow{p} 0$, $\sup_j\left|q^n_{k,j} - q_{k,j}\right| \xrightarrow{p} 0$ and $D_{j,k}(p|\eta_k)$ is continuous in p. The result now follows from the the continuous mapping theorem and lemma B.3, part ii, since $\zeta_{(R,t)}(\cdot;\eta,q,k)$ is continuous in $z_j(\cdot;\eta,q|k)$ for all t, j, k. □

We are now ready for the main result

Proof. For each (R,t), there is no open set in $[0,1]^{J+|\Pi|}$ such that every q in that set violates assumption B.1. Fix a q such that this assumption is satisfied. For this q, it is enough to show the result for fixed (R,t,k) since it belongs to a finite set.

Let

$$\mathcal{E}_k = \left\{\nu : \zeta_{(R,t)}(\nu;\eta,q,k) = 0\right\},$$

where $j = R(k)$. We first show that $|\mathcal{E}_k| \leq 2$. Since

$$\zeta_{(R,t)}(\nu;\eta,q,k) = \min\left\{z_{R(k)}\left(\frac{\nu + t_{R(k)}}{T};\eta_k, q_k\middle| k\right), -\max_{k'<k} z_{R(k')}\left(\frac{\nu + t_{R(k')}}{T};\eta_{k'}, q_{k'}\middle| k'\right)\right\},$$

where both components inside the min are monotonic, continuous functions of ν, it is easy to show that $\mathcal{E}_k$ is the union of at most two convex sets. Further, $\mathcal{E}_k$ is closed since $\zeta_{(R,t)}(\nu;\eta,q,k)$ is continuous in ν. Suppose that there is there is a k and an open interval $(\underline{\nu},\overline{\nu}) \subseteq \mathcal{E}_k$. Then, for all $\nu \in (\underline{\nu},\overline{\nu})$, $D_j\left(\frac{\nu+t_j}{T}\middle|\eta\right)$ is constant. This only occurs if $m(R,t) = 0$, which implies a contradtiction as it violates assumption B.1 at q. Since $\mathcal{E}_k \subseteq \mathbb{R}$, we have that $|\mathcal{E}_k| \leq 2$ and $\left|\cup_{k'\in\{1,..,k\}}\mathcal{E}_{k'}\right| < \infty$.

Fix $\varepsilon > 0$. Construct an open set U that covers $\cup_{k' \in \{1,..,k\}} \mathcal{E}_{k'}$ and has Lebesgue measure less than $\frac{\varepsilon}{2}$. Consider the difference,

$$
\begin{aligned}
& \left| \phi^n_{R(k)} \left((R,t), m^{n-1} \right) - \phi^\infty_{R(k)} \left((R,t), m \right) \right| \\
= & \left| \int \mathbb{E} \left[\varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| m^{n-1}, q^n, \nu \right] d\nu \bigg| \\
\leq & \int \mathbb{E} \left[\left| \varphi_{(R,t)} \left(e; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \right| m^{n-1}, q^n, \nu \right] d\nu \\
\leq & \sup_{\nu \notin U} \mathbb{E} \left[\left| \varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \right| m^{n-1}, q^n, \nu \right] P \left(\nu \notin U \right) \\
& + \sup_{\nu \in U} \mathbb{E} \left[\left| \varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \right| m^{n-1}, q^n, \nu \right] P \left(\nu \in U \right) \\
< & \sup_{\nu \notin U} \mathbb{E} \left[\left| \varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \right| m^{n-1}, q^n, \nu \right] + \frac{\varepsilon}{2}
\end{aligned}
$$

where the last inequality follows from the fact that $P \left(\nu \in U \right) < \frac{\varepsilon}{2}$ and

$$
\sup_{\nu \in U} E \left[\left| \varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \right| m^{n-1} q^n, \nu \right] \leq 1.
$$

We now show that there exists N such that for all $n > N$:

$$
\mathbb{P} \left(\sup_{\nu \notin U} E \left[\left| \varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \right| m^{n-1}, q^n, \nu \right] \geq \frac{\varepsilon}{2} \right) < \varepsilon. \qquad \text{(B.8)}
$$

This would complete the proof as it implies that

$$
\mathbb{P} \left(\left| \phi^n_{R(k)} \left((R,t), m^{n-1} \right) - \phi^\infty_{R(k)} \left((R,t), m \right) \right| > \varepsilon \right) < \varepsilon.
$$

Let $\zeta_\varepsilon = \inf_{\nu \notin U} \left| \zeta_{(R,t)} \left(\nu; \eta, q, k \right) \right|$. Note that $\zeta_\varepsilon > 0$, since $\left| \zeta_{(R,t)} \left(\nu; \eta, q, k \right) \right| > 0$ for all $\nu \notin U$ and $\zeta_{(R,t)} \left(\nu; \eta, q, k \right)$ is continuous with respect to ν. By lemma B.4, there exists N such that for all $n > N$,

$$
\mathbb{P} \left(\sup_{\nu \notin U} \left| \zeta_{(R,t)} \left(\nu; \eta, q, k \right) - \zeta_{(R,t)} \left(\nu; \eta^n, q^n, k \right) \right| > \zeta_\varepsilon \right) < \frac{\varepsilon^2}{2}.
$$

Note that for all $\nu \notin U$, $\left| \zeta \left(\nu; \eta, q, k \right) \right| \geq \zeta_\varepsilon$. Therefore for all $\nu \notin U$,

$$
\left| \varphi_{(R,t)} \left(\nu; \eta^n, q^n, k \right) - \varphi_{(R,t)} \left(\nu; \eta, q, k \right) \right| \neq 0 \Rightarrow \left| \zeta \left(\nu; \eta^n, q^n, k \right) - \zeta \left(\nu; \eta, q, k \right) \right| > \zeta_\varepsilon
$$

since the antecedent requires $\zeta_{(R,t)} \left(\nu; \eta^n, q^n, k \right) \geq 0$ and $\zeta_{(R,t)} \left(\nu; \eta, q, k \right) < -\zeta_\varepsilon$ or $\zeta_{(R,t)} \left(\nu; \eta^n, q^n, k \right) \leq$

0 and $\zeta_{(R,t)}(\nu;\eta,q,k)>\zeta_\varepsilon$. By set inclusion, for all $n>N$,

$$\mathbb{P}\left(\sup_{\nu\notin U}\left|\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)-\varphi_{(R,t)}\left(\nu;\eta,q,k\right)\right|\neq 0\right)<\frac{\varepsilon^2}{2}.$$

Since $\sup_{\nu\notin U}\left|\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)-\varphi_{(R,t)}\left(\nu;\eta,q,k\right)\right|\in\{0,1\}$, the above inequality implies that for all $n>N$,

$$\begin{aligned}
\frac{\varepsilon^2}{2} &> \mathbb{E}\left[\sup_{\nu\notin U}\left|\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)-\varphi_{(R,t)}\left(\nu;\eta,q,k\right)\right|\right]\\
&= \mathbb{E}\left(\mathbb{E}\left[\sup_{\nu\notin U}\left|\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)-\varphi_{(R,t)}\left(\nu;\eta,q,k\right)\right|\middle| m^{n-1},q^n\right]\right)\\
&\geq \mathbb{E}\left(\sup_{\nu\notin U}\mathbb{E}\left[\left|\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)-\varphi_{(R,t)}\left(\nu;\eta,q,k\right)\right|\middle| m^{n-1},q^n\right]\right),
\end{aligned}$$

where the equality follows from the law of iterated expectations and the weak inequality is well-known property of expectations of supremums. Markov inequality implies:

$$\mathbb{P}\left(\sup_{\nu\notin U}\mathbb{E}\left[\left|\varphi_{(R,t)}\left(\nu;\eta^n,q^n,k\right)-\varphi_{(R,t)}\left(\nu;\eta,q,k\right)\right|\middle| m^{n-1},q^n\right]\geq\frac{\varepsilon}{2}\right)<\varepsilon$$

which is exactly equation (B.8). □

C Identification

C.1 Equilibrium Behavior and Testable Restrictions

Our empirical methods are based on the assumption that agent behavior is described by equilibrium play. This section discusses whether this assumption is testable in principle and types of mechanisms for which it may be rejected.

Assumption C.1. *The map $\sigma_i(v_i,t_i)\to\Delta\mathcal{R}_i$ that generates the data is a symmetric limit Bayesian Nash Equilibrium.*

This assumption implies that students have consistent beliefs of the probability that they are assigned to each school in S as a function of their report $R\in\mathcal{R}$. Further, condition 1 implies that $\phi^\infty((R,t),m)$ is identified and can be consistently estimated with knowledge of the mechanism and a large sample from the measure m. Therefore, a student's choice set can be treated as known to the econometrician. This reformulation therefore transforms the problem of an student playing against a distribution of other students to a single agent problem choosing from a known set of options.

A student with utility vector v maximizes expected utility by picking lottery L_R if and only if $v \cdot L_R \geq v \cdot L_{R'}$ for all $L_{R'} \in \mathcal{L}$. The set of students that choose lottery L_R therefore have utilities that belong to the normal cone to $\mathcal{L}$ at L_R:

$$C_R = \left\{ v \in \mathbb{R}^J : \forall L_{R'} \in \mathcal{L}, v \cdot (L_R - L_{R'}) \geq 0 \right\}.$$

This observation immediately yields the result that agents maximize their utility by picking lotteries that are extremal in the set of lotteries.

Proposition C.1. *Let the distribution of indirect utilities admit a density. If $L_{R'}$ is not an extreme point of the convex hull of $\mathcal{L}$, the set of utilities v such that $v \cdot L_R \geq v \cdot L_{R'}$ for all $L_{R'} \in \mathcal{L}$ has measure zero.*

Proof. If L_R is not an extreme point of the convex hull of $\mathcal{L}$, then C_R has Lebesgue-measure zero. Since v admits a density, $\int 1\{v \in C_R\} \mathrm{d}F_V = 0$. $\square$

The result leverages the fact that ties in expected utility for any two lotteries are non-generic, agents whose behavior is consistent with limit-BNE play (typically) pick extremal lotteries. Proposition C.1 also indicates that the fraction of students with behavior that is not consistent with equilibrium play can be identified. This suggests that assumption C.1 is testable. However, we have not yet exploited the structure of assignment probabilities that result from typical assignment mechanisms in discussing testability. We now present a general sufficient condition under which observed behavior can be rationalized as equilibrium play.

Consider a **ranking mechanism** in which reports correspond to rank-orders over the available options. Therefore, a report is a function $R : \{1, \ldots, K\} \to S$ such that (i) for all $k, k' \in \{1, \ldots, K\}$, $R(k) = R(k') \neq 0 \Rightarrow k = k'$ and (ii) $R(k) = 0 \implies R(k') = 0$ if $k' > k$. Let $\mathcal{R}$ be the space of such functions.

Definition C.1. *The ranking mechanism ϕ^∞ is **rank-monotonic** for type t at m, if for all $R, R' \in \mathcal{R}$ and $k \leq K$ we have that $(R(1), \ldots, R(k-1)) = (R'(1), \ldots, R'(k-1))$ implies*

$$\phi^\infty_{R(k)}((R, t), m) \geq \phi^\infty_{R(k)}((R', t), m).$$

*Further, ϕ^∞ is **strictly rank-monotonic** for priority-type t at m if the inequality above is strict if and only if $R(k) \neq R'(k)$, and $\phi^\infty_{R(k)}((R, t), m) > 0$*

Rank-monotonicity is a natural condition that should be satisfied by many single-unit assignment mechanisms. Specifically, it requires that the assignment probability at the k-th ranked school does not depend on schools ranked below it, and that ranking a school higher weakly increases a student's chances of getting assigned to it. Under strict rank-monotonicity, ranking a school higher strictly increases the assignment probability unless this probability is zero.

We now show that in all strictly rank-monotonic ranking mechanisms, all agents that pick a report that gives them a positive probability of assignment at each of their options are behaving in a manner consistent with a limit equilibrium.[2]

Theorem C.1. *Assume that the ranking mechanism ϕ^∞ is strictly rank-monotonic at m for priority type t. The report $R \in \mathcal{R}$ corresponds to an extremal lottery $L_R \in \{\phi^\infty((R,t),m) : R \in \mathcal{R}\}$ if $\phi^\infty_{R(k)}((R,t),m) > 0$ for all k such that $\sum_{k'<k} \phi^\infty_{R(k')}((R,t),m) < 1$.*

Proof. Consider a report $R \in \mathcal{R}$ such that for any $k = 1,2,..,K$, $\sum_{k'<k} \phi^\infty_{R(k')}((R,t),m) < 1$ and $\phi^\infty_{R(k)}((R,t),m) > 0$.

Take any vector of coefficients λ such that:

$$\begin{aligned} \lambda_{\tilde{R}} &\geq 0 \text{ for every } \tilde{R} \in \mathcal{R} \\ \sum_{\tilde{R}\in\mathcal{R}} \lambda_{\tilde{R}} &= 1 \\ \phi^\infty((R,t),m) &= \sum_{\tilde{R}\in\mathcal{R}} \lambda_{\tilde{R}} \phi^\infty\left(\left(\tilde{R},t\right),m\right). \end{aligned}$$

We will show that $\lambda_R = 1$. The proof follows by induction. Consider some report $\tilde{R}$ where $R(1) \neq \tilde{R}(1)$. Strict rank-monotonicity and our assumption on R imply $\lambda_{\tilde{R}} = 0$. We have shown that for $k = 1$, $R(k') \neq \tilde{R}(k')$ for any $k' \leq k \implies \lambda_{\tilde{R}} = 0$. Suppose that this statement is true for all $l \leq k-1$ and that $\sum_{l<k} \phi^\infty_{R(l)}((R,t),m) < 1$. Take any report $\tilde{R}$ where $R(l) \neq \tilde{R}(l)$ for some $l \leq k$. If $l < k$, $\lambda_{\tilde{R}} = 0$ by the inductive hypothesis. If $l = k$, Strict rank-monotonicity and our assumption on R imply $\lambda_{\tilde{R}} = 0$. By induction, $R(l) \neq \tilde{R}(l)$ and $\sum_{l<k} \phi^\infty_{R(l)}((R,t),m) < 1 \implies \lambda_{\tilde{R}} = 0$.

Suppose that there is a $j \in S$ and $\tilde{R} \in \mathcal{R}$ such that $\phi^\infty_j((R,t),m) \neq \phi^\infty_j\left(\left(\tilde{R},t\right),m\right)$; we will show that $\lambda_{\tilde{R}} = 0$. Let $\tilde{k}$ be the minimum k such that $R(k) \neq \tilde{R}(k)$. Rank-monotonicity and the fact that either $\phi^\infty_j((R,t),m) > 0$ or $\phi^\infty_j\left(\left(\tilde{R},t\right),m\right) > 0$ imply that

$$\sum_{l<\tilde{k}} \phi^\infty_{R(l)}\left(\left(\tilde{R},t\right),m\right) = \sum_{l<\tilde{k}} \phi^\infty_{R(l)}((R,t),m) < 1.$$

Thus, our previous results imply that $\lambda_{\tilde{R}} = 0$.

[2]Strict-rank monotonicity does not rule out that two different reports result in the same lottery, e.g., $R_1 = (A,B,C)$ and $R_2 = (A,B,D)$ both result in $\phi^\infty_A = 1 - \phi^\infty_B$, and $\phi^\infty_C = \phi^\infty_D = 0$.

$\square$

The result implies that every report with non-zero assignment probabilities is rationalizable as an optimal report for a priority type if the mechanism is strictly rank-monotonic. Intuitively, this is the case because upgrading any school in the reported rank-order list strictly increases the probability of assignment and there exists a utility vector for which such a report is optimal.

Although the model has testable predictions, we do not develop a statistical test for the null hypothesis that play is consistent with optimal behavior. The technical challenge arises from testing a parameter describing the fraction of agents with non-rationalizable reports on the boundary. The statistical test would have to account for uncertainty in estimating the lotteries. We leave this for future research.

C.2 Characterization of Partially Identified Set

Consider the collection of markets

$$\mathcal{T}(\xi, z) = \{\Gamma_{ib} = (\xi_b, z_{ib}, t_{ib}, m_b, \phi_b^\infty) : (\xi_b, z_{ib}) = (\xi, z)\}.$$

The dependence of the distribution of reports m and the mechanism ϕ on the market index b indicates that we allow variation to be useful in the present exercise. We will consider results that fix (ξ, z) and therefore drop this from the notation. As a reminder, conditioning on z is without loss since it is observed, but this implies that the researcher assumes that the variation considered holds school unobservables ξ fixed.

The next result characterizes what can be learned about $F_V(v)$ from observing data from several large markets in $\mathcal{T}$. Let $N_{\mathcal{L}_\Gamma}(L) = \{v \in \mathbb{R}^J : v \cdot (L - L') \geq 0 \text{ for all } L' \in \mathcal{L}_\Gamma\}$ be the normal cone to $L \in \mathcal{L}_\Gamma$ corresponding to the set $\mathcal{L}_\Gamma$. (We switch notation from using C_R for lottery L_R for clarity since this section uses different sets $\mathcal{L}_\Gamma$, which are not explicitly referred to in the relatively compact notation, C_R.) Further, let $\mathcal{N} = \{\text{int}(N_{\mathcal{L}_\Gamma}(L))\}_{\Gamma \in \mathcal{T}, L \in \mathcal{L}_\Gamma}$ be the collection of (the interiors of) normal cones to lotteries faced by agents in the markets $\mathcal{T}$. For a collection of sets $\mathcal{N}$, let $\mathcal{D}(\mathcal{N})$ be the smallest collection of subsets of $\mathbb{R}^J$ such that

1. $\mathbb{R}^J \in \mathcal{D}(\mathcal{N})$ and $\mathcal{N} \subset \mathcal{D}(\mathcal{N})$
2. For all $N \in \mathcal{D}(\mathcal{N})$, $N^c \in \mathcal{D}(\mathcal{N})$
3. For all countable sequences of sets $N_k \in \mathcal{D}(\mathcal{N})$ such that $N_{k_1} \cap N_{k_2} = \emptyset$, $\bigcup_k N_k \in \mathcal{D}(\mathcal{N})$

The collection $\mathcal{D}(\mathcal{N})$ is sometimes called the minimal Dynkin system containing $\mathcal{N}$.

Theorem C.2. *Given $P(L \in \mathcal{L}_\Gamma | \Gamma)$ for each $\Gamma \in \mathcal{T}$ and $L \in \mathcal{L}_\Gamma$, the quantity*

$$h_D = \int 1\{v \in D\} \mathrm{d}F_V(v)$$

is identified for each $D \in \mathcal{D}(\mathcal{N})$.

Proof. The identified set of conditional distributions $F_V(v)$ is given by

$$\mathscr{F}_I = \left\{ F_V \in \mathscr{F} : \forall L \in \mathcal{L}_\Gamma \text{ and } \Gamma \in \mathcal{T}, P(L \in \mathcal{L}_\Gamma | \Gamma) = \int 1\{v \in N_{\mathcal{L}_\Gamma}(L)\} \mathrm{d}F_V(v) \right\}.$$

Note that for any two distributions F_V and $\tilde{F}_V$ in $\mathscr{F}$, the collection of sets

$$\mathscr{L}(F_V, \tilde{F}_V) = \left\{ A \in \mathcal{F} : \int 1\{v \in A\} \mathrm{d}F_V(v) = \int 1\{v \in A\} \mathrm{d}\tilde{F}_V(v) \right\}$$

is a Dynkin system for the Borel σ-algebra $\mathcal{F}$. Since $\mathcal{D}(\mathcal{N})$ is the minimal Dynkin system where all elements of $\mathscr{F}_I$ agree, $\mathcal{D}(\mathcal{N}) \subseteq \mathscr{L}(F_V, \tilde{F}_V)$ for any two elements F_V and $\tilde{F}_V$. Hence, for all $D \in \mathcal{D}(\mathcal{N})$, we have that

$$h_D = \int 1\{v \in D\} \mathrm{d}F_V(v) = \int 1\{v \in D\} \mathrm{d}\tilde{F}_V(v)$$

is therefore identified. $\square$

The result follows from basic measure theory and characterizes the features of $F_V(v)$ that are identified under such variation in choice environments without any further restrictions. In particular, with the free normalization $\|v_i\| = 1$, the result implies that the mass accumulated on the projection of the sets in $\mathcal{D}(\mathcal{N})$ on the $J-1$ dimensional sphere, $\mathbb{S}^J$, is identified. Typically, this implies only partial identification of $F_V(v)$, but extensive variation in the lotteries could result in point identification.[3]

C.3 Non-Simplicial Cones

In this section, we consider the case when the cone C_R is not spanned by linearly independent vectors. We need that there exists a report for which the normal cone satisfies the following property:

Definition C.2. *A cone C is **salient** if $v \in C \implies -v \notin C$ for all $v \neq 0$.*

[3]Specifically, the $\pi - \lambda$ theorem implies that $F_V(v)$ is identified if and only if the Dynkin-system $\mathcal{D}(\mathcal{N})$ contains a π-system that generates the Borel σ-algebra.

Our results require that the tails of the distribution of utilities are light. Formally, assume that for some $c > 0$, the density of u belongs to the set

$$\mathcal{G}_c \equiv \{g \in \mathbb{L}^1(\mathbb{R}^J) : e^{c|u|}g(u) \in \mathbb{L}^1(\mathbb{R}^J)\},$$

where $\mathbb{L}^1$ is the space of Lebesgue integrable functions.

Theorem C.3. *Assume that $g \in \mathcal{G}_c$ and there is a lottery L_R such that C_R is a salient convex cone with a non-empty interior. If $\zeta = \mathbb{R}^J$, then the distribution of utilities $F_V(v|z^1)$ is identified from*

$$h_{C_R}\left(z^1\right) = P(L_R \in \mathcal{L}|z^1).$$

The key insight is that Fourier transform of an exponential density restricted to any salient cone is non-zero on any open set. We first show a preliminary which specializes results in De Carli (1992, 2012).

Lemma C.1. *Let $f_{\varepsilon,\Gamma}(x) = \chi_\Gamma(x)\, e^{-2\pi\langle\varepsilon,x\rangle}$ for some polygonal, full-dimensional, salient, convex cone Γ and $\varepsilon \in \operatorname{int}(\Gamma^o)$, and let $\hat{f}_{\varepsilon,\Gamma}(\xi)$ be its Fourier Transform. $\hat{f}_{\varepsilon,\Gamma}$ is an entire function. Further, there is no non-empty open subset of $\mathbb{R}^J$ where $\hat{f}_{\varepsilon,\Gamma}$ is zero.*

Proof. Let $\{\Gamma_1 \ldots \Gamma_Q\}$ be a simplicial triangulation of Γ. Let V_q be a matrix $[v_{q1}, v_{q_2}, \ldots, v_{qn}]$ with the linear independent vectors that span cone Γ_q arranged as column vectors. $x \in \Gamma_q \iff x = V_q\alpha$ for some $0 \le \alpha \in \mathbb{R}^J \iff V_q^{-1}x \ge 0$. Normalize V_q so that $|\det V_q| = 1$. Let $f_{\varepsilon,\Gamma}(x) = \chi_\Gamma(x)\, e^{-2\pi\langle\varepsilon,x\rangle}$. This is an integrable function (if ε is in the dual of the cone

Γ). Consider its Fourier transform:

$$
\begin{aligned}
\hat{f}_{\varepsilon,\Gamma}(\xi) &= \int_{\Gamma} \exp\left(-2\pi i \langle \xi - i\varepsilon, x\rangle\right) dx \\
&= \sum_{Q} \int_{\Gamma_q} \exp\left(-2\pi i \langle \xi - i\varepsilon, x\rangle\right) dx \\
&= \sum_{Q} \int_{\mathbb{R}^J} \chi_{\left[x: V_q^{-1} x \geq 0\right]} \exp\left(-2\pi i \langle \xi - i\varepsilon, x\rangle\right) dx \\
&= \sum_{Q} \int_{\mathbb{R}^J_+} \exp\left(-2\pi i \langle \xi - i\varepsilon, V_q a\rangle\right) da \\
&= \sum_{Q} \int_{\mathbb{R}^J_+} \exp\left(-2\pi i \left\langle V_q' \xi - i V_q' \varepsilon, a\right\rangle\right) da \\
&= \sum_{q=1..Q} \prod_{j=1..J} \int_{\mathbb{R}_+} \exp\left(-2\pi i \left(v_{qj}' \xi - i v_{qj}' \varepsilon\right) a\right) da \\
&= \sum_{q=1..Q} \prod_{j=1..J} \int_{\mathbb{R}_+} \exp\left(-a \left[2\pi \left(v_{qj}' \varepsilon\right) + 2\pi i \left(v_{qj}' \xi\right)\right]\right) da \\
&= \sum_{q=1..Q} \prod_{j=1..J} \frac{1}{2\pi} \frac{1}{\left[\left(v_{qj}' \varepsilon\right) + i \left(v_{qj}' \xi\right)\right]},
\end{aligned}
$$

where the last equality follows from the fact that $-a2\pi(v_{qj}'\varepsilon) < 0$. Note that the closed-form expression implies that $\hat{f}_{\varepsilon,\Gamma}(\xi)$ is an entire function for every $\varepsilon \in \Gamma^o / \{0\}$. Therefore, if it is zero in an open subset of $\mathbb{R}^J$ is zero everywhere.

We now show that $\hat{f}_{\varepsilon,\Gamma}(\xi)$ is non-zero on a non-empty open set. Let K be a full-dimensional simplicial convex cone such that $\Gamma \subset K$. K exists because Γ is salient. Let V_K be the corresponding matrix for K. $\kappa_{qj} = V_K^{-1} v_{qj} > 0$ for all $q \in \{1, \ldots, Q\}$ and $j \in \{1, \ldots, J\}$. Consider $\xi = \left(V_K^{-1}\right)' \alpha$,

$$
\begin{aligned}
\hat{f}_{\varepsilon,\Gamma}\left(\left(V_K^{-1}\right)' \alpha\right) &= \left(\frac{1}{2\pi i}\right)^J \sum_{q=1,\ldots,Q} \prod_{j=1,\ldots,J} \frac{1}{\left[\left(\kappa_{qj}' \alpha\right) - i \left(v_{qj}' \varepsilon\right)\right]} \\
&= \left(\frac{1}{2\pi i}\right)^J \sum_{q=1,\ldots,Q} \prod_{j=1,\ldots,J} \frac{\left(\kappa_{qj}' \alpha\right) + \left(v_{qj}' \varepsilon\right) i}{\left[\left(\kappa_{qj}' \alpha\right)^2 + \left(v_{qj}' \varepsilon\right)^2\right]}
\end{aligned}
$$

Each term in the summation has a positive denominator and a numerator that is a polynomial function of α with positive coefficients. It follows that it is not zero everywhere, and therefore there is no open subset of $\mathbb{R}^J$ where $\hat{f}_{\varepsilon,\Gamma}$ is zero. $\square$

We are now ready to prove the main result.

Proof. For a fixed lottery L_R such that C_R is salient, define the linear operator A:

$$Ag(z) = \int_{C_R} g\,(v+z)\,\mathrm{d}v.$$

We need to show that if $A(g' - g'') = 0$ a.e. Then, $g = (g' - g'') = 0$ a.e. The proof is by contradiction.

Since the cone C_R is salient, its dual T_R has a nonempty interior. Let $\varepsilon \in \operatorname{int}(T_R)$, with $|\varepsilon|$ sufficiently small so that $g_\varepsilon(u) = g(u)e^{2\pi\langle\varepsilon,u\rangle} \in \mathbb{L}^1$. Note that $1\{u \in C_R\}e^{-2\pi\langle\varepsilon,u\rangle} \in \mathbb{L}^1$ for every $\varepsilon \in \operatorname{int}(T_R)$ because $\langle\varepsilon, u\rangle > 0$.

Since $A(g' - g'') = 0$ a.e., and $\zeta = \mathbb{R}^J$, we have that for almost all $z \in \mathbb{R}^J$,

$$Ag(z) \;=\; e^{-2\pi\langle\varepsilon,z\rangle}\int 1\{v \in C_R\}\,e^{-2\pi\langle\varepsilon,v\rangle}e^{2\pi\langle\varepsilon,v+z\rangle}g(v+z)dv = 0.$$

Since $e^{-2\pi\langle\varepsilon,z\rangle} > 0$, $Ag = 0$ for almost all $z \iff \hat{f}_{\varepsilon,C_R}(\xi)\cdot\overline{\hat{g}}_\varepsilon(\xi) = 0$, where $\hat{f}_{\varepsilon,C_R}$ is the Fourier Transform of $f_{\varepsilon,C_R}(x) = 1\{x \in C_R\}e^{-2\pi\langle\varepsilon,x\rangle}$ and $\overline{\hat{g}}_\varepsilon$ is the conjugate of the Fourier Transform of $g_\varepsilon(x)$, both continuous functions in $\mathbb{L}^1$. Since $\hat{g}_\varepsilon$ is continuous, the set where $\hat{g}_\varepsilon \neq 0$ is open. Further, since $g \neq 0$, the support of $\hat{g}_\varepsilon$ is non-empty. It follows that there is an open Z_ϵ where $\hat{g}_\varepsilon$ is different from zero, and therefore, $\hat{f}_{\varepsilon,C_R}(\xi) = 0$ for all $\xi \in Z_\epsilon$. This contradicts the fact that $\hat{f}_{\varepsilon,C_R}$ is an entire function, as shown in lemma C.1 below.

Finally, since $g(u)$ is known for almost all u, we have that $F_V(v|z^1) = \int_{-\infty}^{v-z^1} g(u)\mathrm{d}u$ is identified. □

D Estimation Appendix

D.1 Consistency of Two-Step Estimation

Theorem D.1 (Consistency)**.** *Suppose there exists a function Q_0 such that (i) θ and ϕ are elements of a compact set (ii) $\|\hat{\phi}(R,t) - \phi^\infty((R,t),m)\|_\infty \xrightarrow{p} 0$ (iii) $\sup_{\theta,\phi}|Q_n(\theta,\phi) - Q_0(\theta,\phi)| \xrightarrow{p} 0$ (iv) $Q_0(\theta,\phi)$ is jointly continuous in θ and ϕ (v) $Q_0(\theta,\phi_0)$ is uniquely minimized at θ_0, then $\hat{\theta} \xrightarrow{p} \theta_0$.*

Proof. Hypotheses (i) - (iv) and the continuous mapping theorem imply that $\sup_{\theta\in\Theta}|Q_n(\theta,\hat{\phi}) - Q_0(\theta,\phi_0)| \xrightarrow{p} 0$. The conclusion follows by (i), (v), and Newey and McFadden (1994), theorem 2.1. □

D.2 Gibbs' Sampler: Implementation Details

We specify a multivariate probit model following McCulloch and Rossi (1994) (section 4.3). The utility of student i for school j is given by

$$v_{ij} = \sum_{k=1}^{K} \delta_{jk} x_{ijk} - d_{ij} + \varepsilon_{ij} \tag{D.9}$$

and the utility of the outside option is normalized to zero: $v_{i0} = 0$. d_{ij} is the road distance between student i's home and school j; x_{ijk} student-school specific covariates; δ_{kj} are school specific parameters to be estimated. The normalization of $v_{i0} = 0$ is without loss of generality, and the scale normalization is embedded in the assumption that the coefficient on d_{ij} is -1.

The vector of error terms is distributed multivariate normal:

$$\varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iJ}) \sim N(0, \Sigma).$$

While utilities are unobserved, they are related to the observed action of student i through the requirement that the utility vector lies in the cone associated with the chosen report:

$$y_i = R \implies v_i \in C_R.$$

Let X_i be a $J \times (K \times J)$ block-diagonal matrix that is constructed placing the K-row vector covariates $x_{ij} = [x_{ijk}]_{k=1}^{K}$ in each of the J blocks; $\delta = vec\left(\{\delta_{jk}\}\right)$, a KJ-column vector; and D_i a $J \times J$ diagonal matrix with d_{ij} in the j-th position. The system in equation (D.9) can be compactly written as:

$$v_i = X_i \delta - D_i + \varepsilon_i$$

The unobserved utilities v_i are treated as unknown parameters along with δ and Σ. We specify independent prior distributions for δ and Σ:

$$\begin{aligned} p(\delta, \Sigma) &= p(\delta) p(\Sigma), \\ \delta &\sim N(\overline{\delta}, A^{-1}), \\ \Sigma &\sim IW(\nu_0, V_0), \end{aligned}$$

where IW is the inverse Wishart distribution.

The Gibbs sampler proceeds as follows:

0. Start with initial values Σ^0 and $v^0 = \{v_i^0\}_{i=1}^{N}$ so that $v_i^0 \in C_{R_i}$ for all $i = 1 \ldots N$.

1. Draw $\delta^1|v^0, \Sigma^0$ from a $N\left(\tilde{\delta}, V\right)$,

$$\begin{aligned}
V &= \left(X^{*\prime}X^{*} + A\right)^{-1}, \tilde{\delta} = V\left(X^{*\prime}v^{*} + A\overline{\delta}\right) \\
X^{*} &= \begin{bmatrix} X_1^{*} \\ \ldots \\ X_S^{*} \end{bmatrix} \\
X_i^{*\prime} &= C'X_i, v_i^{*} = C'v_i^0 \\
\Sigma^0 &= C'C
\end{aligned}$$

2. Draw $\Sigma^1|v^0, \delta^1$ from a $IW\left(\nu_0 + N, V_0 + S\right)$

$$\begin{aligned}
S &= \sum_{i=1}^{n} \varepsilon_i \varepsilon_i', \\
\varepsilon_i &= v_i^0 - X_i \delta^1
\end{aligned}$$

3. Draw $v^1|\delta^1, \Sigma^1, y$ iterating over students and schools. Take student i and the cone associated with the report y_i:

$$C_{y_i} = \left\{v \in \mathbb{R}^J : \Gamma_i v \geq 0\right\}$$

where $\Gamma_i = (L_{y_i}' - L_{R_1}', \ldots, L_{y_i}' - L_{R_{|\mathcal{R}|}'})'$.[4] For each school $j = 1 \ldots J$, draw

$$v_{ij}^1 | \left\{v_{ik}^1\right\}_{k=1}^{j-1}, \left\{v_{ik}^0\right\}_{k=j+1}^{J}, \delta^1, \Sigma^1$$

from a truncated normal $TN\left(\mu_{ij}, \sigma_{ij}^2, a_{ij}, b_{ij}\right)$, where

$$\begin{aligned}
\mu_{ij} &= \sum_{k=1}^{K} \delta_{jk}^1 x_{ijk} - d_{ij} \\
\sigma_{ij}^2 &= \Sigma_{jj}^1 - \Sigma_{j(-j)}^1 \left[\Sigma_{(-j)(-j)}^1\right]^{-1} \Sigma_{(-j)j}^1
\end{aligned}$$

and the truncation points a_{ij} and b_{ij} guarantee the draw v_{ij}^1 is such that

$$v = \left[\left\{v_{ik}^1\right\}_{k=1}^{j-1}, v_{ij}^1, \left\{v_{ik}^0\right\}_{k=j+1}^{J}\right]'$$

[4]For the specification that assumes truthful reporting, Γ_i, is a matrix that encodes the inequalities implied by the rank order list $R_i = (R_i(1), \ldots, R_i(K))$. Hence, $\Gamma_i v_i > 0$ if and only if $v_{iR_i(1)} > v_{iR_i(2)} > \ldots > v_{iR_i(K)}$, $v_{i0} < v_{iR_i(K)}$ and $v_{ij} < v_{iR(K)}$ if $j \notin R_i$.

lies in the interior of C_{y_i}. To calculate these truncation points, define A_i^j as matrix Γ_i with its jth row removed, B_i^j as its jth row and $v^j = \left[\{v_{ik}^1\}_{k=1}^{j-1}, \{v_{ik}^0\}_{k=j+1}^{J}\right]'$.

$$
\begin{aligned}
a_{ij} &= \max_{j \in \left\{j : B_i^j > 0\right\}} \frac{-A_i^j v^j}{B_i^j} \\
b_{ij} &= \min_{j \in \left\{j : B_i^j < 0\right\}} \frac{-A_i^j v^j}{B_i^j}
\end{aligned}
$$

4. Set $\Sigma^0 = \Sigma^1$ and $v^0 = v^1$, store, and repeat the steps 1-3 to obtain $(\delta^k, \Sigma^k, v^k)$ given $(\delta^{k-1}, \Sigma^{k-1}, v^{k-1})$ and the priors.

D.3 Gibbs' Sampler for the Naïve-Sophisticate Mixture Model

We extend the Gibbs' sampler described earlier to allow for two types of agents. The model assumes that naïve agents report truthfully while sophisticates pick the report that maximizes their expected utility. For a rank-order list $R = (R(1), R(2), \ldots, R(K))$ of length K, let $\tilde{C}_R$ be the region in utility space such that $v_i \in \tilde{C}_R \implies v_{iR(1)} > v_{iR(2)} > \ldots > v_{iR(K)} > v_{ij}$ for all $j \notin R_i$, and $v_{iR(K)} > v_{i0}$. Note that $\tilde{C}_R$ is a convex cone in $\mathbb{R}^J$. Let π_i be an indicator for whether a student is naïve. Therefore, the model specifies the observed report of the agent given v_i and π_i as follows:

$$
\begin{aligned}
y_i = R, \pi_i = 0 &\implies v_i \in C_R \\
y_i = R, \pi_i = 1 &\implies v_i \in \tilde{C}_R.
\end{aligned}
$$

Our Gibb's sampler uses data augmentation on π_i in addition to v_i. Let $\bar{\pi}$ be the fraction of naïve agents in the economy. We let $\bar{\pi}$ be a vector to allow for free-lunch and paid-lunch students to have differing proportions of naïve and sophisticated agents. We specify independent prior distributions for $\delta, \bar{\pi}$ and Σ:

$$
\begin{aligned}
p(\delta, \Sigma) &= p(\delta) p(\bar{\pi}) p(\Sigma), \\
\delta &\sim N(\overline{\delta}, A^{-1}), \\
\bar{\pi}_l &\sim Beta\,(\alpha_0, \beta_0) \\
\Sigma &\sim IW(\nu_0, V_0),
\end{aligned}
$$

where IW is the inverse Wishart distribution and $l \in \{\text{Paid Lunch, Free Lunch}\}$. The Gibbs' sampler proceeds as follows:

0. Start with initial values Σ^0, $\pi^0 = \{\pi_i^0\}_{i=1}^N$, and $v^0 = \{v_i^0\}_{i=1}^N$ so that $v_i^0 \in \tilde{C}_{R_i}$ for all $i = 1...N$.

1-2. Update (Σ, δ) according to steps 1-2 in Appendix D.2.

3. Update $\bar{\pi}^1|\pi^0$. For $l \in \{\text{Paid Lunch, Free Lunch}\}$, draw $\bar{\pi}_l$ from

$$Beta\left(\alpha_0 + |\mathcal{N}_l| - \sum_{i\in\mathcal{N}_l}\pi_i^0, \beta_0 + \sum_{i\in\mathcal{N}_l}\pi_i^0\right),$$

where $\mathcal{N}_l$ is the set of students in paid/free-lunch group l.

4. Draw $v^1|\delta^1, \Sigma^1, \bar{\pi}^1, y$ iterating over students and schools. For the observed report y_i for student i, consider the cones

$$\begin{aligned}\tilde{C}_{y_i} &= \left\{v \in \mathbb{R}^J : v_{y_i(1)} > v_{y_i(2)} > \ldots > v_{y_i(K)} > v_{ij} \text{ for all } j \in \{0, \ldots, J\}\backslash R_i\right\}\\ C_{y_i} &= \left\{v \in \mathbb{R}^J : \Gamma_i v \geq 0\right\},\end{aligned}$$

where $\Gamma_i = (L'_{y_i} - L'_{R_1}, \ldots, L'_{y_i} - L_{R'_{|\mathcal{R}|}})'$. Let $\bar{\pi}_i^1 = \bar{\pi}_l^1$, for l equal to the paid lunch status of i. For each school $j = 1...J$, draw

$$v_{ij}^1|\left\{v_{ik}^1\right\}_{k=1}^{j-1}, \left\{v_{ik}^0\right\}_{k=j+1}^{J}, \delta^1, \Sigma^1, \bar{\pi}_i^1$$

from a mixture of two truncated normals $TN\left(\mu_{ij}, \sigma_{ij}^2, \tilde{a}_{ij}, \tilde{b}_{ij}\right)$ and $TN\left(\mu_{ij}, \sigma_{ij}^2, a_{ij}, b_{ij}\right)$ with weights $\bar{\pi}_i^1$ and $(1-\bar{\pi}_i^1)$. $\mu_{ij}, \sigma_{ij}^2, a_{ij}$ and b_{ij} are defined as in step 3 in Appendix D.2. The truncation points $\left(\tilde{a}_{ij}, \tilde{b}_{ij}\right)$ guarantee that draws from $TN\left(\mu_{ij}, \sigma_{ij}^2, \tilde{a}_{ij}, \tilde{b}_{ij}\right)$ lay in the interior of $\tilde{C}_{y_i}$.

5. Update $\pi^1|v^1, \bar{\pi}^1$. For each student i, draw π_i^1 from a binomial distribution with parameter $\bar{\pi}_i^1$ if $v_i^1 \in C_{R_i} \cap \tilde{C}_{R_i}$. If $v_i^1 \in C_{R_i}\backslash\tilde{C}_{R_i}$, set $\pi_i^1 = 0$. If $v_i^1 \in \tilde{C}_{R_i}\backslash C_{R_i}$, set $\pi_i^1 = 1$.

6. Repeat steps 1-5 to obtain $(\delta^k, \Sigma^k, v_i^k, \pi_i^k, \bar{\pi}^k)$ given $(\delta^{k-1}, \Sigma^{k-1}, v_i^{k-1}, \pi_i^{k-1}, \bar{\pi}^{k-1})$.

We parametrize v_i as in Appendix D.2 and assume identical distributions for naïves are sophisticates.

D.4 Priors

We use very diffuse priors to minimize their influence on our estimates and as a reflection of our prior uncertainty about the values of the parameters of the model. We set the prior distribution of $\delta \sim N(\overline{\delta}, A^{-1})$

$$
\begin{aligned}
\overline{\delta} &= 0 \\
A^{-1} &= 100 \times I
\end{aligned}
$$

and the prior of $\Sigma \sim IW(\nu_0, V_0)$

$$
\begin{aligned}
\nu_0 &= 100 \\
V_0 &= I.
\end{aligned}
$$

We experimented with more diffuse priors ($A^{-1} = 200 \times I, \nu_0 = 50$) without noticeable changes in our main results.

For the mixture model, we set the prior of $\bar{\pi}_l = Beta\left(\alpha_0, \beta_0\right)$, with $\alpha_0 = \beta_0 = 1$ for $l \in$ {Paid Lunch, Free Lunch}.

D.5 Convergence Diagnosis

The Gibbs' sampler produces a markov chain with the posterior distribution of the parameters as its invariant distribution. Since the chain is ergodic, it ultimately converges to this distribution irrespective of the starting point. However, it is essential to burn-in a large set of initial draws since they are influenced by the starting point, and to check that the chains have converged. To ensure mixing, we simulate three chains of length 400,000, burn-in the first half. We monitor convergence by examining the trace plots of the various co-efficients and use Geweke's means test across and within the chains to ensure mixing. Finally, we use the Raftery-Lewis Diagnosis Test to check that the chain has been simulated for long enough to ensure that the 2.5th percentile of the vast majority of parameters are estimated within a tolerance of 0.005 with 95% probability.

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Table D.1: Estimated Preference Parameters: Sophisticated Reports

	Constant	Paid Lunch	Sibling	Black	Asian	Hispanic	Other Ethn	Spanish	Portuguese	Other Lang	Unobs. s.d.
Graham Parks	0.97	1.01	4.57	-0.21	0.02	-0.71	-0.13	-0.47	-8.04	-0.08	1.73
	[0.22]	[0.21]	[0.55]	[0.23]	[0.24]	[0.33]	[0.40]	[0.57]	[6.90]	[0.20]	[0.13]
Haggerty	1.66	0.20	4.66	-0.17	-0.46	-0.39	0.22	-1.64	-15.85	0.37	1.34
	[0.18]	[0.16]	[0.48]	[0.19]	[0.20]	[0.30]	[0.32]	[0.71]	[6.49]	[0.18]	[0.07]
Baldwin	1.82	-0.12	3.14	-0.06	0.04	-0.14	0.52	-1.51	-4.20	-0.18	0.88
	[0.12]	[0.11]	[0.32]	[0.13]	[0.13]	[0.17]	[0.23]	[0.58]	[3.04]	[0.12]	[0.06]
Morse	1.35	-0.36	3.95	0.46	0.37	-0.59	0.70	-0.84	0.29	0.37	1.17
	[0.16]	[0.13]	[0.39]	[0.15]	[0.16]	[0.26]	[0.27]	[0.48]	[1.02]	[0.14]	[0.06]
Amigos	0.50	-0.19	15.49	0.12	-0.26	1.27	0.52	0.74	-0.01	-0.76	1.41
	[0.21]	[0.17]	[4.57]	[0.19]	[0.23]	[0.23]	[0.32]	[0.33]	[1.11]	[0.21]	[0.09]
Cambridgeport	1.37	-0.33	4.93	-0.05	-0.17	-0.08	-0.04	-0.61	-6.66	0.00	1.04
	[0.13]	[0.12]	[0.74]	[0.13]	[0.15]	[0.18]	[0.26]	[0.32]	[4.58]	[0.12]	[0.06]
King Open	1.10	-0.19	5.79	0.34	0.13	0.04	0.08	-0.65	1.32	-0.24	1.20
	[0.14]	[0.12]	[0.59]	[0.13]	[0.14]	[0.18]	[0.26]	[0.32]	[0.81]	[0.13]	[0.06]
Peabody	0.97	-0.47	4.00	0.51	0.21	-0.21	-0.25	0.00	-6.17	0.00	1.49
	[0.17]	[0.15]	[0.45]	[0.17]	[0.18]	[0.26]	[0.32]	[0.44]	[5.12]	[0.16]	[0.08]
Tobin	0.60	-1.10	5.26	0.63	0.64	0.16	-0.15	0.56	-3.99	0.44	1.84
	[0.29]	[0.23]	[0.64]	[0.26]	[0.29]	[0.37]	[0.47]	[0.56]	[4.66]	[0.25]	[0.13]
Flet Mayn	-0.01	-1.90	4.67	1.03	-0.64	0.53	0.86	0.10	-4.89	0.58	1.79
	[0.26]	[0.25]	[0.60]	[0.23]	[0.36]	[0.34]	[0.48]	[0.48]	[3.12]	[0.23]	[0.13]
Kenn Long	0.95	-0.72	2.84	0.30	0.49	0.38	-0.62	0.09	-7.17	0.04	1.31
	[0.16]	[0.14]	[0.30]	[0.16]	[0.18]	[0.21]	[0.51]	[0.33]	[4.94]	[0.16]	[0.08]
MLK	0.31	-0.81	2.82	0.70	0.54	0.25	0.12	0.13	-7.46	0.32	1.51
	[0.22]	[0.16]	[0.43]	[0.18]	[0.20]	[0.25]	[0.35]	[0.39]	[5.28]	[0.17]	[0.15]
King Open Ola	-0.28	-0.54	15.53	0.45	-0.95	-2.48	-10.25	-3.74	5.43	-1.92	1.25
	[0.35]	[0.22]	[4.91]	[0.27]	[0.88]	[1.07]	[3.11]	[2.53]	[1.18]	[0.96]	[0.19]

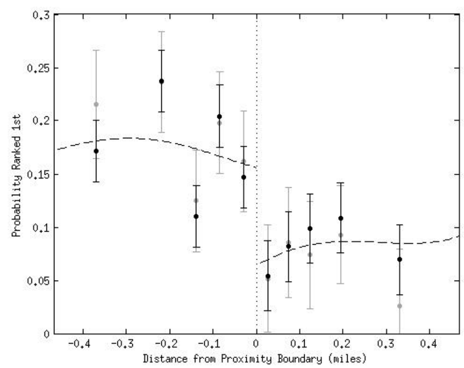
Notes: Demand estimates under the assumption of sophisticated reporting. N=1958. Excluded ethnicity is white and excluded language is english. The table reports means and standard deviations of the posterior distribution of each parameter. The distance coefficient is normalized to -1; therefore, all magnitudes are in equivalent miles

Table D.2: Estimated Preference Parameters: Truthful Reporting

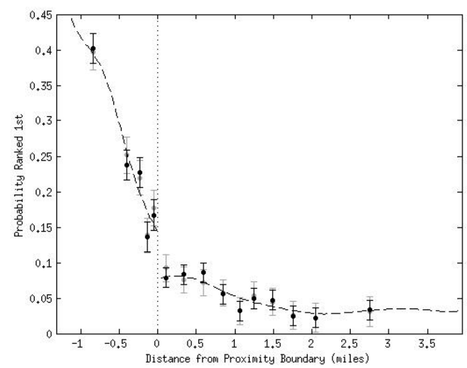
	Constant	Paid Lunch	Sibling	Black	Asian	Hispanic	Other Ethn	Spanish	Portuguese	Other Lang	Unobs s.d.
Graham Parks	2.23	1.18	3.31	-0.68	0.07	-0.60	-0.46	-0.46	-2.87	-0.14	1.77
	[0.16]	[0.14]	[0.31]	[0.15]	[0.17]	[0.22]	[0.31]	[0.33]	[0.84]	[0.15]	[0.10]
Haggerty	2.59	0.91	4.70	-0.92	-0.21	-0.76	-0.27	-0.97	-2.09	0.13	1.98
	[0.18]	[0.16]	[0.45]	[0.18]	[0.20]	[0.26]	[0.35]	[0.43]	[1.01]	[0.17]	[0.10]
Baldwin	2.33	1.06	3.56	-0.58	0.15	-0.44	-0.11	-0.92	-1.59	-0.39	1.87
	[0.16]	[0.14]	[0.32]	[0.15]	[0.17]	[0.22]	[0.31]	[0.35]	[0.63]	[0.15]	[0.11]
Morse	1.92	0.65	3.82	0.03	0.35	-0.44	-0.01	-0.42	-2.21	0.19	1.98
	[0.18]	[0.14]	[0.34]	[0.16]	[0.20]	[0.24]	[0.36]	[0.36]	[0.77]	[0.16]	[0.09]
Amigos	1.19	0.77	11.99	-0.54	-0.32	0.67	-0.28	0.65	-0.85	-0.91	1.65
	[0.19]	[0.15]	[3.01]	[0.17]	[0.21]	[0.22]	[0.35]	[0.33]	[0.56]	[0.19]	[0.10]
Cambridgeport	1.94	0.99	4.71	-0.50	-0.35	-0.47	-0.39	-0.65	-2.61	-0.05	1.71
	[0.15]	[0.13]	[0.60]	[0.15]	[0.17]	[0.21]	[0.31]	[0.32]	[0.70]	[0.15]	[0.09]
King Open	1.94	0.74	4.52	-0.11	-0.06	-0.27	-0.35	-0.61	-0.65	-0.25	1.59
	[0.15]	[0.12]	[0.49]	[0.14]	[0.17]	[0.20]	[0.32]	[0.30]	[0.45]	[0.14]	[0.08]
Peabody	1.88	0.34	3.60	-0.02	0.28	-0.37	-0.19	-0.19	-1.90	-0.02	1.69
	[0.17]	[0.14]	[0.39]	[0.16]	[0.18]	[0.23]	[0.33]	[0.35]	[0.84]	[0.15]	[0.08]
Tobin	1.79	-0.31	4.41	0.01	0.37	-0.20	-0.21	0.23	-0.45	0.17	1.85
	[0.19]	[0.16]	[0.48]	[0.18]	[0.21]	[0.28]	[0.40]	[0.41]	[0.76]	[0.18]	[0.09]
Flet Mayn	1.12	-0.36	2.99	0.61	-0.08	0.09	0.31	-0.12	-10.35	0.05	1.65
	[0.18]	[0.16]	[0.40]	[0.17]	[0.21]	[0.24]	[0.36]	[0.32]	[5.25]	[0.16]	[0.10]
Kenn Long	1.86	0.04	2.70	-0.01	0.09	-0.10	-0.44	-0.16	-0.68	-0.15	1.44
	[0.16]	[0.13]	[0.26]	[0.15]	[0.18]	[0.21]	[0.37]	[0.30]	[0.43]	[0.15]	[0.08]
MLK	1.15	0.12	2.50	0.40	0.43	0.10	-0.13	0.27	-1.04	0.15	1.52
	[0.18]	[0.13]	[0.36]	[0.15]	[0.19]	[0.22]	[0.33]	[0.30]	[0.58]	[0.15]	[0.10]
King Open Ola	-0.43	0.19	12.98	0.20	-2.11	-2.97	-3.13	-3.02	4.72	-4.67	0.67
	[0.33]	[0.19]	[3.32]	[0.22]	[1.12]	[0.63]	[2.58]	[2.14]	[0.63]	[1.63]	[0.13]

Notes: Demand estimates under the assumption of truthful reporting. N=2128. Excluded ethnicity is white and excluded language is english. The table reports means and standard deviations of the posterior distribution of each parameter. The distance coefficient is normalized to -1; therefore, all magnitudes are in equivalent miles

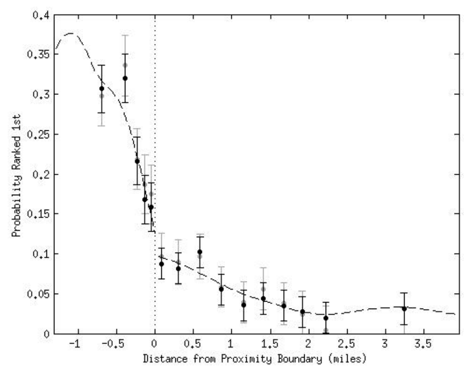
Figure D.1: Effect of Proximity Priority on Ranking Behavior



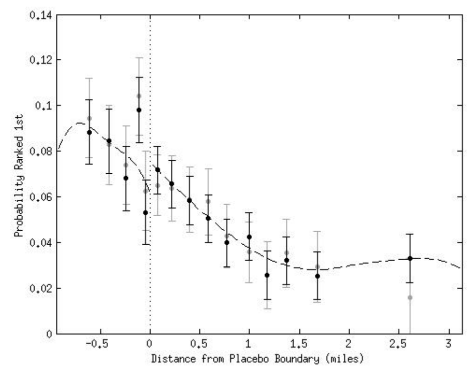
(a) Second and Third Closest Schools



(b) First Rank: Paid Lunch Students



(c) First Rank: Free Lunch Students



(d) Pacebo at Unprioritized Schools

Notes: The graphs are bin-scatter plots (based on distance) with equally sized bins on either side of the boundary. For each student, we construct a boundary distance, $\bar{d}_i$, based on her distance to the schooling options. For a given school-student pair, the horizontal axis represents $d_{ij} - \bar{d}_i$. The vertical axis is the probability that a student ranks the school in the relevant distance bin. Range plots depict 95% confidence intervals. Black plot points are based on the raw data, while the grey points control for school fixed effects. Dashed lines represent local linear fits estimated on either side of the boundary based on bandwidth selection rules recommended in Bowman and Azzalini (1997) (page 50). Panels (a) through (c) use the average distance between the second and third closest schools as the boundary. A student is given proximity priority at the schools to the left of the boundary and does not receive priority at schools to the right. Panel (a) only considers the two closest schools. Panel (d) drops the two closest schools and considers a placebo boundary at the mid-point of the fourth and fifth closest schools. All panels plot the probability that a school is ranked first. Distances as calculated using ArcGIS. Proximity priority recorded by Cambridge differs from these calculations in about 20% of the cases. Graphs are qualitatively similar when using only students with consistent calculated and recorded priorities. Details in data appendix.