Online appendices for

A MACROECONOMIC MODEL WITH FINANCIALLY CONSTRAINED PRODUCERS AND INTERMEDIARIES

Vadim Elenev
Tim Landvoigt
Stijn Van Nieuwerburgh

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A Model Appendix

A.1 Borrower-entrepreneur problem

A.1.1 Technology

The exogenous laws of motion for the TFP level Z_t^A is (lower case letters denote logs):

$$\log Z_t^A = (1-\rho_A)z^A + \rho_A \log Z_{t-1}^A + \epsilon_t^A \qquad \epsilon_t^A \sim iid\, \mathcal{N}(0, \sigma^A)$$

Denote $\mu_{ZA} = e^{z^A + \frac{(\sigma^A)^2}{2(1-\rho_A^2)}}$.

Idiosyncratic productivity of borrower-entrepreneur i at date t is denoted

$$\omega_{i,t} \sim iid \text{ Gamma}(\gamma_{0,t}, \gamma_{1,t}),$$

where the parameters $\gamma_{0,t}$ and $\gamma_{1,t}$ are chosen such that

$$\mathrm{E}(\omega_{i,t}) = 1,$$
$$\mathrm{Var}(\omega_{i,t}) = \sigma^2_{\omega,t}.$$

Individual output is

$$Y_{i,t} = \omega_{i,t} Z_t^A K_t^{1-\alpha} L_t^\alpha.$$

Aggregate production is

$$Y_t = \int_\Omega Y_{i,t} dF(\omega_i) = \int_\Omega \omega dF(\omega) Z_t^A K_t^{1-\alpha} (L_t)^\alpha = Z_t^A K_t^{1-\alpha}(L_t)^\alpha.$$

Individual producer profit is

$$\pi_{i,t} = Y_{i,t} - \sum_j w^j L^j - A_t.$$

Therefore, the default cutoff at $\pi_{i,t} = 0$ is

$$\omega_t^* = \frac{\underline{\pi} + \sum_j w_t^j L_t^j + A_t}{Y_t}. \tag{23}$$

A.1.2 Preliminaries

We start by defining some preliminaries.

Borrower Defaults

$$\Omega_A(\omega_t^*) = 1 - F_{\omega,t}(\omega_t^*)$$
$$\Omega_K(\omega_t^*) = \int_{\omega_t^*}^\infty \omega dF_{\omega,t}(\omega)$$

where $F_{\omega,t}(\cdot)$ is the CDF of $\omega_{i,t}$.

It is useful to compute the derivatives of $\Omega_K(\cdot)$ and $\Omega_A(\cdot)$:

$$\frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*} = \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} \omega f_\omega(\omega) d\omega = -\omega_t^* f_\omega(\omega_t^*),$$
$$\frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*} = \frac{\partial}{\partial \omega_t^*} \int_{\omega_t^*}^{\infty} f_\omega(\omega) d\omega = -f_\omega(\omega_t^*),$$

where $f_\omega(\cdot)$ is the p.d.f. of $\omega_{i,t}$.

Capital Adjustment Cost Let

$$\Psi(X_t, K_t^B) = \frac{\psi}{2}\left(\frac{X_t}{K_t^B} - \delta_K\right)^2 K_t^B.$$

Then partial derivatives are

$$\Psi_X(X_t, K_t^B) = \psi\left(\frac{X_t}{K_t^B} - \delta_K\right) \tag{24}$$
$$\Psi_K(X_t, K_t^B) = -\frac{\psi}{2}\left(\left(\frac{X_t}{K_t^B}\right)^2 - \delta_K^2\right) \tag{25}$$

A.1.3 Optimization Problem

We consider the producers's problem in the current period after aggregate TFP and idiosyncratic productivity shocks have been realized.

Let $\mathcal{S}_t^B = \left(Z_t^A, \sigma_{\omega,t}, W_t^I, W_t^S, B_{t-1}^G\right)$ represent state variables exogenous to the borrower-entrepreneur's decision.

Then the borrower problem is

$$V^B(K_t^B, A_t^B, \mathcal{S}_t^B) = \max_{\{C_t^B, K_{t+1}^B, X_t, A_{t+1}^B, L_t^j\}} \Big\{(1-\beta_B)\left(C_t^B\right)^{1-1/\nu} + \\ + \beta_B \mathrm{E}_t\left[\left(V^B(K_{t+1}^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\Big\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C_t^B = (1-\tau_\Pi^B)\Omega_K(\omega_t^*)Y_t + (1-\tau_t^B)w_t^B \bar{L}^B + G_t^{T,B} + p_t[X_t + \Omega_A(\omega_t^*)(1-\tilde{\delta}_K)K_t^B] \\ + q_t^m A_{t+1}^B - \Omega_A(\omega_t^*)A_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) \\ - p_t K_{t+1}^B - X_t - \Psi(X_t, K_t^B) - (1-\tau_\Pi^B)\Omega_A(\omega_t^*)\sum_{j=B,S} w_t^j L_t^j + D_t^I \tag{26}$$
$$F A_t^B \leq \Phi p_t \Omega_A(\omega_t^*)(1-\tilde{\delta}_K)K_t^B, \tag{27}$$

where we have define after-tax depreciation $\tilde{\delta}_K = (1-\tau_\Pi^B)\delta_K$.

Denote the value function and the partial derivatives of the value function as:

$$\begin{aligned} V_t^B &\equiv V(K_t^B, A_t^B, \mathcal{S}_t^B), \\ V_{A,t}^B &\equiv \frac{\partial V(K_t^B, A_t^B, \mathcal{S}_t^B)}{\partial A_t^B}, \\ V_{K,t}^B &\equiv \frac{\partial V(K_t^B, A_t^B, \mathcal{S}_t^B)}{\partial K_t^B}. \end{aligned}$$

Denote the certainty equivalent of future utility as:

$$CE_t^B = \mathrm{E}_t\left[\left(V^B(K_{t+1}^B, A_{t+1}^B, \mathcal{S}_{t+1}^B)\right)^{1-\sigma_B}\right]^{\frac{1}{1-\sigma_B}}.$$

Marginal Cost of Default Before deriving optimality conditions, it is useful to compute the marginal consumption loss due to an increased default threshold ω_t^*

$$\begin{aligned} \frac{\partial C_t^B}{\partial \omega_t^*} =& \frac{\partial \Omega_K(\omega_t^*)}{\partial \omega_t^*}(1-\tau_\Pi^B)Y_t \\ &+ \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*}\left[(1-\tilde{\delta}_K)p_t K_t^B - A_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) - (1-\tau_\Pi^B)\sum_j w_t^j L_t^j\right] \\ =& -f_\omega(\omega_t^*)\left[(1-\tau_\Pi^B)\omega_t^* Y_t + (1-\tilde{\delta}_K)p_t K_t^B - A_t^B(1-(1-\theta)\tau_\Pi^B + \delta q_t^m) - (1-\tau_\Pi^B)\sum_j w_t^j L_t^j\right] \\ =& -f_\omega(\omega_t^*)Y_t \underbrace{\left[\frac{(1-\tilde{\delta}_K)p_t K_t^B - A_t^B(\theta\tau_\Pi^B + \delta q_t^m)}{Y_t}\right]}_{=\mathcal{F}_t} \\ =& -f_\omega(\omega_t^*)Y_t\mathcal{F}_t. \end{aligned}$$

The function $\mathcal{F}_t$ has an intuitive interpretation as the marginal loss, expressed in consumption units per unit of aggregate output, to producers from an increase in the default threshold. The first term is the loss of capital due to defaulting members. The second term represents gains due to debt erased in foreclosure.

A.1.4 First-order conditions

Loans The FOC for loans A_{t+1}^B is:

$$\begin{aligned} q_t^m \frac{(u_t^B)^{1-1/\nu}}{C_t^B}(1-\beta_B)(V_t^B)^{1/\nu} = \\ \lambda_t^B F - \beta_B \mathrm{E}_t[(V_{t+1}^B)^{-\sigma_B} V_{A,t+1}^B](CE_t^B)^{\sigma_B - 1/\nu}(V_t^B)^{1/\nu} \end{aligned} \tag{28}$$

where λ_t^B is the Lagrange multiplier on the constraint in (27).

Capital Similarly, the FOC for new capital K^B_{t+1} is:

$$\begin{aligned}
p_t \frac{(1-\beta_B)(V^B_t)^{1/\nu}(u^B_t)^{1-1/\nu}}{C^B_t} &= \\
&\beta_B \mathrm{E}_t[(V^B_{t+1})^{-\sigma_B} V^B_{K,t+1}](CE^B_t)^{\sigma_B - 1/\nu}(V^B_t)^{1/\nu}
\end{aligned} \tag{29}$$

Investment The FOC for investment X_t is:

$$[1+\Psi_X(X^B_t, K^B_t) - p_t] \frac{(1-\beta_B)(U^B_t)^{1-1/\nu}(V^B_t)^{1/\nu}}{C^B_t} = 0,$$

which simplifies to

$$1+\Psi_X(X^B_t, K^B_t) = p_t. \tag{30}$$

Labor Inputs Defining $\gamma_B = 1 - \gamma_I - \gamma_S$, aggregate labor input is

$$L_t = \prod_{j=B,I,S} (L^j_t)^{\gamma_j}.$$

We further compute

$$\frac{\partial \omega^*_t}{\partial L^j_t} = \left(\frac{w^j_t}{Y_t} - \omega^*_t \frac{\mathrm{MPL}^j_t}{Y_t} \right),$$

defining the marginal product of labor of type j as

$$\mathrm{MPL}^j_t = \alpha \gamma_j Z^A_t \frac{L_t}{L^j_t} \left(\frac{K^B_t}{L_t} \right)^{1-\alpha}.$$

The FOC for labor input L^j_t is then

$$\frac{(1-\beta_B)(u^B_t)^{1-1/\nu}(V^B_t)^{1/\nu}}{C^B_t} \left[(1-\tau^B_\Pi)\Omega_K(\omega^*_t)\mathrm{MPL}^j_t - (1-\tau^B_\Pi)\Omega_A(\omega^*_t) w^j_t + \frac{\partial \omega^*_t}{\partial L^j_t} \frac{\partial C^B_t}{\partial \omega^*_t} \right] = 0,$$

which yields

$$(1-\tau^B_\Pi)\Omega_K(\omega^*_t)\mathrm{MPL}^j_t = (1-\tau^B_\Pi)\Omega_A(\omega^*_t) w^j_t + f_\omega(\omega^*_t) \left(w^j_t - \omega^*_t \mathrm{MPL}^j_t \right) \mathcal{F}_t. \tag{31}$$

A.1.5 Marginal Values of State Variables and SDF

Loans Taking the derivative of the value function with respect to A^B_t gives:

$$\begin{aligned}
V^B_{A,t} &= \left[-\left(1-(1-\theta)\tau^B_\Pi + \delta q^m_t\right) \Omega_A(\omega^*_t) + \frac{\partial \omega^*_t}{\partial A^B_t} \frac{\partial C^B_t}{\partial \omega^*_t} \right] \frac{(1-\beta_B)(u^B_t)^{1-1/\nu}(V^B_t)^{1/\nu}}{C^B_t} \\
&= -\left[\left(1-(1-\theta)\tau^B_\Pi + \delta q^m_t\right) \Omega_A(\omega^*_t) + f_\omega(\omega^*_t)\mathcal{F}_t \right] \frac{(1-\beta_B)(u^B_t)^{1-1/\nu}(V^B_t)^{1/\nu}}{C^B_t},
\end{aligned} \tag{32}$$

where we used the fact that $\frac{\partial \omega^*_t}{\partial A^B_t} = \frac{1}{Y_t}$.

Capital Taking the derivative of the value function with respect to K_t^B gives:

$$V_{K,t}^B = \Bigg[p_t \Omega_A(\omega_t^*) \left(1 - (1 - \tau_\Pi^B)\delta_K\right) + (1 - \tau_\Pi^B)(1 - \alpha)\Omega_K(\omega_t^*) Z_t^A \left(\frac{K_t^B}{L_t}\right)^{-\alpha} - \Psi_K(X_t^B, K_t^B) + \frac{\partial C_t^B}{\partial \omega_t^*}\frac{\partial \omega_t^*}{\partial K_t^B}$$
$$+ \tilde{\lambda}_t^B \Phi p_t (1 - \tilde{\delta}_K)\left(\Omega_A(\omega_t^*) + K_t^B \frac{\partial \Omega_A(\omega_t^*)}{\partial \omega_t^*}\frac{\partial \omega_t^*}{\partial K_t^B}\right)\Bigg] \frac{(1 - \beta_B)(u_t^B)^{1-1/\nu}(V_t^B)^{1/\nu}}{C_t^B},$$

where $\tilde{\lambda}_t^B$ is the original multiplier λ_t^B divided by the marginal value of wealth. Taking the derivative

$$\frac{\partial \omega_t^*}{\partial K_t^B} = -\frac{\omega_t^*}{Y_t}(1 - \alpha) Z_t^A \left(\frac{K_t^B}{L_t}\right)^{-\alpha},$$

we get

$$V_{K,t}^B = \Bigg\{p_t \Omega_A(\omega_t^*)(1 - \tilde{\delta}_K)\left(1 + \Phi \tilde{\lambda}_t^B\right) + (1 - \tau_\Pi^B)(1 - \alpha)\Omega_K(\omega_t^*) Z_t^A \left(\frac{K_t^B}{L_t}\right)^{-\alpha} - \Psi_K(X_t^B, K_t^B)$$
$$+(1 - \alpha) f_\omega(\omega_t^*)\omega_t^* \left[Z_t^A \left(\frac{K_t^B}{L_t}\right)^{-\alpha} \mathcal{F}_t + \tilde{\lambda}_t^B \Phi p_t (1 - \tilde{\delta}_K)\right]\Bigg\} \frac{(1 - \beta_B)(u_t^B)^{1-1/\nu}(V_t^B)^{1/\nu}}{C_t^B}. \quad (33)$$

SDF We can define the stochastic discount factor (SDF) from t to $t + 1$ of borrowers:

$$\mathcal{M}_{t,t+1}^B = \beta_B \left(\frac{C_{t+1}^B}{C_t^B}\right)^{-1/\nu_B} \left(\frac{V_{t+1}^B}{CE_t^B}\right)^{1/\nu_B - \sigma_B}. \quad (34)$$

A.1.6 Euler Equations

Loans Substituting in for $V_{A,t+1}^B$ in (28) and using the SDF expression, we get the recursion:

$$q_t^m = \tilde{\lambda}_t^B F + \mathrm{E}_t \left\{\mathcal{M}_{t,t+1}^B \left[\Omega_A(\omega_{t+1}^*) \left(1 - (1 - \theta)\tau_\Pi^B + \delta q_{t+1}^m\right) + f_\omega(\omega_{t+1}^*)\mathcal{F}_{t+1}\right]\right\}. \quad (35)$$

Capital Substituting in for $V_{K,t+1}^B$ and using the SDF expression, we get the recursion:

$$p_t = \mathrm{E}_t \Bigg[\mathcal{M}_{t,t+1}^B \Bigg\{p_{t+1}\Omega_A(\omega_{t+1}^*)(1 - \tilde{\delta}_K)\left(1 + \Phi \tilde{\lambda}_{t+1}^B\right) + (1 - \tau_\Pi)(1 - \alpha)\Omega_K(\omega_{t+1}^*) Z_{t+1}^A \left(\frac{K_{t+1}^B}{L_{t+1}}\right)^{-\alpha}$$
$$-\Psi_K(X_{t+1}^B, K_{t+1}^B) + (1 - \alpha) f_\omega(\omega_{t+1}^*)\omega_{t+1}^* \left(Z_{t+1}^A \left(\frac{K_{t+1}^B}{L_{t+1}}\right)^{-\alpha} \mathcal{F}_{t+1} + (1 - \tilde{\delta}_K)\Phi \tilde{\lambda}_{t+1}^B p_{t+1}\right)\Bigg\}\Bigg]. \quad (36)$$

A.2 Intermediaries

A.2.1 Aggregation

Here we show that three assumptions we make are sufficient to obtain aggregation to a representative intermediary. These assumptions are (i) that the intermediary objective is linear in the idiosyncratic profit shock $\epsilon_{t,i}$, (ii) that idiosyncratic shocks only affect the contemporaneous payout (but not net worth), and (iii) that defaulting intermediaries are replaced by new intermediaries with equity equal to that of non-defaulting intermediaries.

Denote by $w^I_{t,i}$ the beginning-of-period wealth of intermediary i which did not default. Further denote by $\mathcal{S}^I_t = (Z^A_t, \sigma_{\omega,t}, K^B_t, A^B_t, W^I_t, W^S_t, B^G_{t-1})$ all aggregate state variables exogenous to the individual intermediary problem, where W^I_t is aggregate intermediary wealth.

In this case, we can define the optimization problem of the non-defaulting intermediary with profit shock realization $\epsilon_{t,i}$ recursively as

$$\hat{V}^I_{ND}(w^I_{t,i}, \epsilon_{t,i}, \mathcal{S}^I_t) = \max_{d^I_{t,i}, B^I_{t,i}, A^I_{t+1,i}} d^I_{t,i} - \epsilon_{t,i} + \mathrm{E}_t \left[\mathcal{M}^B_{t,t+1} \max \left\{ \hat{V}^I_{ND}(w^I_{t+1,i}, \epsilon_{t+1,i}, \mathcal{S}^I_{t+1}), 0 \right\} \right] \tag{37}$$

subject to the budget constraint (13), the regulatory capital constraint (11), and the definition of wealth (10). Since the objective function is linear (assumption (i)) in the profit shock $\epsilon_{t,i}$, we can equivalently define a value function $V^I(w^I_{t,i}, \mathcal{S}^I_t) = \hat{V}^I_{ND}(w^I_{t,i}, \epsilon_{t,i}, \mathcal{S}^I_t) + \epsilon_{t,i}$, and write the objective as

$$V^I(w^I_{t,i}, \mathcal{S}^I_t) = \max_{d^I_{t,i}, B^I_{t,i}, A^I_{t+1,i}} d^I_{t,i} + \mathrm{E}_t \left[\mathcal{M}^B_{t,t+1} \max \left\{ V^I(w^I_{t+1,i}, \mathcal{S}^I_{t+1}) - \epsilon_{t+1,i}, 0 \right\} \right] \tag{38}$$

subject to the same set of constraints. Conditional on the same state variables $(w^I_{t,i}, \mathcal{S}^I_t)$, the objective functions in (37) and (38) imply the same optimal choices $(d^I_{t,i}, B^I_{t,i}, A^I_{t+1,i})$, independent of the realization of the current profit shock $\epsilon_{t,i}$. Thus conjecturing that all non-defaulting banks start the period with identical wealth $w_{t,i} = W^I_t$, these banks will also have identical wealth at the beginning of the next period, W^I_{t+1}, since idiosyncratic shocks do not affect next-period net worth directly (assumption (ii)). Hence absent default, all banks have identical wealth W^I_t.

What about defaulting banks? By construction, the realization of the profit shock is irrelevant for banks that defaulted and were reseeded with initial capital. Here we assume that equity holders (borrower households) seed all newly started banks with identical capital W^{Def}_t. Therefore, all banks newly started to replace defaulting banks are identical and solve the problem

$$V^I(W^{Def}_t, \mathcal{S}^I_t) = \max_{d^{Def}_t, B^{Def}_t, A^{Def}_{t+1}} d^{Def}_t + \mathrm{E}_t \left[\mathcal{M}^B_{t,t+1} \max \left\{ V^I(\hat{W}^{Def}_{t+1}, \mathcal{S}^I_{t+1}) - \epsilon_{t+1,i}, 0 \right\} \right], \tag{39}$$

again subject to the same set of constraints, conformably rewritten for the different choice variables. Clearly, if $W^{Def}_t = W^I_t$, which is assumption (iii), then the new banks will choose the same portfolio $(d^{Def}_t, B^{Def}_t, A^{Def}_{t+1}) = (d^I_t, B^I_t, A^I_{t+1})$ as the non-defaulting banks. This means that new banks replacing defaulted banks will also have the same wealth at the beginning of next period, W^I_{t+1}. Together, this means that all banks have the same beginning-of-period wealth W^I_t.

A.2.2 Statement of stationary problem

Wealth W^I_t is the wealth of all intermediaries after firm and intermediary bankruptcies and recapitalization of defaulting intermediaries by borrowers.

At the end of each period, all intermediaries face the following optimization problem over dividend payout and portfolio composition (see equation (12) in the main text):

$$V^I(W^I_t, \mathcal{S}^I_t) = \max_{d^I_t, B^I_t, A^I_{t+1}} d^I_t + \mathrm{E}_t \left[\mathcal{M}^B_{t,t+1} F_{\epsilon,t+1} \left(V^I_{t+1}(W^I_{t+1}, \mathcal{S}^I_{t+1}) - \epsilon^{I,-}_{t+1} \right) \right] \tag{40}$$

subject to:

$$W_t^I \geq d_t^I + \Sigma(d_t^I) + q_t^m A_{t+1}^I + (q_t^f + \tau^\Pi r_t^f - \mathrm{I}_{\{B_t^I<0\}}\kappa) B_t^I, \tag{41}$$

$$W_{t+1}^I = \left[\left(\tilde{M}_{t+1} + \Omega_A(\omega_{t+1}^*)\delta q_{t+1}^m\right) A_{t+1}^I + B_t^I\right], \tag{42}$$

$$q_t^f B_t^I \geq -\xi q_t^m A_{t+1}^I, \tag{43}$$

$$A_{t+1}^I \geq 0, \tag{44}$$

$$\mathcal{S}_{t+1}^I = h(\mathcal{S}_t^I). \tag{45}$$

For the evolution of intermediary wealth in (42), we have defined the total after-tax payoff per unit of the bond

$$\tilde{M}_{t+1} = (1-(1-\theta)\tau_\Pi^I)\Omega_A(\omega_{t+1}^*) + M_{t+1}/A_{t+1}^B,$$

where M_{t+1} is the total recovery value of bankrupt borrower firms seized by intermediaries, as defined in (9).

Since the idiosyncratic bank profit shocks are independent of the aggregate state of the economy, an individual bank's probability of continuing (i.e. not defaulting) conditional on the aggregate state, but before realization of the idiosyncratic shock is:

$$\text{Prob}\left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I > 0\right) = \text{Prob}\left(\epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)\right) = F_\epsilon\left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)\right).$$

By the law of large numbers, $F_\epsilon\left(V^I(W_t^I, \mathcal{S}_t^I)\right)$ is also the aggregate survival rate of intermediaries, i.e. $1 - F_\epsilon\left(V^I(W_t^I, \mathcal{S}_t^I)\right)$ is the intermediary default rate.

Hence we can express the intermediary problem as:

$$V_t^I(W_t^I, \mathcal{S}_t^I) = \max_{d_t^I, B_t^I, A_{t+1}^I} d_t^I + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^B F_\epsilon\left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)\right)\left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-}\right)\right].$$

The conditional expectation, $\epsilon_t^{I,-} = \mathrm{E}_\epsilon(\epsilon \,|\, \epsilon \leq V^I(W_t^I, \mathcal{S}_t^I))$, is the expected idiosyncratic loss conditional on not defaulting.

A.2.3 First-order conditions

Dividend Adjustment Cost Let

$$\Sigma(d_t^I) = \frac{\sigma^I}{2}(d_t^I - \bar{d})^2.$$

The derivative is

$$\Sigma'(d_t^I) = \sigma^I(d_t^I - \bar{d}).$$

Dividend Payout To take the FOC for dividends d_t^I, eliminate B_t^I by substituting the budget constraint into the transition law for wealth to get

$$W_{t+1}^I = (\tilde{M}_{t+1} + \delta\Omega_A(\omega_{t+1}^*)q_{t+1}^m)A_{t+1}^I + \frac{W_t^I - d_t^I - \Sigma(d_t^I) - q_t^m A_{t+1}^I}{q_t^f + \tau_\Pi^I r_t^f - \kappa}, \tag{46}$$

and for the leverage constraint

$$-\frac{W_t^I - d_t^I - \Sigma(d_t^I) - q_t^m A_{t+1}^I}{q_t^f + \tau_\Pi^I r_t^f - \kappa} q_t^f \leq \xi q_t^m A_{t+1}^I. \tag{47}$$

Now we can differentiate the objective function with respect to d_t^I

$$\frac{1}{1+\Sigma'(d_t^I)} = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa}\left[q_t^f \lambda_t^I + \mathrm{E}_t\left\{\mathcal{M}_{t,t+1}^B \frac{\partial}{\partial W_{t+1}^I}\left(F_{\epsilon,t+1}\left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-}\right)\right)\right\}\right],$$

where λ_t^I denotes the Lagrange multiplier on the leverage constraint.

To compute the derivative in the expectation, rewrite the expression as

$$F_{\epsilon,t+1}\left(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-}\right) = F_{\epsilon,t} V_t^I(W_t^I, \mathcal{S}_t^I) - \int_{-\infty}^{V_t^I(W_t^I, \mathcal{S}_t^I)} \epsilon dF_\epsilon(\epsilon).$$

Differentiating with respect to W_t^I gives (by application of Leibniz' rule)

$$V_t^I V_{W,t}^I f_{\epsilon,t} + V_{W,t}^I F_{\epsilon,t} - V_t^I V_{W,t}^I f_{\epsilon,t} = V_{W,t}^I F_{\epsilon,t}.$$

Substituting in this result, the FOC becomes

$$\frac{1}{1+\Sigma'(d_t^I)} = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa}\left[q_t^f \lambda_t^I + \mathrm{E}_t\left\{\mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1}\right\}\right].$$

Loans Using the same approach as for the dividend payout FOC, the FOC for loans A_{t+1}^I is

$$\begin{aligned}&\frac{q_t^m}{q_t^f + \tau^\Pi r_t^f - \kappa}\left[q_t^f \lambda_t^I + \mathrm{E}_t\left\{\mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1}\right\}\right] \\ &\qquad = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa}\left[\xi q_t^m \lambda_t^I + \mathrm{E}_t\left\{\mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1}\left(\tilde{M}_{t+1} + \delta\Omega_A(\omega_{t+1}^*) q_{t+1}^m\right)\right\}\right].\end{aligned}$$

Noting that the LHS is equal to the RHS of the dividend FOC above, this can be written more compactly as

$$\frac{1}{1+\Sigma'(d_t^I)} = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa}\left[\xi q_t^m \lambda_t^I + \mathrm{E}_t\left\{\mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1}\left(\tilde{M}_{t+1} + \delta\Omega_A(\omega_{t+1}^*) q_{t+1}^m\right)\right\}\right].$$

A.2.4 Marginal value of wealth and SDF

First take the envelope condition

$$V_{W,t}^I = \frac{1}{q_t^f + \tau^\Pi r_t^f - \kappa}\left[q_t^f \lambda_t^I + \mathrm{E}_t\left\{\mathcal{M}_{t,t+1}^B V_{W,t+1}^I F_{\epsilon,t+1}\right\}\right].$$

Combining this with the FOC for dividends above yields

$$V_{W,t}^I = \frac{1}{1+\Sigma'(d_t^I)}. \tag{48}$$

We can define a stochastic discount factor for intermediaries as

$$\mathcal{M}^I_{t,t+1} = \mathcal{M}^B_{t,t+1} \frac{1 + \Sigma'(d^I_t)}{1 + \Sigma'(d^I_{t+1})} F_{\epsilon,t+1}. \tag{49}$$

A.2.5 Euler Equations

Using the definition of the SDF $\mathcal{M}^I_{t,t+1}$ above, we can write the FOC for dividend payout and new loans more compactly as:

$$q^f_t + \tau^\Pi r^f_t - \kappa = q^f_t \tilde{\lambda}^I_t + \mathrm{E}_t \left[\mathcal{M}^I_{t,t+1} \right], \tag{50}$$

$$q^m_t = \xi \tilde{\lambda}^I_t q^m_t + \mathrm{E}_t \left[\mathcal{M}^I_{t,t+1} \left(\tilde{M}_{t+1} + \delta q^m_{t+1} \Omega_A(\omega^*_{t+1}) \right) \right], \tag{51}$$

where $\tilde{\lambda}^I_t$ is the original multiplier λ^I_t divided by the marginal value of wealth.

A.3 Savers

A.3.1 Statement of stationary problem

Let $\mathcal{S}^S_t = \left(Z^A_t, \sigma_{\omega,t}, K^B_t, A^B_t, W^I_t, B^G_{t-1} \right)$ be the saver's state vector capturing all exogenous state variables. The problem of the saver is:

$$V^S(W^S_t, \mathcal{S}^S_t) = \max_{\{C^S_t, B^S_t\}} \left\{ (1 - \beta_S) \left[C^S_t \right]^{1-1/\nu} + \beta_S \mathrm{E}_t \left[\left(V^S(W^S_{t+1}, \mathcal{S}^S_{t+1}) \right)^{1-\sigma_S} \right]^{\frac{1-1/\nu}{1-\sigma_S}} \right\}^{\frac{1}{1-1/\nu}}$$

subject to

$$C^S_t = (1 - \tau^S_t) w^S_t \bar{L}^S + G^{T,S}_t + W^S_t - q^f_t B^S_t \tag{52}$$

$$W^S_{t+1} = B^S_t \tag{53}$$

$$B^S_t \geq 0 \tag{54}$$

$$\mathcal{S}^S_{t+1} = h(\mathcal{S}^S_t) \tag{55}$$

As before, we will drop the arguments of the value function and denote the marginal value of wealth as:

$$V^S_t \equiv V^S_t(W^S_t, \mathcal{S}^S_t),$$

$$V^S_{W,t} \equiv \frac{\partial V^S_t(W^S_t, \mathcal{S}^S_t)}{\partial W^S_t},$$

Denote the certainty equivalent of future utility as:

$$CE^S_t = \mathrm{E}_t \left[\left(V^S(W^S_t, \mathcal{S}^S_t) \right)^{1-\sigma_S} \right]^{\frac{1}{1-\sigma_S}}.$$

A.3.2 First-order conditions

The first-order condition for the short-term bond position is:

$$q_t^f (C_t^S)^{-1/\nu}(1-\beta_S)(V_t^S)^{1/\nu} = \lambda_t^S + \beta_S \mathrm{E}_t[(V_{t+1}^S)^{-\sigma_S} V_{W,t+1}^S](CE_t^S)^{\sigma_S - 1/\nu}(V_t^S)^{1/\nu} \tag{56}$$

where λ_t^S is the Lagrange multiplier on the no-borrowing constraint (54).

A.3.3 Marginal Values of State Variables and SDF

The marginal value of saver wealth is:

$$V_{W,t}^S = (C_t^S)^{-1/\nu}(1-\beta_S)(V_t^S)^{1/\nu},. \tag{57}$$

Defining the SDF in the same fashion as we did for borrowers, we get:

$$\mathcal{M}_{t,t+1}^S = \beta_S \left(\frac{V_{t+1}^S}{CE_t^S}\right)^{1/\nu_S - \sigma_S} \left(\frac{C_{t+1}^S}{C_t^S}\right)^{-1/\nu_S}.$$

A.3.4 Euler Equations

Combining the first-order condition for short-term bonds (56) with the marginal value of wealth, and the SDF, we get the Euler equation for the short-term bond:

$$q_t^f = \tilde{\lambda}_t^S + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^S\right] \tag{58}$$

where $\tilde{\lambda}_t^S$ is the original multiplier λ_t^S divided by the marginal value of wealth.

A.4 Equilibrium

The optimality conditions describing the problem are (26), (30), (35), (36), and (31) for borrowers, (41), (50), and (51) for intermediaries, and (52) and (58) for depositors. We add complementary slackness conditions for the constraints (27) for borrowers, (43) and (44) for intermediaries, and (54) for depositors. Together with the market clearing conditions (18), (19), (20), and (21) these equations fully characterize the economy.

B Computational Method

The equilibrium of dynamic stochastic general equilibrium models is usually characterized recursively. If a stationary Markov equilibrium exists, there is a minimal set of state variables that summarizes the economy at any given point in time. Equilibrium can then be characterized using two types of functions: transition functions map today's state into probability distributions of tomorrow's state, and policy functions determine agents' decisions and prices given the current state. Brumm, Kryczka, and Kubler (2018) analyze theoretical existence properties in this class of models and discuss the literature. Perturbation-based solution methods find local approximations to these functions around the "deterministic steady-state". For applications in finance, there are often two problems with local solution methods. First, portfolio restrictions such as leverage constraints may be occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will therefore inaccurately capture nonlinear dynamics when constraints go from slack to binding. Guerrieri and Iacoviello (2015) propose a solution using local methods. Secondly, the portfolio allocation of agents across assets with different risk profiles is generally indeterminate at the non-stochastic steady state. This means that it is generally impossible to solve for equilibrium dynamics using local methods since the point around which to perturb the system is not known.

Global projection methods (Judd (1998)) avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space. Additional advantages of global nonlinear methods are greater flexibility in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs.

B.1 Solution Procedure

The projection-based solution approach used in this paper has three main steps:

Step 1. **Define approximating basis for the policy and transition functions.** To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our general solution framework provides an object-oriented MATLAB library that allows approximation of arbitrary multivariate functions using linear interpolation, splines, or polynomials. For the model in this paper, splines or polynomials of various orders achieved inferior results due to their lack of global shape preservation.

Step 2. **Iteratively solve for the unknown functions.** Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate.

Step 3. **Simulate the model for many periods using approximated functions.** Verify that the simulated time path stays within the bounds of the state space for which policy and transition functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

We will now provide a more detailed description for each step.

Step 1 The state space consists of

- two exogenous state variables $[Z_t^A, \sigma_{\omega,t}]$, and
- five endogenous state variables $[K_t^B, A_t^B, W_t^I, W_t^S, B_t^G]$.

We first discretize Z_t^A into a N^{Z_A}-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points $\{Z_j^A\}_{j=1}^{N^{Z_A}}$ and the $N^{Z_A} \times N^{Z_A}$ Markov transition matrix Π_{Z^A} between them to match the volatility and persistence of HP-detrended GDP. The dispersion of idiosyncratic productivity shocks $\sigma_{\omega,t}$ can take on two realizations $\{\sigma_{\omega,L}, \sigma_{\omega,H}\}$ as described in the calibration section. The 2 x 2 Markov transition matrix between these states is given by Π_{σ_ω}. We assume independence between both exogenous shocks. Denote the set of the $N^x = 2N^{Z_A}$ values the exogenous state variables can take on as $\mathcal{S}_x = \{Z_j^A\}_{j=1}^{N^{Z_A}} \times \{\sigma_{\omega,L}, \sigma_{\omega,H}\}$, and the associated Markov transition matrix $\Pi_x = \Pi_{Z^A} \otimes \Pi_{\sigma_\omega}$.

One endogenous state variable can be eliminated for computational purposes since its value is implied by the agents' budget constraints, conditional on any four other state variables. We eliminate saver wealth W_t^S, which can be computed as

$$W_t^S = \Omega_A(\omega_t^*)(1 + \delta q_t^m)A_t^B + M_t - W_t^I + B_t^G.$$

Our solution algorithm requires approximation of continuous functions of the endogenous state variables. Define the "true" endogenous state space of the model as follows: if each endogenous state variable $S_t \in \{K_t^B, A_t^B, W_t^I, B_t^G\}$ can take on values in a continuous and convex subset of the reals, characterized by constant state boundaries, $[\bar{S}_l, \bar{S}_u]$, then the endogenous state space $\mathcal{S}_n = [\bar{K}_l^B, \bar{K}_u^B] \times [\bar{A}_l^B, \bar{A}_u^B] \times [\bar{W}_l^I, \bar{W}_u^I] \times [\bar{B}_l^G, \bar{B}_u^G]$. The total state space is the set $\mathcal{S} = \mathcal{S}_x \times \mathcal{S}_n$.

To approximate any function $f : \mathcal{S} \to \mathcal{R}$, we form an univariate grid of (not necessarily equidistant) strictly increasing points for each endogenous state variables, i.e., we choose $\{K_j^B\}_{j=1}^{N_K}$, $\{A_k^B\}_{k=1}^{N_A}$, $\{W_m^I\}_{m=1}^{N_W}$, and $\{B_n^G\}_{n=1}^{N_G}$. These grid points are chosen to ensure that each grid covers the ergodic distribution of the economy in its dimension, and to minimize computational errors, with more details on the choice provided below. Denote the set of all endogenous-state grid points as $\hat{\mathcal{S}}_n = \{K_j^B\}_{j=1}^{N_K} \times \{A_k^B\}_{k=1}^{N_A} \times \{W_m^I\}_{m=1}^{N_W} \times \{B_n^G\}_{n=1}^{N_G}$, and the total discretized state space as $\hat{\mathcal{S}} = \mathcal{S}_x \times \hat{\mathcal{S}}_n$. This discretized state space has $N^S = N^x \cdot N^K \cdot N^A \cdot N^W \cdot N^G$ total points, where each point is a 5 x 1 vector as there are 5 distinct state variables. We can now approximate the smooth function f if we know its values $\{f_j\}_{j=1}^{N^S}$ at each point $\hat{s} \in \hat{\mathcal{S}}$, i.e. $f_j = f(\hat{s}_j)$ by multivariate linear interpolation.

Our solution method requires approximation of of three sets of functions defined on the domain of the state variables. The first set of unknown functions $\mathcal{C}_P : \mathcal{S} \to \mathcal{P} \subseteq \mathcal{R}^{N^C}$, with N^C being the number of policy variables, determines the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the prices, agents' choice variables, and the Lagrange multipliers on the portfolio constraints. Specifically, the 12 policy functions are bond prices $q^m(\mathcal{S})$, $q(\mathcal{S})$, investment $X(\mathcal{S})$, consumption $c^B(\mathcal{S})$, $c^S(\mathcal{S})$, the bank dividend $d^I(\mathcal{S})$, wages $w^B(\mathcal{S})$, $w^S(\mathcal{S})$, the Lagrange multipliers for the bank leverage constraint $\lambda^I(\mathcal{S})$ and no-shorting constraint $\mu^I(\mathcal{S})$, the multiplier for borrowers' leverage constraint $\lambda^B(\mathcal{S})$, and finally the multiplier on the savers' no-shorting constraint $\mu^S(\mathcal{S})$. There is an equal number of these unknown functions and nonlinear functional equations, to be listed under step 2 below.

The second set of functions $\mathcal{C}_T : \mathcal{S} \times \mathcal{S}_x \to \mathcal{S}_n$ determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous shocks. There is one transition function for each endogenous state variable, corresponding to the transition law for each state variable, also to be listed below in step 2.

The third set are forecasting functions $\mathcal{C}_F : \mathcal{S} \to \mathcal{F} \subseteq \mathcal{R}^{N^F}$, where N^F is the number of forecasting variables. They map the state into the set of variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with the policy functions, but include additional functions. In particular, the forecasting functions for our model are the bond price $q^m(\mathcal{S})$, investment $X(\mathcal{S})$, consumption $c^B(\mathcal{S})$, $c^S(\mathcal{S})$, the bank dividend $d^I(\mathcal{S})$, the value functions of households $V^S(\mathcal{S})$, $V^B(\mathcal{S})$, and banks $V^I(\mathcal{S})$, the wage bill $w(\mathcal{S}) = w^B(\mathcal{S}) + w^S(\mathcal{S})$, and the Lagrange multiplier on the borrowers' leverage constraint $\lambda^B(\mathcal{S})$.

Step 2 Given an initial guess $\mathcal{C}^0 = \{\mathcal{C}_P^0, \mathcal{C}_T^0, \mathcal{C}_F^0\}$, the algorithm to compute the equilibrium takes the following steps.

A. **Initialize** the algorithm by setting the current iterate $\mathcal{C}^m = \{\mathcal{C}_P^m, \mathcal{C}_T^m, \mathcal{C}_F^m\} = \{\mathcal{C}_P^0, \mathcal{C}_T^0, \mathcal{C}_F^0\}$.

B. **Compute forecasting values.** For each point in the discretized state space, $s_j \in \hat{\mathcal{S}}$, $j = 1, \ldots, N^S$, perform the steps:

   i. Evaluate the transition functions at s_j combined with each possible realization of the exogenous shocks $x_i \in \mathcal{S}_x$ to get $s_j^{'}(x_i) = \mathcal{C}_T^m(s_j, x_i)$ for $i = 1, \ldots, N^x$, which are the values of the endogenous state variables given the current state s_j and for each possible future realization of the exogenous state.

   ii. Evaluate the forecasting functions at these future state variable realizations to get $f_{i,j}^0 = \mathcal{C}_F^m\left(s_j^{'}(x_i), x_i\right)$.

   The end result is a $N^x \times N^S$ matrix $\mathscr{F}^m$, with each entry being a vector

$$f_{i,j}^m = [q_{i,j}^m, c_{i,j}^B, c_{i,j}^S, d_{i,j}^I, V_{i,j}^B, V_{i,j}^S, V_{i,j}^I, X_{i,j}, w_{i,j}, \lambda_{i,j}^B] \tag{F}$$

   of the next-period realization of the forecasting functions for current state s_j and future exogenous state x_i.

C. **Solve system of nonlinear equations.** At each point in the discretized state space, $s_j \in \hat{\mathcal{S}}$, $j = 1, \ldots, N^S$, solve the system of nonlinear equations that characterize equilibrium in the equally many "policy" variables, given the forecasting matrix $\mathscr{F}^m$ from step B. This amounts to solving a system of 12 equations in 12 unknowns

$$\hat{P}_j = [\hat{q}_j^m, \hat{q}_j, \hat{X}_j, \hat{c}_j^B, \hat{c}_j^S, \hat{d}_j^I, \hat{w}_j^B, \hat{w}_j^S, \hat{\lambda}_j^I, \hat{\mu}_j^I, \hat{\lambda}_j^B, \hat{\mu}_j^S] \tag{P}$$

at each s_j. The equations are

$$\hat{q}^m_j = \hat{\lambda}^B_j F + \mathrm{E}_{s'_{i,j}|s_j}\left\{\hat{\mathcal{M}}^B_{i,j}\left[\Omega_A(\omega^*_{i,j})\left(1-(1-\theta)\tau_\Pi + \delta q^m_{i,j}\right) + f_\omega(\omega^*_{i,j})\mathcal{F}_{i,j}\right]\right\} \tag{E1}$$

$$\hat{p}_j = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}^B_{i,j}\left\{p_{i,j}\Omega_A(\omega^*_{i,j})(1-\tilde{\delta}_K)\left(1+\Phi\lambda^B_{i,j}\right) + (1-\tau_\Pi)(1-\alpha)\Omega_K(\omega^*_{i,j})Z^A_i\left(\frac{K^B_{i,j}}{L_{i,j}}\right)^{-\alpha}\right.\right.$$
$$\left.\left. -\Psi_K(X_{i,j},K^B_{i,j}) + (1-\alpha)f_\omega(\omega^*_{i,j})\omega^*_{i,j}\left(Z^A_i\left(\frac{K^B_{i,j}}{L_{i,j}}\right)^{-\alpha}\mathcal{F}_{i,j} + (1-\tilde{\delta}_K)\Phi\lambda^B_{i,j}p_{i,j}\right)\right\}\right] \tag{E2}$$

$$(1-\tau^B_\Pi)\Omega_K(\hat{\omega}^*_j)\mathrm{M\hat{P}L}^B_j = (1-\tau^B_\Pi)\Omega_A(\hat{\omega}^*_j)\hat{w}^B_j + f_\omega(\hat{\omega}^*_j)\left(\hat{w}^B_j - \hat{\omega}^*_j\mathrm{M\hat{P}L}^B_j\right)\hat{\mathcal{F}}_j \tag{E3}$$

$$(1-\tau^B_\Pi)\Omega_K(\hat{\omega}^*_j)\mathrm{M\hat{P}L}^S_j = (1-\tau^B_\Pi)\Omega_A(\hat{\omega}^*_j)\hat{w}^S_j + f_\omega(\hat{\omega}^*_j)\left(\hat{w}^S_j - \hat{\omega}^*_j\mathrm{M\hat{P}L}^S_j\right)\hat{\mathcal{F}}_j \tag{E4}$$

$$\hat{q}^f_j + \tau^\Pi\hat{r}^f_j - \kappa = \hat{q}^f_j\hat{\lambda}^I_j + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}^I_{i,j}\right] \tag{E5}$$

$$\hat{q}^m_j = \xi\hat{\lambda}^I_j\hat{q}^m_j + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}^I_{i,j}\left(\tilde{M}_{i,j} + \delta q^m_{i,j}\Omega_A(\omega^*_{i,j})\right)\right] \tag{E6}$$

$$\hat{q}^f_j = \hat{\mu}^S_j + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}^S_{i,j}\right] \tag{E7}$$

$$\left(\Phi\hat{p}_j\Omega_A(\hat{\omega}^*_j)(1-\tilde{\delta}_K)K^B_j - F\hat{A}^B_j\right)\hat{\lambda}^B_j = 0 \tag{E8}$$

$$\left(\xi\hat{q}^m_j\hat{A}^I_j + \hat{q}^f_j\hat{B}^I_j\right)\hat{\lambda}^I_j = 0 \tag{E9}$$

$$\hat{A}^I_j\hat{\mu}^I_j = 0 \tag{E10}$$

$$\hat{B}^S_j\hat{\mu}^S_j = 0 \tag{E11}$$

$$B^G_j = \hat{B}^S_j + \hat{B}^I_j \tag{E12}$$

(E1) and (E2) are the Euler equations for borrower-entrepreneurs from (35) and (36). (E3) and (E4) are the intratemporal optimality conditions for labor demand by borrower-entrepreneurs from (31). (E5) and (E6) are the Euler equations for banks from (50) and (51). (E7) is the savers' Euler equation (58). (E8) and (E9) are the leverage constraints ((27) and (43)) for borrowers and banks, respectively. (E10) and (E11) are the no-shorting constraints ((44) and (54)) for banks and savers, respectively. Finally, (E12) is the market clearing condition for riskfree debt, (18).

Expectations are computed as weighted sums, with the weights being the probabilities of transitioning to exogenous state x_i, conditional on state s_j. Hats ($\hat{\cdot}$) in (E1) – E(12) indicate variables that are direct functions of the vector of unknowns (P). These are effectively the choice variables for the nonlinear equation solver that finds the solution to the system (E1) – (E12) at each point s_j. All variables in the expectation terms with subscript $_{i,j}$ are direct functions of the forecasting variables (F).

These values are *fixed* numbers when the system is solved, as they we pre-computed in step B. For example, the stochastic discount factors $\hat{\mathcal{M}}^h_{i,j}$, $h = B, I, S$, depend on both the solution and the forecasting vector, e.g. for savers

$$\hat{\mathcal{M}}^S_{i,j} = \beta_S\left(\frac{V^S_{i,j}}{CE^S_j}\right)^{1/\nu_S-\sigma_S}\left(\frac{c^S_{i,j}}{\hat{c}^S_j}\right)^{-1/\nu_S},$$

since they depend on future consumption and indirect utility, but also current consumption. To compute the expectation of the right-hand side of equation (E7) at point s_j, we first look up the corresponding column j in the matrix containing the forecasting values that we computed in step B, $\mathscr{F}^m$. This column contains the N^x vectors, one for each possible realization of the exogenous state, of the forecasting values defined in (F). From these vectors, we need saver consumption $c^S_{i,j}$ and the saver value function $V^S_{i,j}$. Further, we need current consumption $\hat{c}^S_j$, which is a policy variable chosen by the nonlinear equation solver. Denoting the probability of moving from current exogenous state x_j to state x_i as $\pi_{i,j}$, we compute the certainty equivalent

$$CE^S_j = \left[\sum_{x_i \,|\, x_j} \pi_{i,j} (V^S_{i,j})^{1-\sigma_S} \right]^{\frac{1}{1-\sigma_S}},$$

and then complete expectation of the RHS of (E7)

$$\mathrm{E}_{s'_{i,j}|s_j} \left[\hat{\mathcal{M}}^S_{i,j} \right] = \sum_{x_i \,|\, x_j} \pi_{i,j} \beta_S \left(\frac{V^S_{i,j}}{CE^S_j} \right)^{1/\nu_S - \sigma_S} \left(\frac{c^S_{i,j}}{\hat{c}^S_j} \right)^{-1/\nu_S}.$$

The mapping of solution and forecasting vectors (P) and (F) into the other expressions in equations (E1) – E(12) follows the same principles and is based on the definitions in model appendix A. For example, the borrower default threshold is a function of current wages and state variables based on (23)

$$\hat{\omega}^*_j = \frac{\underline{\pi} + \hat{w}^B_j L^B + \hat{w}^S_j L^S + A^B_j}{Z^A_i (K^B_j)^{1-\alpha} (L)^\alpha},$$

and the capital price is a linear function of investment from the first-order condition (30)

$$\hat{p}_j = 1 + \psi \left(\frac{\hat{X}_j}{K^B_j} - \delta_K \right).$$

The system (E1) – (E12) implicitly uses the budget constraints of all agents, and the market clearing condition for corporate debt. First, one can solve for new debt issued by borrowers from their budget constraint (26)

$$\begin{aligned}\hat{A}^B_j = \frac{1}{q^m_j} \Big[\hat{c}^B_j - \Big(& (1-\tau^B_\Pi)\Omega_K(\hat{\omega}^*_j)\hat{Y}_j + (1-\tau^B)\hat{w}^B_j \bar{L}^B + \hat{G}^{T,B}_j + \hat{p}_j[\hat{X}_j + \Omega_A(\hat{\omega}^*_j)(1-\tilde{\delta}_K)K^B_j] \\ & - \Omega_A(\hat{\omega}^*_j) A^B_j (1 - (1-\theta)\tau^B_\Pi + \delta \hat{q}^m_j) \\ & - \hat{p}_j \hat{K}^B_j - \hat{X}_j - \Psi(\hat{X}_j, K^B_j) - (1-\tau^B_\Pi)\Omega_A(\hat{\omega}^*_t) \sum_{n=B,S} \hat{w}^j_j \hat{L}^n_t + \hat{D}^I_j \Big) \Big].\end{aligned}$$

All expressions on the right-hand side of the above equation are direct functions of the state or policy variables. Market clearing for corporate debt implies $\hat{A}^I_j = \hat{A}^B_j$, and thus deposits issued by banks follow from their budget constraint (46)

$$\hat{B}^I_j = \frac{1}{\hat{q}^f_j + \tau^\Pi \hat{r}^f_j - \kappa} \left[W^I_j - \left(\hat{d}^I_j + \Sigma(\hat{d}^I_j) + \hat{q}^m_j \hat{A}^I_j \right) \right].$$

Similarly, deposits bought by savers follow from their budget constraint (52)

$$\hat{B}_j^S = \frac{1}{\hat{q}_j^f}\left[\hat{c}_j^S - \left((1-\tau^S)\hat{w}_j^S \bar{L}^S + \hat{G}_j^{T,S} + W_j^S\right)\right].$$

Note that we could exploit the linearity of the market clearing condition in (E12) to eliminate one more policy variable, $\hat{c}_j^S$, from the system analytically. However, in our experience the algorithm is more robust when we explicitly include consumption of all agents as policy variables, and ensure that these variables stay strictly positive (as required with power utility) when solving the system. To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton's method, using policy functions $\mathcal{C}_P^m$ as initial guess. More on these issues in subsection B.2 below.

The final output of this step is a $N^S \times 12$ matrix $\mathscr{P}^{m+1}$, where each row is the solution vector $\hat{P}_j$ that solves the system (E1) – E(12) at point s_j.

D. **Update forecasting, transition and policy functions.** Given the policy matrix $\mathscr{P}^{m+1}$ from step B, update the policy function directly to get $\mathcal{C}_P^{m+1}$. All forecasting functions with the exception of the value functions are also equivalent to policy functions. Value functions are updated based on the recursive definitions

$$\hat{V}_j^S = \left\{(1-\beta_S)\left[\hat{c}_j^S\right]^{1-1/\nu} + \beta_S \mathrm{E}_{s'_{i,j}|s_j}\left[\left(V_{i,j}^S\right)^{1-\sigma_S}\right]^{\frac{1-1/\nu}{1-\sigma_S}}\right\}^{\frac{1}{1-1/\nu}} \tag{V1}$$

$$\hat{V}_j^B = \left\{(1-\beta_B)\left[\hat{c}_j^B\right]^{1-1/\nu} + \beta_B \mathrm{E}_{s'_{i,j}|s_j}\left[\left(V_{i,j}^B\right)^{1-\sigma_B}\right]^{\frac{1-1/\nu}{1-\sigma_B}}\right\}^{\frac{1}{1-1/\nu}} \tag{V2}$$

$$\hat{V}_j^I = \hat{d}_j^I + \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}_{i,j}^B F_{\epsilon,i,j}\left(V_{i,j}^I - \epsilon_{i,j}^{I,-}\right)\right], \tag{V3}$$

using the same notation as defined above under step C. Note that each value function combines current solutions from $\mathscr{P}^{m+1}$ (step C) for consumption and dividend with forecasting values from $\mathscr{F}^m$ (step B). Using these updated value functions, we get $\hat{\mathcal{C}}_F^{m+1}$.

Finally, update transition functions for the endogenous state variables using the following laws of motion, for current state s_j and future exogenous state x_i as defined above:

$$K_{i,j}^B = (1-\delta_K)K_j^B + \hat{X}_j \tag{T1}$$

$$A_{i,j}^B = \hat{A}_j^B \tag{T2}$$

$$W_{i,j}^I = \left(\tilde{M}_{i,j} + \delta q_{i,j}^m \Omega_A(\omega_{i,j}^*)\right)\hat{A}_j^I + \hat{B}_j^I \tag{T3}$$

$$B_{i,j}^G = \frac{1}{\hat{q}_j^f}\left(B_j^G + \hat{G}_j - \hat{T}_j\right). \tag{T4}$$

(T1) is simply the law of motion for aggregate capital, and (T2) follows trivially from the direct mapping of policy into state variable for borrower debt. (T3) is the law of motion for bank net worth (42), which again combines inputs from old forecasting functions $\mathscr{F}^m$ and new policy solutions $\mathscr{P}^{m+1}$. (T4) is the government budget constraint (17). Updating according to (T1) – (T4) gives the next set of functions $\hat{\mathcal{C}}_T^{m+1}$.

E. **Check convergence.** Compute distance measures $\Delta_F = ||\mathcal{C}_F^{m+1} - \mathcal{C}_F^m||$ and $\Delta_T = ||\mathcal{C}_T^{m+1} - \mathcal{C}_T F^m||$. If $\Delta_F < \mathrm{Tol}_F$ and $\Delta_T < \mathrm{Tol}_T$, stop and use $\mathcal{C}^{m+1}$ as approximate solution. Otherwise reset policy functions to the next iterate i.e. $\mathscr{P}^m \to \mathscr{P}^{m+1}$ and reset forecasting and transition functions to a convex combination of their previous and updated values i.e. $\mathcal{C}^m \to \mathcal{C}^{m+1} = D \times \mathcal{C}^m + (1-D) \times \hat{\mathcal{C}}^{m+1}$, where D is a dampening parameter set to a value between 0 and 1 to reduce oscillation in function values in successive iterations. Next, go to step B.

Step 3 Using the numerical solution $\mathcal{C}^* = \mathcal{C}^{m+1}$ from step 2, we simulate the economy for $\bar{T} = T_{ini} + T$ period. Since the exogenous shocks follow a discrete-time Markov chain with transition matrix Π_x, we can simulate the chain given any initial state x_0 using $\bar{T}-1$ uniform random numbers based on standard techniques (we fix the seed of the random number generator to preserve comparability across experiments). Using the simulated path $\{x_t\}_{t=1}^{\bar{T}}$, we can simulate the associated path of the endogenous state variables given initial state $s_0 = [x_0, K_0^B, A_0^B, W_0^I, W_0^S, B_0^G]$ by evaluating the transition functions

$$[K_{t+1}^B, A_{t+1}^B, W_{t+1}^I, W_{t+1}^S, B_{t+1}^G] = \mathcal{C}_T^*(s_t, x_{t+1}),$$

to obtain a complete simulated path of model state variables $\{s_t\}_{t=1}^{\bar{T}}$. To remove any effect of the initial conditions, we discard the first T_{ini} points. We then also evaluate the policy and forecasting functions along the simulated sample path to obtain a complete sample path $\{s_t, P_t, f_t\}_{t=1}^{\bar{T}}$.

To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions, and restart the procedure at step 1. Secondly, we compute relative errors for all equations of the system (E1) – E(12) and the transition functions (T1) – (T4) along the simulated path. For equations involving expectations (such as (E1)), this requires evaluating the transition and forecasting function as in step 2B at the current state s_t. For each equation, we divide both sides by a sensibly chosen endogenous quantity to yield "relative" errors; e.g., for (E1) we compute

$$1 - \frac{1}{\hat{q}_j^m}\left(\hat{\lambda}_j^B F + \mathrm{E}_{s'_{i,j}|s_j}\left\{\hat{\mathcal{M}}_{i,j}^B\left[\Omega_A(\omega_{i,j}^*)\left(1-(1-\theta)\tau_\Pi + \delta q_{i,j}^m\right) + f_\omega(\omega_{i,j}^*)\mathcal{F}_{i,j}\right]\right\}\right),$$

using the same notation as in step 2B. These errors are small by construction when calculated at the points of the discretized state grid $\hat{\mathcal{S}}$, since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points, at which the functions $\mathcal{C}^*$ are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

B.2 Implementation

Solving the system of equations. We solve system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB's fsolve). This nonlinear equation solver uses a variant of Newton's method to find a "zero" of the system. We employ several simple modifications of the system (E1) – E(12) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solver are notoriously bad at dealing with complementary slackness conditions associated with constraint, such as (E8) – E(11). Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose. For example, consider the bank's Euler Equation for risk-free bonds and the Kuhn-Tucker condition for its leverage constraint:

$$\hat{q}^f_j(1-\hat{\lambda}^I_j) + \tau^\Pi \hat{r}^f_j - \kappa = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}^I_{i,j}\right]$$
$$\left(\xi \hat{q}^m_j \hat{A}^I_j + \hat{q}^f_j \hat{B}^I_j\right)\hat{\lambda}^I_j = 0$$

Now define an auxiliary variable $h_j \in \mathcal{R}$ and two functions of this variable, such that $\hat{\lambda}^{I,+}_j = \max\{0, h_j\}^3$ and $\hat{\lambda}^{I,-}_j = \max\{0, -h_j\}^3$. Clearly, if $h_j < 0$, then $\hat{\lambda}^{I,+}_j = 0$ and $\hat{\lambda}^{I,-}_j > 0$, and vice versa for $h_j > 0$. Using these definitions, the two equations above can be transformed to:

$$\hat{q}^f_j(1-\hat{\lambda}^{I,+}_j) + \tau^\Pi \hat{r}^f_j - \kappa = \mathrm{E}_{s'_{i,j}|s_j}\left[\hat{\mathcal{M}}^I_{i,j}\right] \tag{K1}$$
$$\xi \hat{q}^m_j \hat{A}^I_j + \hat{q}^f_j \hat{B}^I_j - \hat{\lambda}^{I,-}_j = 0 \tag{K2}$$

The solution variable for the nonlinear equation solver corresponding to the multiplier is h_j. The solver can choose positive h_j to make the constraint binding ($\hat{\lambda}^{I,-}_j = 0$), in which case $\hat{\lambda}^{I,+}_j$ takes on the value of the Lagrange multiplier. Or the solver can choose negative h_j to make the constraint non-binding ($\hat{\lambda}^{I,+}_j = 0$), in which case $\hat{\lambda}^{I,-}_j$ can take on any value that makes (K2) hold.

Similarly, certain solution variables are restricted to positive values due to the economic structure of the problem. For example, with power utility consumption must be positive. To avoid that the solver tries out negative consumption values (and thus utility becomes ill-defined), we use $\log(\hat{c}^n_j)$, $n = B, S$, as solution variable for the solver. This means the solver can make consumption arbitrarily small, but not negative.

The nonlinear equation solver needs to compute the Jacobian of the system at each step. Numerical central-difference (forward-difference) approximation of the Jacobian can be inaccurate and is computationally costly because it requires $2N+1$ ($N+1$) evaluations of the system, with N being the number of variables, whereas analytically computed Jacobians are exact and require only one evaluation. We follow Elenev (2016) in "pre-computing" all forecasting functions in step 2B of the algorithm, so that we can calculate the Jacobian of the system analytically. To do so, we employ the Symbolic Math Toolbox in MATLAB, passing the analytic Jacobian to fsolve at the beginning of step 2C. This greatly speeds up calculations.

Grid configuration. We choose to include borrower wealth W^B_t as state variable instead of borrower debt A^B_t, defined as

$$W^B_t = p_t K^B_t - q^m_t A^B_t,$$

such that the total set of endogenous state variables is $[K^B_t, W^B_t, W^I_t, W^S_t, B^G_t]$. Keeping track of borrower wealth W^B_t instead of debt A^B_t turns out to have better properties for numerical approximation and the same information content. The reason is that borrower wealth is much more stable in the dynamics of the model than borrower debt, since borrower debt and capital are strongly correlated reflecting borrowers' optimal investment and leverage choices. Recall that one endogenous state variable can be eliminated because of the adding-up property of budget constraints in combination with market clearing. We choose to eliminate saver wealth W^S. The grid points in each state dimension are as follows

- **Z^A:** We discretize Z^A_t into a 5-state Markov chain using the Rouwenhorst (1995) method. The procedure chooses the productivity grid points $\{Z^A_j\}_{j=1}^5$ and the 5×5 Markov transition matrix Π_{Z^A} between them to match the volatility and persistence of HP-detrended GDP. This yields the possible realizations: $[0.957, 0.978, 1.000, 1.022, 1.045]$.

- σ_ω**:** $[0.095, 0.175]$ (see calibration)
- K^B**:** $[1.84, 1.98, 2.05, 2.10, 2.26, 2.45, 2.70]$
- W^B**:** $[1.00, 1.16, 1.20, 1.23, 1.24, 1.285, 1.33, 1.35, 1.375, 1.41, 1.50, 1.60, 1.70]$
- W^I**:**

$$[-0.02, -0.01, 0, 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.045, 0.05, 0.055, 0.06, 0.065, \dots \\ \dots, 0.07, 0.075, 0.08, 0.10, 0.125, 0.15, 0.25, 0.3, 0.38]$$

- B^G**:** $[-0.2000, -0.02, 0, 0.1833, 0.4667, 0.7500, 1.0333, 1.3167, 1.4000]$

The total state space grid has 204,750 points. As pointed out by several previous studies such as Kubler and Schmedders (2003), portfolio constraints lead to additional computational challenges since portfolio policies may not be smooth functions of state variables due to occasionally binding constraints. Hence we cluster grid points in areas of the state space where constraints transition from slack to binding. Our policy functions are particularly nonlinear in bank net worth W_t^I, since the status of the bank leverage constraint (binding or not binding) depends predominantly on this state variable. To achieve acceptable accuracy, we have to specify a very dense grid for W^I, as can be seen above. Also note that the lower end of the W^I grid includes some negative values. Negative realizations of W^I can occur in severe financial crisis episodes. Recall that W^I is the beginning-of-period net worth of all banks. Depending on the realization of their idiosyncratic payout shock, banks decide whether or not to default. Thus the model contains two reasons why banks may not default despite initial negative net worth: (i) positive idiosyncratic shocks, and (ii) positive franchise value. The lower bound of W^I needs to be low enough such that bank net worth is not artificially truncated during crises, but it must not be so low that, given such low initial net worth, banks cannot be recapitalized to get back to positive net worth. Thus the "right" lower bound depends on the strength of the equity issuance cost and other parameters. Finding the right value for the lower bound is a matter of experimentation.

Generating an initial guess and iteration scheme. To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic "steady-state" of the model under the assumption that the bank leverage constraint is binding and government debt/GDP is 40%. We then initialize all functions to their steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, the steady state delivers a good enough initial guess.

In case the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these "failed" points using as initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually find solutions at all points.

To further speed up computation time, we run the initial 100 iterations with a coarser state space grid (19,500 points total). After these iterations, the algorithm is usually close to convergence; however, the accuracy during simulation would be too low. Therefore, we initialize the finer (final) solution grid using the policy, forecasting, and transition function obtained after 100 coarse grid iterations. We then run the algorithm for at most 40 more iterations on the fine grid.

To determine convergence, we check absolute errors in the value functions of households and banks, (V1) – V(3). Out of all functions we approximate during the solution procedure, these exhibit the slowest convergence. We stop the solution algorithm when the maximum absolute difference between two iterations, and for all three functions and all points in the state space, falls below 1e-3 and the mean distance falls below 1e-4. For appropriately chosen grid boundaries, the algorithm will converge within the final 40 iterations.

In some cases, our grid boundaries are wider than necessary, in the sense that the simulated economy never visits the areas near the boundary on its equilibrium path. Local convergence in those areas is usually very slow, but not relevant for the equilibrium path of the economy. If the algorithm has not achieved convergence after the 40 additional iterations on the fine grid, we nonetheless stop the procedure and simulate the economy. If the resulting simulation produces low relative errors (see step 3 of the solution procedure), we accept the solution. After the 140 iterations described above, our simulated model economies either achieve acceptable accuracy in relative errors, or if not, the cause is a badly configured state grid. In the latter case, we need to improve the grid and restart the solution procedure. Additional iterations, beyond 100 on the coarse and 40 on the fine grid, do not change any statistics of the simulated equilibrium path for any of the simulations we report.

We implement the algorithm in MATLAB and run the code on a high-performance computing (HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 28 CPU cores of a single HPC node. From computing the initial guess and analytic Jacobian to simulating the solved model, the total running time for the benchmark calibration is about 2 hours and 40 minutes. Calibrations that exhibit more financial fragility and/or macro volatility converge up to 15% slower.

Simulation. To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 500 periods. The starting point of the simulation is the ergodic mean of the state variables. As described in detail above, we verify that the simulated time path stays within the bounds of the state space for which the policy functions were computed. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse response function (IRF) graphs, we simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution non-parametrically. We fix the initial exogenous shock realization to mean productivity ($Z^A = 1$) and low uncertainty ($\sigma_{\omega,low}$). The "impulse" in the second period is either only a bad productivity shock ($Z^A = 0.978$) for non-financial recessions, or both low Z^A and a high uncertainty shock ($\sigma_{\omega,hi}$) for financial recessions. For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition.

B.3 Evaluating the solution

Equation errors. Our main measure to assess the accuracy of the solution are relative equation errors calculated as described in step 3 of the solution procedure. Table 6 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 100^{th} percentiles during the 10,000 period simulation of the model. Median and 75th percentile errors are small for all equations. Equations (E5) – (E6) and (E9) have elevated maximum errors. These errors are caused by a bad approximation of the bank's Lagrange multiplier λ^I in rarely occurring states. It is possible to reduce these errors by placing more grid points in those areas of the state space. In our experience, adding points to eliminate the tail errors has little to no effect on any of the results we report. Since it increases computation

Table 6: Computational Errors

Equation		Percentile				
		50th	75th	95th	99th	Max
E1	(35)	0.0004	0.0009	0.0019	0.0033	0.0316
E2	(36)	0.0003	0.0005	0.0011	0.0017	0.0051
E3	(31), B	0.0001	0.0002	0.0002	0.0003	0.0004
E4	(31), S	0.0001	0.0002	0.0002	0.0003	0.0004
E5	(50)	0.0038	0.0079	0.0140	0.0180	0.1302
E6	(51)	0.0042	0.0091	0.0185	0.0212	0.1389
E7	(58)	0.0007	0.0014	0.0026	0.0036	0.0119
E8	(27)	0.0041	0.0065	0.0137	0.0228	0.0581
E9	(43)	0.0005	0.0011	0.0027	0.0048	0.1069
E10	(44)	0.0002	0.0006	0.0010	0.0015	0.0079
E11	(54)	0.0055	0.0080	0.0181	0.0288	0.0783
E12	(18)	0.0005	0.0006	0.0007	0.0009	0.0369

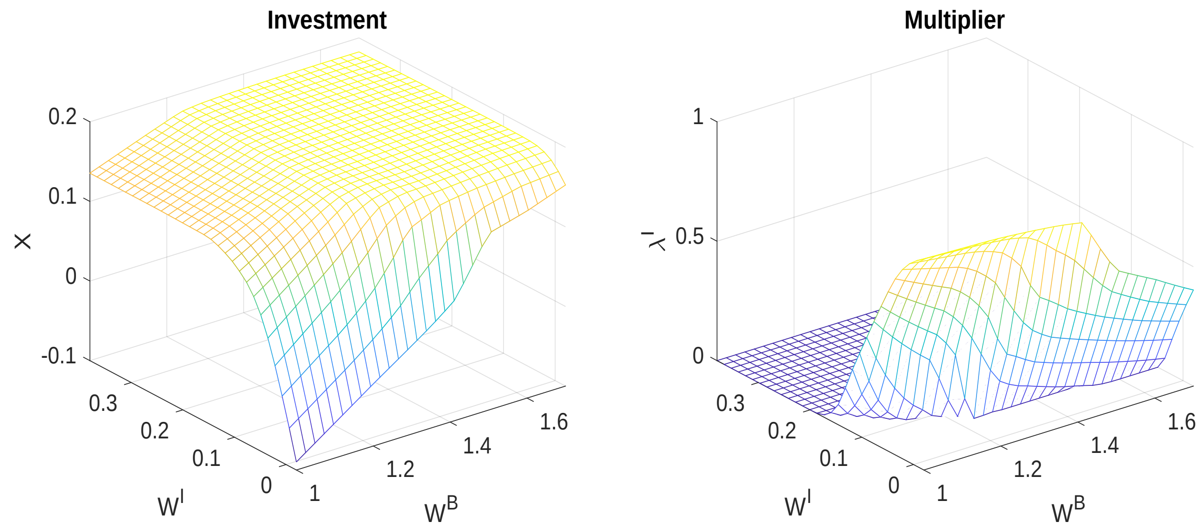
The table reports median, 75th percentile, 95th percentile, 99th percentile, and maximum absolute value errors, evaluated at state space points from a 10,000 period simulation of the benchmark model. Each row contains errors for the respective equation of the nonlinear system (E1) – (E12) listed in step 2 of the solution procedure. The table's second column contains corresponding equation numbers in the main text and appendix A.

times nonetheless, we chose the current grid configuration.

Policy function plots. We further visually inspect policy functions to gauge whether the approximated functions have the smoothness and monotonicity properties implied by our choices of utility and adjustment cost functions. Such plots also allow us to see the effect of binding constraints on prices and quantities. For example, figure 7 shows investment by firms and the Lagrange multiplier on the bank's leverage constraint. It is obvious from the graphs that a binding intermediary constraint restricts investment. The intermediary constraint becomes binding for low values of intermediary net worth. Further note the interaction with borrower-entrepreneur net worth: holding fixed intermediary net worth, the constraint is more likely to become biding for low borrower wealth.

State space histogram plots. We also create histogram plots for the endogenous state variables, overlaid with the placement of grid points. These types of plots allow us to check that the simulated path of the economy does not violate the state grid boundaries. It further helps us to determine where to place grid points. Histogram plots for the benchmark economy are in figure 8.

Figure 7: Plot of optimal investment and Lagrange multiplier on bank leverage constraint



The left panel plots investment by borrower-entrepreneurs as function of borrower-entrepreneur wealth W^B and bank net worth W^I. The right panel plots the Lagrange multiplier on the bank leverage constraint for the same state variables. Both plots are for the benchmark economy. The other state variables are fixed to the following values: $Z^A = 1$, $\sigma_\omega = \bar{\sigma}_{\omega,L}$, $K^B = 2.3$, $B^G = 0.5$.

C Calibration Appendix

C.1 Parameter Sensitivity Analysis

In a complex, non-linear structural general equilibrium model like ours, it is often difficult to see precisely which features of the data drive the ultimate results. This appendix follows the approach advocated by Andrews, Gentzkow, and Shapiro (2017) to report how key moments are affected by changes in the model's key parameters, in the hope of improving the transparency of the results. Structural identification of parameters and sensitivity of results are two sides of the same coin.

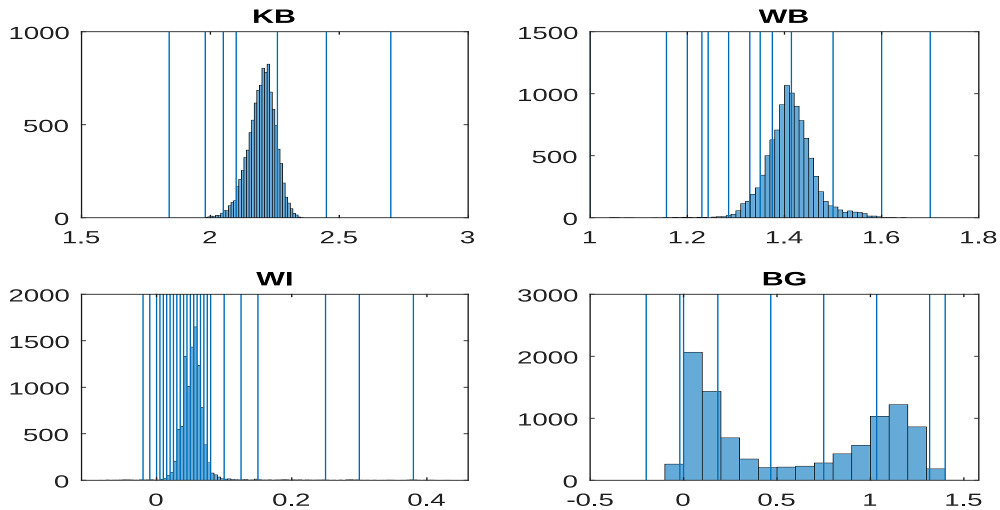
Consider a generic vector of moments $\mathbf{m}$ which depends on a generic parameter vector θ. Let ι_i be a selector vector of the same length as θ taking a value of 1 in the i'th position and zero elsewhere. Denote the parameter choices in the benchmark calibration by a superscript b. For each parameter θ_i, we solve the model once for $\theta^b \circ e^{\iota_i \varepsilon}$ and once for $\theta^b \circ e^{-\iota_i \varepsilon}$. We then report the symmetric finite difference:

$$\frac{\log\left(\mathbf{m}(\theta^{\mathbf{b}} \circ \mathbf{e}^{\iota_\mathbf{i}\varepsilon})\right) - \log\left(\mathbf{m}(\theta^{\mathbf{b}} \circ \mathbf{e}^{\iota_\mathbf{i}\varepsilon})\right)}{2\varepsilon}$$

We set $\varepsilon = .01$, or 1% of the benchmark parameter value. The resulting quantities are elasticities of moments to structural parameters.

To avoid excessive reporting, we focus on 8 key parameters and 13 key moments. The parameters are: (1) the equity adjustment cost parameter σ_I, (2) the cost of default parameter ζ, (3) the mortgage duration parameter δ, (4) the capital adjustment cost parameter ϕ, (5) the idiosyncratic bank profit risk σ_ε, (6) the dispersion of TFP shocks in the normal state $\sigma_{\omega,L}$, and (7) the dispersion of TFP shocks in the crisis state $\sigma_{\omega,H}$, and (8) the risk aversion coefficient (of both borrowers and savers, $\sigma_B = \sigma_S$). Each panel of Figure 9 lists the same 14 moments and shows the elasticity of the moments to one of the eight parameters. As an aside, the movements in the excess bond return in response to

Figure 8: Histogram plots of endogenous state variables



The plots show histograms for capital and borrower-entrepreneur wealth in the top row, and intermediary net worth and and government debt in the bottom row, for the 10,000 period simulation of the benchmark economy. The vertical lines indicate the values of grid points.

multiple parameters appear to be large but they are only large relative to a fairly small baseline level of excess returns of 30 basis points per year. For consistency, we report percentage changes, which are unit-free, in every moment.

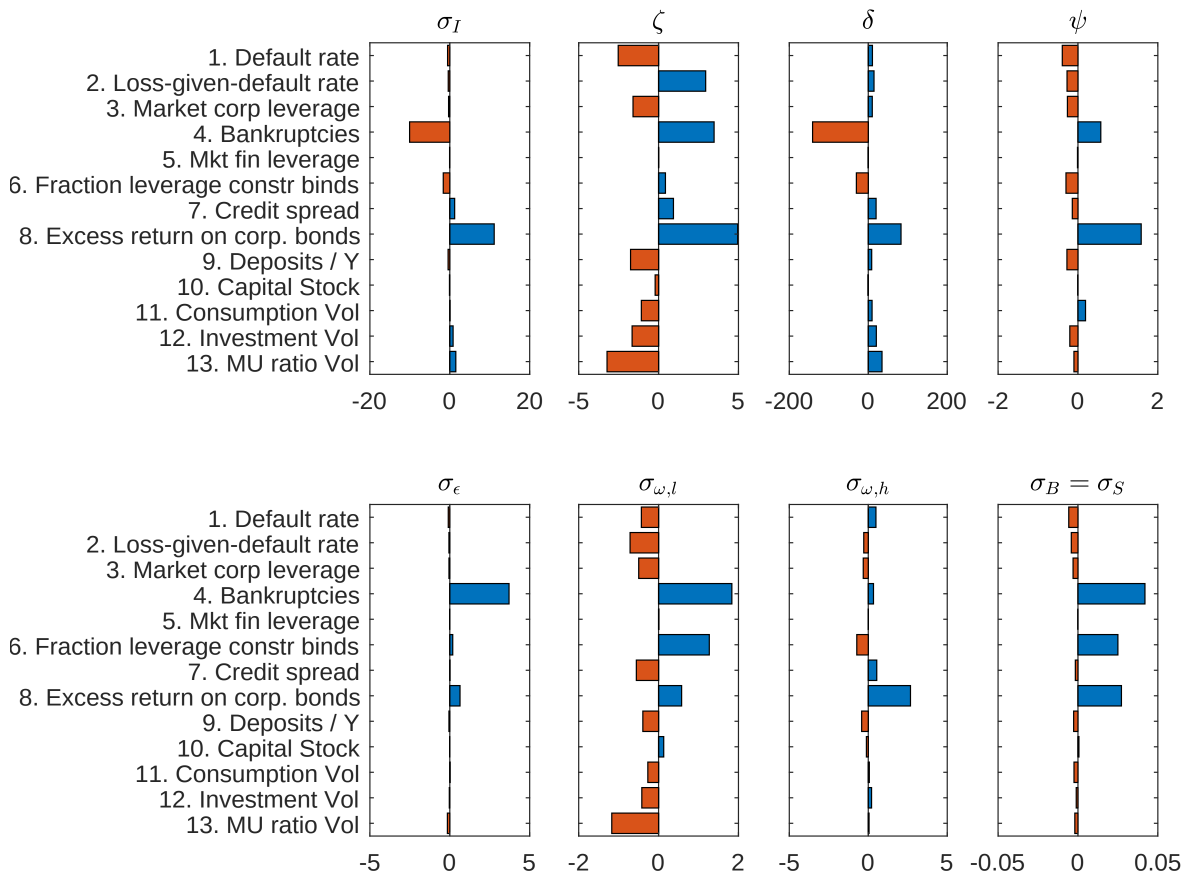
A higher equity adjustment cost in the first panel, strongly increases the excess return on corporate bonds, the moment chosen to pin down this parameter. It also strongly decreases bank bankruptcies. Increasing σ^I is akin to an increase in the risk aversion of banks, consistent with the discussion in Section D.6 below. Higher risk aversion naturally results in a larger equilibrium compensation for bearing credit risk and a tendency for banks to stay farther away from their borrowing constraint.

A higher value for the bankruptcy cost parameter ζ naturally results in higher losses given default, the moment chosen to pin down this parameter. While there is a modest decline in the default rate, the overall loss rate still goes up. There are more bank bankruptcies and a higher excess return on corporate bonds, given the increased quantity of credit risk. Corporate leverage declines in the wake of costlier credit. With less corporate debt and unchanged financial sector leverage, the banking sector shrinks (Deposits/Y). Lower corporate debt also results in a lower capital stock and a less volatile economy, which improves risk sharing (MU vol goes down).

An increase in the corporate debt maturity parameter δ most directly affects bond duration, the elasticity of corporate bond prices to interest rates (not reported). An increase in bond duration increases the excess return on bonds. With increased duration, firms become better duration-matched since the duration of their capital assets is high. As a result, firm leverage slightly increases despite the higher cost of debt.

The fourth panel explores changes in the capital adjustment cost parameter ψ. Higher capital adjustment costs naturally reduce investment volatility. They raise consumption volatility. The increase in capital adjustment costs increases the volatility in the price of capital (not reported), which causes risk-averse firms to de-lever. Lower leverage reduces the quantity of default risk as well as the credit spread unconditionally, but makes realized excess returns lower in crises, eroding bank capital and increasing expected excess returns enough to increase them unconditionally as well. With more risk

Figure 9: Parameter Sensitivity Analysis



in bad times, the banking sector shrinks (deposits/Y).

Panel five increases the volatility of idiosyncratic bank profit shocks σ_ϵ. That most directly affects bank bankruptcies, which is how the parameter is calibrated. The banks' leverage constraint binds more frequently. It increases the credit spread and excess bond return. A riskier banking sector shrinks.

Panel six (seven) studies an increase in the idiosyncratic productivity dispersion in normal (crisis) times. The elasticities tend to have an opposite pattern since the former change narrows the gap between the low and the high state thereby reducing the aggregate risk in the economy, while the latter change increases the gap. The reduction in aggregate risk is consistent with a reduction in macroeconomic volatility and an improvement in risk sharing (a reduction in MU vol). Financial firms respond to the safer macro-economic environment and the higher excess bond returns by increasing their risk taking, which results in higher financial sector leverage and bankruptcies.

Panel eight studies an increase in the risk aversion of both types of households in the economy, from the benchmark value of one. The intertemporal elasticity of substitution stays unchanged at one. The effect of this change is orders of magnitude smaller than the effect of other parameter changes. Corporate leverage and defaults go down. The financial leverage constraint becomes binding more frequently, as intermediating has become more profitable as witnessed by the increase in the excess bond return.

C.2 Long-term Corporate Bonds

Our model's corporate bonds are geometrically declining perpetuities, and as such have no principal. The issuer of one unit of the bond at time t promises to pay the holder 1 at time $t+1$, δ at time $t+2$, δ^2 at time $t+3$, and so on. Issuers must hold enough capital to collateralize the face value of the bond, given by $F = \frac{\theta}{1-\delta}$, a constant parameter that does not depend on any state variable of the economy. Real life bonds have a finite maturity and a principal payment. They also have a vintage (year of issuance), whereas our bonds combine all vintages in one variable. This appendix explains how to map the geometric bonds in our model into real-world bonds by choosing values for δ and θ.

Our model's corporate loan/bond refers to the entire pool of all outstanding corporate loans/bonds. To proxy for this pool, we use investment-grade and high-yield indices constructed by Bank of America Merill Lynch (BofAML) and Barclays Capital (BarCap). For the BofAML indices (Datastream Codes LHYIELD and LHCCORP for investment grade and high-yield corporate bonds, respectively) we obtain a time series of monthly market values, durations (the sensitivity of prices to interest rates), weighted-average maturity (WAM), and weighted average coupons (WAC) for January 1997 until December 2015. For the BarCap indices (C0A0 and H0A0 for investment grade and high-yield corporate bonds, respectively), we obtain a time series of option-adjusted spreads over the Treasury yield curve.

First, we use market values of the BofAML investment grade and high-yield portfolios to create an aggregate bond index and find its mean WAC c of 5.5% and WAM T of 10 years over our time period. We also add the time series of OAS to the constant maturity treasury rate corresponding to that period's WAM to get a time series of bond yields r_t. Next, we construct a plain vanilla corporate bond with a semiannual coupon and maturity equal to the WAC and WAM of the aggregate bond index, and compute the price for \$1 par of this bond for each yield:

$$P^c(r_t) = \sum_{i=1}^{2T} \frac{c/2}{(1+r_t)^{i/2}} + \frac{1}{(1+r_t)^T}$$

We can write the steady-state price of a geometric bond with parameter δ as

$$P^G(r_t) = \frac{1}{1+r_t}\left[1+\delta P^G(r_t)\right]$$

Solving for $P^G(y_t)$, we get

$$P^G(r_t) = \frac{1}{1+r_t-\delta}$$

The calibration determines how many units X of the geometric bond with parameter δ one needs to sell to hedge one unit of plain vanilla bond P^c against parallel shifts in interest rates, across the range of historical yields:

$$\min_{\delta,X} \sum_{t=1997.1}^{2015.12} \left[P^c(r_t) - XP^G(r_t;\delta)\right]^2$$

We estimate $\delta = 0.937$ and $X = 12.9$, yielding an average pricing error of only 0.41%. This value for δ implies a time series of durations $D_t = -\frac{1}{P_t^G}\frac{dP_t^G}{dr_t}$ with a mean of 6.84.

To establish a notion of principal for the geometric bond, we compare it to a duration-matched zero-coupon bond i.e. borrowing some amount today (the principal) and repaying it D_t years from now. The principal of this loan is just the price of the corresponding D_t maturity zero-coupon bond $\frac{1}{(1+r_t)^{D_t}}$

We set the "principal" F of one unit of the geometric bond to be some fraction θ of the undiscounted sum of all its cash flows $\frac{\theta}{1-\delta}$, where

$$\theta = \frac{1}{N} \sum_{t=1997.1}^{2015.12} \frac{1}{(1+r_t)^{D_t}}$$

We get $\theta = 0.582$ and $F = 9.18$.

C.3 LTV constraint

The cost of bankruptcy induces banks to limit leverage. In the computation of the model solution, we additionally impose a hard constraint on leverage. This is a standard leverage constraint:

$$FA_{t+1}^{B} \leq \Phi p_t (1 - (1 - \tau_{\Pi}^{B})\delta_K)\Omega_A(\omega_t^*)K_t^B. \tag{59}$$

The borrowing constraint in (59) caps the face value of debt at the end of the period, FA_{t+1}^{B}, to a fraction of the market value of the available capital units after default and depreciation, $p_t(1 - (1 - \tau_{\Pi}^{B})\delta_K)\Omega_A(\omega_t^*)K_t^B$, where Φ is the maximum leverage ratio. With such a constraint, declines in capital prices (in bad times) tighten borrowing constraints, as in Kiyotaki and Moore (1997). The constraint (59) imposes a hard upper bound on borrower leverage.

We set the maximum LTV ratio parameter $\Phi = 0.45$. This value is just large enough so that the LTV constraint never binds during expansions and non-financial recessions. In the simulation of the benchmark model, the borrower's LTV constraint binds in 3% of financial recessions. The LTV constraint limits corporate borrowing as a fraction of the market value of capital. We set Φ to match the volatility of corporate debt-to-GDP of the non-financial sector, which is 5.2% in the data and 4.3% in the model.

We have verified that relaxing this constraint to the extent that it is never binding does not significantly affect the results. For example, setting the maximum leverage ratio to $\Phi = .55$ yields almost identical results. We include the constraint for comparability with the existing literature that has emphasized the financial accelerator operating through capital prices. In our setup the main force limiting corporate leverage is a standard trade-off between the benefits and costs of debt finance.

C.4 Measuring Labor Income Tax Revenue

We define income tax revenue as current personal tax receipts (line 3) plus current taxes on production and imports (line 4) minus the net subsidies to government sponsored enterprises (line 30 minus line 19) minus the net government spending to the rest of the world (line 25 + line 26 + line 29 - line 6 - line 9 - line 18). Our logic for adding the last three items to personal tax receipts is as follows. Taxes on production and export mostly consist of federal excise and state and local sales taxes, which are mostly paid by consumers. Net government spending on GSEs consists mostly of housing subsidies received by households which can be treated equivalently as lowering the taxes that households pay. Finally, in the data, some of the domestic GDP is sent abroad in the form of net government expenditures to the rest of the world rather than being consumed domestically. Since the model has no foreigners, we reduce personal taxes for this amount, essentially rebating this lost consumption back to domestic agents.

C.5 Taxation of Savers' Financial Income

Savers earn financial income from two sources. First, they earn interest on their private lending i.e. deposits in the financial intermediaries. This income is ultimately a claim on the capital rents in the economy and should be taxed at the same rate τ_K as borrowers' and intermediaries' net income.

Second, they earn interest on their public lending i.e. government bonds. In the data, Treasury coupons are taxed at the household's marginal tax rate, τ in the model. However, the tax revenue collected by the government from interest income on its own bonds is substantially lower than τB_t^G because (a) Treasury coupons are exempt from state and local taxes, and (b) more than half of privately owned Treasury debt is held by foreigners, who also do not pay federal income taxes.

In the model, there is one tax rate τ_D at which all of the saver's interest income is taxed. We choose τ_D to satisfy

$$\tau_D(\hat{B}^I + \hat{B}^G) = \tau^K(\hat{B}^I - \hat{B}^I_{\text{pension}}) + \tau \frac{\hat{\tau}^{\text{federal}}}{\hat{\tau}^{\text{total}}}(\hat{B}^G - \hat{B}^G_{\text{foreign}} - \hat{B}^I_{\text{pension}})$$

where hats denote quantities in the data. Specifically, the revenue from taxes collected at rate τ_D on all private safe debt and government debt must equal the sum of tax revenues collected on taxable private safe debt (private safe debt not held in tax-advantaged pension funds) at rate τ_K, and tax revenues collected on taxable public debt (Treasury debt not held by foreigners, the Fed, or pension funds) taxed at rate $\tau \frac{\hat{\tau}^{\text{federal}}}{\hat{\tau}^{\text{total}}}$.

We measure all quantities at December 31, 2014. Private debt stocks are taken from the Financial Accounts of the United States. Treasury debt stocks are taken from the Treasury Bulletin. Federal and total personal tax revenues are taken from the BEA's National Income and Product Accounts. There is approximately \$13 trillion each outstanding of private and public debt. Almost all private debt is taxable, but only \$4 trillion of public debt is. Federal taxes constitute approximately 80% of all personal income tax revenue. Using the calibration for τ_K and τ, we get

$$\tau_D \approx \frac{20\% \times \$13T + 29.5\% \times 0.8 \times \$4T}{\$13T + \$13T}$$

or $\tau_D = 13.4\%$ precisely.

C.6 Stationarity of Government Debt

In our numerical work, we guarantee the stationarity of the ratio of government debt to GDP by gradually decreasing personal tax rates τ_t when debt-to-GDP falls below $\underline{b^G} = 0.1$ –the profligacy region– and by gradually increasing personal tax rates when debt-to-GDP exceed $\overline{b^G} = 1.2$ –the austerity region. Specifically, taxes are gradually and smoothly lowered with a convex function until they hit zero at debt to GDP of -0.1. Tax rates are gradually and convexly increased until they hit 60% at a debt-to-GDP ratio of 150%. Our simulations never reach the -10% and +150% debt/GDP states. The simulation spends 24% of the time in the profligacy and 15% of the time in the austerity region. The fraction of time spent in these regions has no effect on the overall resources of the economy.

Achieving stationarity of government debt requires primary surpluses, since the government must also service the debt. Generating primary surpluses requires slightly overshooting on personal and corporate tax revenue relative to the data, since the U.S. government has historically had an average primary surplus of (just about) zero. Put differently, the actual U.S. fiscal path is unsustainable, i.e., incompatible with a stationary model.

C.7 Measuring Intermediary Sector Leverage

Our notion of the intermediary sector is the levered financial sector. We take book values of assets and liabilities of these sectors from the Financial Accounts of the United States (formerly Flow of Funds). We subtract holding and funding company equity investments in subsidiaries from those subsidiaries' liabilities. Table 7 reports the assets, liabilities, and leverage of each sector as of 2014, as well as the average leverage from 1953 to 2014. We find that the average leverage ratio of the levered financial sector was 91.5%. This is our calibration target.

Krishnamurthy and Vissing-Jorgensen (2015) identify a similar group of financial institutions as net suppliers of safe, liquid assets. Their financial sector includes money market mutual funds (who do not perform maturity transformation) and equity REITS (who operate physical assets) but excludes life insurance companies (which are highly levered). The financial sector definition of Krishnamurthy and Vissing-Jorgensen (2015) suggests a similar ratio of 90.9%. As an aside, we note that Krishnamurthy and Vissing-Jorgensen (2015) report lower total assets and liabilities than in our reconstruction of their procedure because they net out positions within the financial sector by instrument while we do not.

Table 7: Balance Sheet Variables and Prices

			Dec 2014		Avg 53-14
Table	**Sector**	**Assets**	**Liabilities**	**Leverage**	**Leverage**
L.111	U.S.-Chartered Depository Institutions	$ 13,647	$ 12,161	0.891	0.921
L.112	Foreign Banking Offices in U.S.	$ 2,093	$ 2,086	0.996	1.065
L.113	Banks in U.S.-Affiliated Areas	$ 92	$ 88	0.953	1.080
L.114	Credit Unions	$ 1,066	$ 958	0.899	0.916
	Subtotal: Banks	$ 16,898	$ 15,292	0.905	0.928
L.125	Government-Sponsored Enterprises (GSEs)	$ 6,400	$ 6,387	0.998	0.971
L.126	Agency- and GSE-Backed Mortgage Pools	$ 1,649	$ 1,649	1.000	1.000
L.127	Issuers of Asset-Backed Securities (ABS)	$ 1,424	$ 1,424	1.000	1.003
L.129.m	Mortgage Real Estate Investment Trusts	$ 568	$ 483	0.851	0.955
L.128	Finance Companies	$ 1,501	$ 1,376	0.916	0.873
L.130	Security Brokers and Dealers	$ 3,255	$ 1,345	0.413	0.808
L.131	Holding Companies	$ 4,391	$ 2,103	0.479	0.441
L.132	Funding Corporations	$ 1,305	$ 1,305	1.000	1.000
	Subtotal: Other Liquidity Providers	$ 20,492	$ 16,070	0.784	0.872
L.116	Life Insurance Companies	$ 6,520	$ 5,817	0.892	0.932
	Total	$ 43,910	$ 37,179	0.847	**0.915**
L.121	Money-Market Mutual Funds	$ 2,725	$ 2,725	1.000	1.000
L.129.e	Equity Real Estate Investment Trusts	$ 157	$ 539	3.427	2.577
	Total (K-VJ Definition)	$ 40,271	$ 33,549	0.833	0.909

D Results Appendix

D.1 Pure Uncertainty Shock

Figure 10 compares the dynamics of important macro-economic aggregates and balance sheet variables in a financial recession (red lines) to the effect of a pure second-moment shock. The IRF plots are generated as explained in the main text. The red line in the plots of figure 10 is identical to the red lines in figures 2 and 3 in the main text, as both are caused by the same combination of a low TFP realization and an increase in σ_ω in period 1. The blue lines in figure 10 show dynamics after the economy is hit only by the increase in σ_ω, with stable TFP. The plots show that this pure uncertainty shock has much smaller negative effects on output, consumption and investment than the combination that causes a financial recession. This feature of our model is consistent with the empirical finding that uncertainty shocks alone have at most moderate negative effects on output and investment, see for example Bachmann and Bayer (2013) or Vavra (2014).

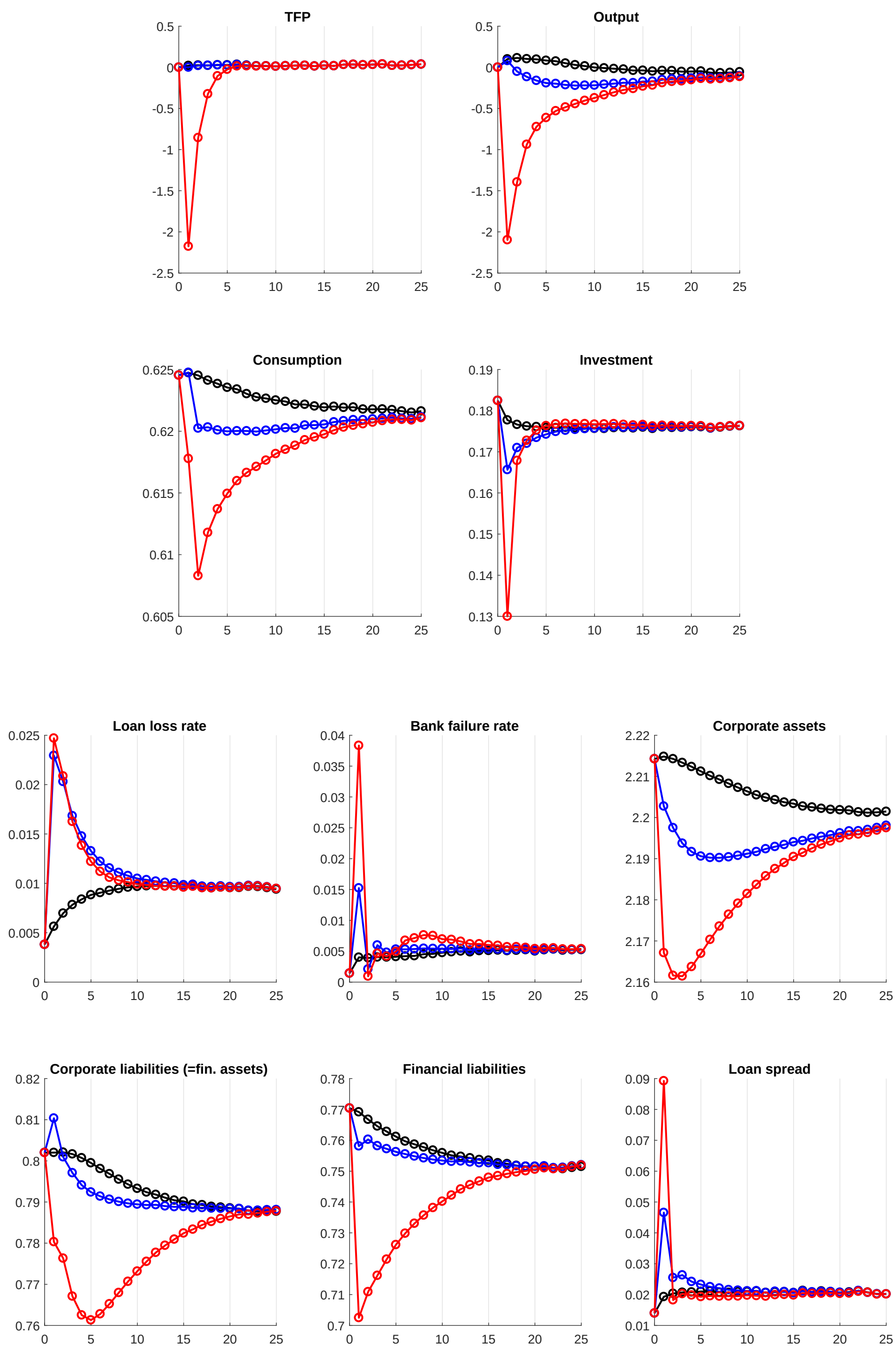
A closer look at the balance sheet variables in the bottom panel reveals that the fundamental difference between both types of shocks lies in the response of intermediaries. The losses suffered on loans during a financial crisis are only marginally larger than those from the uncertainty shock. However, the financial sector does not shrink after the uncertainty shock. Rather, firms raise more debt (bottom left panel) despite a temporarily smaller capital stock (top right panel), effectively increasing leverage. Banks reduce deposit funding only marginally (bottom middle). The spikes in bank failure rate and credit spread are less than half of those experienced in a financial recession. We can conclude that only the combination of TFP and uncertainty shock activates the intermediary-based financial accelerator.

Why are financial recessions so much worse despite similar losses from borrower defaults for banks? Figure 11 shows that the dynamics of the corporate bond price (top right) are the key amplifying force. This price drops sharply in financial recessions, causing large market value losses for intermediaries. This large drop in price is driven by two main forces. First, the negative TFP shock reduces bank demand for corporate bonds, as seen in Figure 3 in the main text. Second, the losses on corporate bonds caused by the uncertainty shock reduce bank capital, and thus amplify the first effect of reduced demand on prices. The stronger financial accelerator means that intermediary net worth falls only half as much in an uncertainty shock episode compared to a financial recession (bottom right). As a result, intermediaries are not forced to shrink as they are in a financial recession. Continuity in lending to borrower-entrepreneurs prevents a sharp reduction in investment and the capital price (bottom left) despite intermediary losses on loans. In the third period of a financial recession, intermediary wealth overshoots as banks earn large spreads due to the sharp drop in the risk-free rate. Intermediaries deplete this extra wealth to gradually expand lending again as the production sector recovers to normal levels of capital. These dynamics are not present in an uncertainty shock episode, since lending never contracted to begin with.

D.2 Drivers of Financial Leverage

This appendix explores what model ingredients contribute quantitatively to the high financial leverage that the benchmark model is able to generate. Specifically, we turn off the three financial frictions, one at the time: (1) the bankruptcy option for banks, (2) equity adjustment costs ($\sigma^I = 0$), and (3) the tax shield for financial firms. Table 8 contains the results. The main finding is that financial leverage is affected very little by these financial frictions. In other words, the main driver of the high financial leverage is the wedge between the subjective time discount rate of borrowers and savers. This wedge

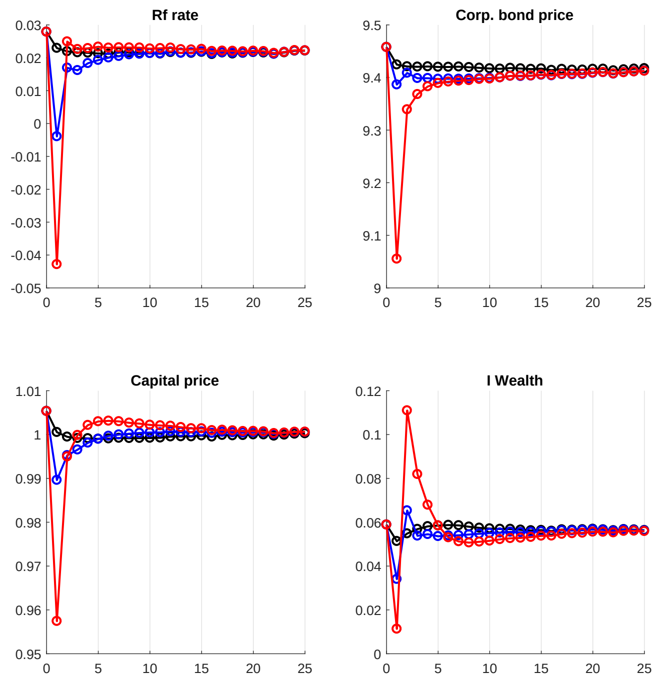
Figure 10: Financial Recession vs. Uncertainty Shock: Macro Quantities and Balance Sheets



Blue line: uncertainty shock, **red line:** financial recession, **black line:** no shock

creates a strong incentive to channel savings from depositors to non-financial firms, i.e., for financial intermediation.

Figure 11: Financial Recession vs. Uncertainty Shock: Prices



Blue line: uncertainty shock, **red line:** financial recession, **black line:** no shock

Furthermore, we see that when banks are not allowed to default, they stay away from their leverage constraint more often. Without equity adjustment costs, it becomes much cheaper to recapitalize banks for their shareholders. This acts like a reduction in risk aversion for bank shareholders and their leverage constraint becomes binding all the time. The slightly higher financial leverage results in significantly more bank bankruptcies. The effective reduction in risk aversion also lowers the required compensation for risk banks receive, as shown in the lower credit spread and excess return on corporate bonds, despite a slightly higher loss rate on corporate loans. The cheaper cost of debt in turn incentivizes non-financial firms to increase leverage. In sum, a reduction in the cost of equity finance for banks has a stronger effect on non-financial leverage than on financial leverage.

The model without tax shield features higher credit spread and excess return and lower loss rates. The banks manage to pass through the loss of their tax shield to their customers, the non-financial firms. Their compensation per unit of risk increases, providing incentives to increase financial leverage (modestly). The increased cost of credit coincides with lower corporate leverage.

D.3 Credit Spread and Risk Premium

One important quantitative success of the model is its ability to generate a high unconditional credit spread while matching the observed amount of default risk. The credit spread is also highly volatile (2.94% standard deviation) and more than twice as high in financial recessions than in expansions. The rise in the credit spread in financial recessions to 4.28% reflects not only the increase in the quantity default risk but also an increase in the price of credit risk. The model generates a high and counter-cyclical price of credit risk, which itself comes from the high and counter-cyclical "shadow

Table 8: Drivers of Financial Sector Leverage

	Bench	No bankruptcy	$\sigma^I = 0$	No tax shield
Mkt fin leverage (in %)	93.3	93.3	94.0	93.9
Book fin leverage (in %)	97.1	97.3	99.7	96.6
% fin leverage constr binds	61.3	50.7	100.0	78.8
Bankruptcies (in %)	0.54	0.00	1.71	1.21
Credit spread (in %)	2.05	2.05	1.88	2.16
Excess ret. corp. bonds (in %)	1.09	1.01	0.74	1.43
Loss Rate (in %)	0.96	1.05	1.17	0.79
Market corp leverage (in %)	35.8	36.9	38.8	33.6
Book corp leverage (in %)	35.2	36.2	37.2	33.3

SDF" for the intermediary sector.

The intermediary SDF is given by:

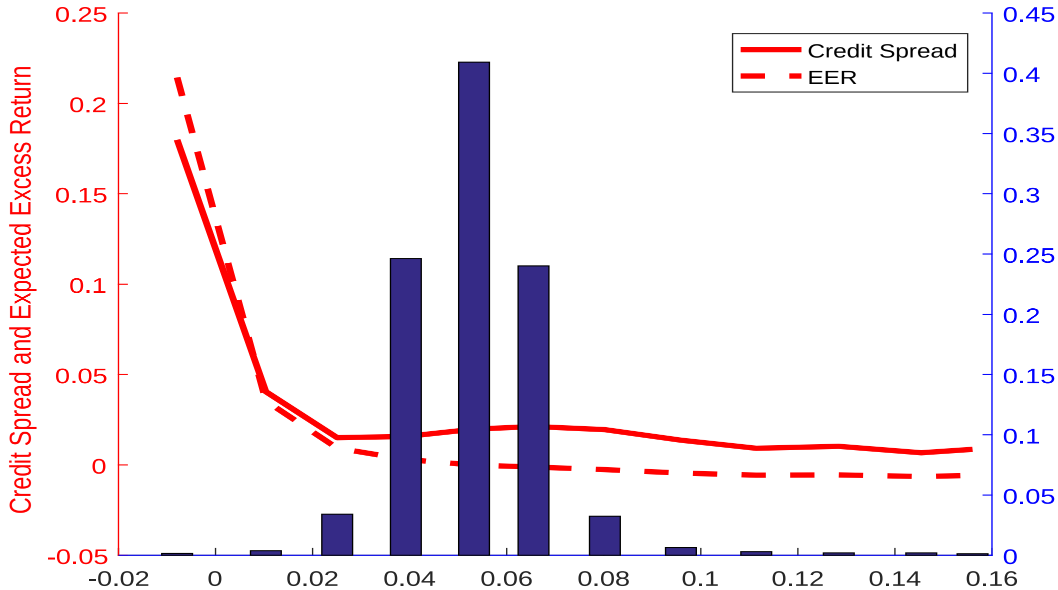
$$\mathcal{M}^I_{t,t+1} = \mathcal{M}^B_{t,t+1} \left(\frac{1 + \sigma^I (d^I_{t+1} - \bar{d})}{1 + \sigma^I (d^I_t - \bar{d})} \right)^{-1} F_{\epsilon,t+1},$$

where $\mathcal{M}^B_{t,t+1}$ is the borrower SDF, $F_{\epsilon,t+1}$ is the probability of intermediary failure in $t+1$, and $\frac{1}{1+\sigma^I(d^I_t - \bar{d})}$ is the marginal value of wealth to intermediaries in t.

Figure 12 shows the histogram of the intermediary wealth share plotted against two different measures of credit risk compensation earned by intermediaries. The solid red line plots the credit spread, the difference between the yield r^m_t on corporate bonds and the risk-free rate. We compute the bond yield as $r^m_t = \log\left(\frac{1}{q^m_t} + \delta\right)$. This is a simple way of transforming the price of the long-term bond into a yield; however, note that this definition assumes a default-free payment stream $(1, \delta, \delta^2, \ldots)$ occurring in the future. Consistent with the result in He and Krishnamurty (2013), the credit spread is high when the financial intermediary's wealth share is low. Since our model has defaultable debt, the increase in the credit spread reflects both risk-neutral compensation for expected defaults and a credit risk premium.

To shed further light on the source of the high credit spread, we compute the expected excess return (EER) on corporate loans earned by the intermediary. The EER consists both of the credit risk premium, defined as the (negative) covariance of the intermediary's stochastic discount factor with the corporate bond's excess return, and an additional component that reflects the tightness of the intermediary's leverage constraint. This component arises because the marginal agent in the market for risk-free debt is the saver household, while corporate bonds are priced by the constrained intermediary. The market risk free rate is lower than the "shadow" risk free rate implied by the intermediary SDF. Given log preferences, most of the action in the EER comes from the constraint tightness component. When intermediary wealth is relatively high, the leverage constraint is not binding and the EER is approximately zero. Low levels of intermediary wealth result from credit losses, and the lowest levels occur during financial crises. At these times, credit risk increases and the intermediary becomes constrained. In the worst crisis episodes when intermediary wealth reaches zero or drops below zero, the EER reaches 20 percent.

Figure 12: The Credit Spread and the Financial Intermediary Wealth Share



Solid line: credit spread; **dashed line**: expected excess return

D.4 Counter-cyclical Capital Requirements

D.5 Policy transitions

The tables above only compare the ergodic distributions of economies with different policy parameters. How does an unanticipated policy change to a tighter or looser capital requirement affect output, consumption, and the welfare of borrowers and savers in the short term? Figure 14 plots the evolution of these variables after a policy change from the benchmark to either a higher ($\xi = .90$) or a lower ($\xi = .97$) capital requirement. In the long run, output, consumption, and agent welfare converge to their ergodic means in tables 4 and 5. In the short run, consumption "overshoots" in both cases. Tightening the capital requirement by 4 p.p. leads a contraction in GDP as investment drops. But lower investment also causes a consumption boom in the short run as the economy transitions to a permanently lower capital stock.

D.6 Effect of Equity Adjustment Cost

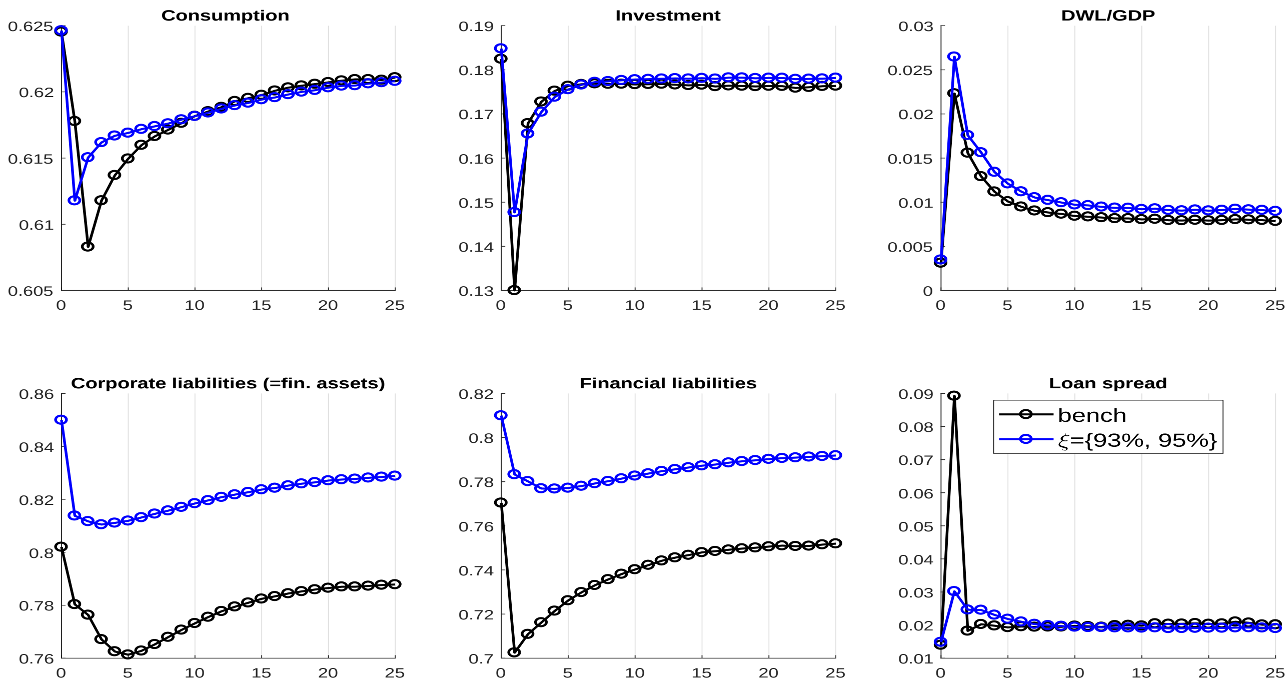
Table 9 shows the effect of larger or smaller equity adjustment costs (σ^I) relative to the benchmark economy. The overall take-away from this comparison is that larger equity frictions in the intermediation have a similar effect to tightening the intermediaries' capital requirement. Higher marginal equity adjustments costs (columns $\sigma^I = 6$, $\sigma^I = 7$) lead to a smaller non-financial sector, both in terms of assets and liabilities. Corporate leverage declines, and as a result, fewer firms default, causing an overall decline in loss rates on corporate loans.

Even though intermediaries face less credit risk, they reduce their own leverage and their constraint becomes binding much less frequently as σ^I is increased. Consequently, intermediary failures are almost completely eliminated at $\sigma^I = 7$. An important difference to the macro-prudential policy exercise with tighter capital requirements is the effect on bank profitability. In both cases (tighter

Table 9: Effect of Varying σ^I

	Bench ($\sigma^I = 5$)	$\sigma^I = 3$	$\sigma^I = 4$	$\sigma^I = 6$	$\sigma^I = 7$
			Borrowers		
1. Mkt value of capital / Y (in %)	225.0	226.6	226.4	221.4	219.0
2. Mkt value of corp debt / Y (in %)	80.6	84.4	83.9	72.2	64.5
3. Book val of corp debt / Y (in %)	79.1	82.1	81.7	72.2	65.1
4. Market corp leverage (in %)	35.8	37.3	37.1	32.6	29.4
5. Book corp leverage (in %)	35.2	36.2	36.1	32.6	29.7
6. % leverage constr binds	0.32	0.77	0.62	0.01	0.00
7. Default rate (in %)	2.25	2.40	2.37	1.96	1.70
8. Loss-given-default rate (in %)	43.09	45.10	44.86	38.66	32.86
9. Loss Rate (in %)	0.96	1.07	1.06	0.75	0.53
			Intermediaries		
10. Mkt fin leverage (in %)	93.3	93.9	93.8	92.7	91.6
11. Book fin leverage (in %)	97.1	98.7	98.5	94.9	92.7
12. % leverage constr binds	61.30	93.61	82.39	30.66	20.70
13. Bankruptcies (in %)	0.54	1.45	1.26	0.07	0.03
14. Wealth I / Y (in %)	5.6	5.2	5.3	5.7	5.8
15. Franchise value (in %)	33.9	20.0	21.1	75.8	90.2
			Savers		
16. Deposits / Y (in %)	76.9	81.1	80.5	68.5	60.5
17. Government debt / Y	60.2	115.7	110.6	19.0	15.4
			Prices		
18. Risk-free rate (in %)	2.19	2.24	2.24	2.23	2.23
19. Corporate bond rate 9in %)	4.24	4.15	4.16	4.42	4.52
20. Credit spread (in %)	2.05	1.91	1.92	2.19	2.30
21. Excess ret. corp. bonds (in %)	1.09	0.87	0.90	1.45	1.72
			Welfare		
22. Aggr. welfare $\mathcal{W}^{pop}$	0.620	-0.38%	-0.39%	+0.28%	+0.52%
23. Aggr. welfare $\mathcal{W}^{cev}$	0%	+16.20%	+13.62%	-24.24%	-31.92%
24. Value function, B	0.285	-2.50%	-2.28%	+2.84%	+4.29%
25. Value function, S	0.336	+1.43%	+1.22%	-1.89%	-2.69%
26. DWL/GDP	0.008	+18.89%	+15.31%	-18.68%	-30.84%
			Size of the Economy		
27. GDP	0.978	+0.29%	+0.25%	-0.65%	-1.09%
28. Capital stock	2.199	+1.00%	+0.87%	-2.24%	-3.72%
29. Aggr. Consumption	0.621	-0.07%	-0.05%	+0.02%	-0.00%
30. Consumption, B	0.291	-2.71%	-2.45%	+2.70%	+4.42%
31. Consumption, S	0.343	+2.17%	+1.98%	-2.25%	-3.75%
			Volatility		
32. Mkt value corp debt gr	0.029	-1.62%	-6.25%	+10.61%	+78.99%
33. Deposits gr	0.049	-56.63%	-56.80%	-1.82%	+86.46%
34. Dividend gr	2.370	+7.71%	+3.93%	-30.67%	-38.58%
35. Investment gr	29.56%	-63.82%	-63.66%	-31.95%	+40.37%
36. Consumption gr	2.17%	-12.69%	-14.68%	-0.94%	+27.08%
37. Consumption gr, B	3.12%	-5.24%	-6.17%	-6.34%	+8.37%
38. Consumption gr, S	4.08%	-40.84%	-40.96%	-5.96%	+45.08%
39. log (MU B / MU S)	0.052	-25.85%	-27.18%	-9.44%	+29.32%

Figure 13: Financial Recessions with Counter-cyclical Capital Requirements



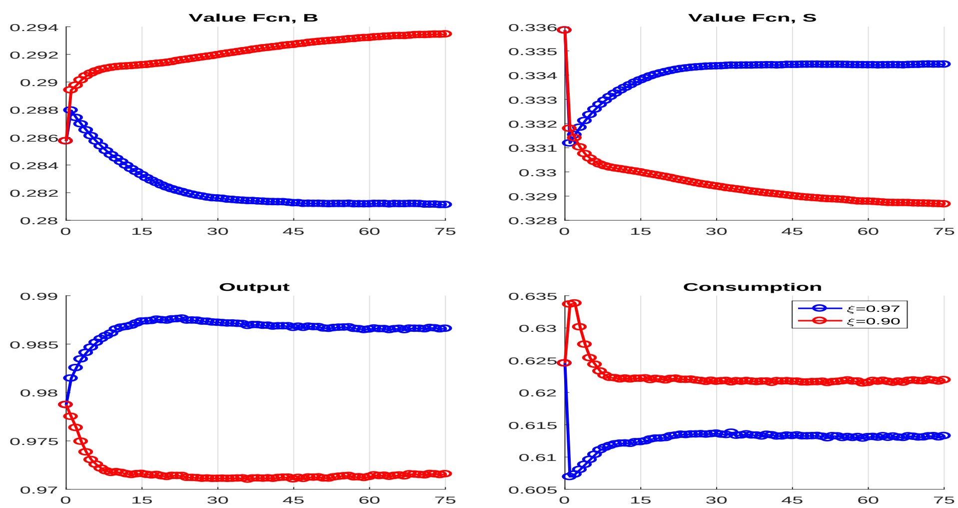
Blue line: responses to financial recession in economy with counter-cyclical capital requirements; **Black line:** responses to financial recession in benchmark economy. The underlying shocks in the two cases are identical.

capital constraint and higher equity adjustment cost), intermediaries effectively become more risk averse and require larger compensation for bearing risk, as evidenced by the large increase in the excess return on loans (row 21). However, increasing σ^I increases the franchise value of intermediaries (row 15), since it raises the risk premium while at the same time not requiring banks to raise more equity. Hence greater σ^I raises the return on bank equity, while lower ξ does not.

The overall welfare effects of larger equity adjustment frictions are comparable to the effects of tighter ξ. Locally the reduction in bankruptcies of producers and intermediaries dominates the reduction in the size of the capital stock, leading to a small aggregate welfare gain based on the population-weighted measure (row 22). Like tighter capital regulation, greater σ^I benefits equity owners of producers at the expense of savers.

The effects on macroeconomic volatility are nonlinear based on the same opposing forces that are at play with tighter capital constraints: since higher intermediation frictions increase the cost of debt funding, producers reduce the debt share of financing, which makes financial recessions less severe. At the same time, greater intermediation frictions hamper banks' ability to absorb aggregate risk through their balance sheet. The net effect, at least locally around the benchmark level of σ^I, is that aggregate investment and consumption growth become less volatile with lower equity adjustment costs ($\sigma^I = 3$, $\sigma^I = 4$), and risk sharing improves (MU ratio in row 40 becomes less volatile). Interestingly, this is also the case for slightly higher adjustment costs ($\sigma^I = 6$). However, as we increase σ^I to 7, the impairment of banks' risk-bearing capacity dominates the reduction in risk: both aggregate consumption and investment growth are more volatile, and risk sharing between borrowers and savers becomes worse (row 39).

Figure 14: Transition Dynamics After Change in Capital Requirement



Blue line: lower capital requirement; **red line**: higher capital requirement

D.7 Sensitivity of Macro-prudential Policy

In this appendix we study how sensitive the macro-prudential policy conclusions are to specific model ingredients/parameter constellations. In each experiment, we compare the effects of relaxing bank capital requirements by two percentage points versus tightening them by two percentage points, around the benchmark model. In other words, we study a four percentage point relaxation from $\xi = .92$ to $\xi = .96$. The first column of Table 10 reports the results from this particular relaxation for the benchmark model. Firm loss rates and bank bankruptcies both increase, the size of the banking sector and the economy as a whole increase, investment volatility falls modestly while consumption growth volatility rises modestly, and aggregate welfare falls since the gains to the savers are insufficient to offset the losses to the borrowers. All these results are in line with our discussion in the main text.

Table 10: Sensitivity of Macro-prudential Policy Experiment

	Benchmark	No bankruptcy	$\sigma^I = 0$	No tax shield	Higher β_B	Lower β_S
			Financial Fragility			
Loss rate	+0.32%	+0.42%	+0.07%	+0.03%	+0.15%	+0.20%
Bankruptcies	+3.72%	0.00%	+5.24%	+3.01%	+2.92%	+3.21%
			Size of the Economy			
GDP	+0.83%	+0.72%	+0.54%	+0.21%	+0.26%	+0.30%
Deposits / GDP	+21.61%	+24.24%	+9.58%	+5.72%	+14.04%	+14.17%
			Macro Volatility			
Investment vol	-0.05%	-0.33%	+0.50%	+0.20%	+0.16%	-0.04%
Consumption vol	+0.05%	-0.15%	+0.37%	+0.13%	+0.17%	+0.02%
MU vol	-1.65%	-1.64%	+0.54%	-0.07%	-0.19%	-1.55%
			Welfare			
Borrower	-5.61%	-3.16%	-2.36%	-3.69%	-0.94%	-4.74%
Saver	+3.08%	+2.14%	+0.91%	+1.95%	+0.22%	+2.53%
Aggregate	-1.01%	-0.35%	-0.59%	-0.69%	-0.30%	-0.87%

The other columns of Table 10 study the same change in macro-prudential policy in a model without bankruptcy option (column 2), in a model without equity issuance costs (column 3), in a model without tax shield for banks (column 4), in a model with more patient borrowers (column 5, β_B increases by 0.15), and less patient savers (columns 6, β_S decreases by 0.15). The latter two changes decrease the wedge between the patience of borrowers and savers and reduce the need for intermediation services.

The main finding is that the aggregate welfare changes from macro-prudential policy are robust to these parameter variations. In all experiments, welfare decreases in response to the four percentage point increase in maximum allowable financial sector leverage from 92% to 96%. The range of estimates is -0.35%, when banks are not allowed to fail (and hence cannot be bailed out), to -1.01%. In all cases, we see more fragility in the form of higher corporate loss rates and higher bank bankruptcies (except of course when banks are not allowed to go bankrupt). When it is easier and cheaper for shareholders to recapitalize banks, the size of the banking sector is naturally less sensitive to a change in macro-prudential regulation. The basic trade-off between a larger banking sector and size of the economy and more financial fragility is also present in every model. The quantitative slope of that trade-off depends on the model details. The only results that are more fragile are those on macro-economic volatility. That should not come as a surprise since, even in the benchmark model, macro-economic volatility is non-monotonic in ξ. Risk sharing tends to improve (MU vol falls) as macro-prudential policy is relaxed, reflecting the financial sector's improved ability to absorb aggregate risk when it is larger. The one exception is when bank equity can be costlessly adjusted, which is also the model when the banking sector size changes the least and macro-economic volatility increases the most.